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Generalized Dynamical Equations of Motion for Particles of Non- Zero Masses for Static Homogeneous Spherical Gravitational Fields

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Abstract

In this paper the generalized dynamical gravitational scalar potential exterior to the body is applied to the well-known Newton's dynamical gravitational equations of motion to obtain generalizations of Newton's dynamical equations of motion. The generalized dynamical gravitational equations of motion are applied to the motion of the planets in the equatorial plane to obtain a generalized dynamical planetary equation of motion. The results are that the generalized Newtonian dynamical equations of motion and the dynamical planetary equation of motion are augmented by correction terms of all orders of c^{-2} which are not found in Newton's dynamical equations of motion or Einstein's geometrical equations of motion.

Keywords: Static Homogeneous Spherical Bodies, Generalized Gravitational Scalar Potential, Generalized Dynamical equations of Motion, Equatorial plane Planetary Equation of Motion

1.0 Introduction

In the year 1686 Newton published his dynamical theory of gravitation. According to Newton's dynamical gravitational theory of gravitation all interactions in nature manifest through force. The first great significance of Newton's laws of motion and gravitation is their success in explaining the experimental facts of the solar system. One serious limitation of Newton's dynamical laws of motion and gravitation is that they are formulated in terms of the invariant rest masses of particles and bodies and consequently they cannot be applied to a photon which has no measurable rest mass [1, 2, 3, 4]. At the end of the 19th century there were several attempts to generalize or extend Newton's dynamical gravitational theory of gravitation in order to provide better agreement to all physical theories. It is well known that Newton's dynamical equations of motion [5, 6, 7] is given by

$$\bar{a} = -\nabla f \tag{1}$$

where, $\bar{a}$ = Pure Newtonian acceleration vector, ∇ = Gradient operator and f = Newton's dynamical gravitational scalar potential exterior to the body. In this paper, we show how to formulate a generalized dynamical Newtonian acceleration vector to derive generalizations of Newton's dynamical gravitational equations of motion and Newton's dynamical planetary equation of motion based on our generalized dynamical gravitational scalar potential exterior to the body.

2.0 Theoretical Analysis

Newton's Dynamical Equation of Motion with Generalized Dynamical Gravitational Scalar Potential

The generalized dynamical gravitational scalar potential exterior to the body (f^+) acting on a particle in spherical polar coordinates (r, θ, ϕ) is given as [8]

$$f^+(r) = -\frac{k}{r}\left(1 - \frac{k}{c^2 R}\right) - \frac{k^2}{c^2 r^2} \tag{2}$$

where,

$$k = GM \tag{3}$$

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Newton's dynamical equation of motion with generalized dynamical gravitational scalar potential is given [3, 4] by

$$\underline{a} = -\underline{\nabla} f^{+} \tag{4}$$

where,

f^{+} = Gravitationalscalarpotentialexteriortothebody
$\underline{a}$ = pure Newtonian AccelerationVector
$\underline{\nabla}$= gradientoperator

For a static homogeneous spherical body, the gravitational field will depend on only the radial distance r, therefore, the gradient operator

$$\underline{\nabla} f^{+} = -\hat{r}\frac{\partial f}{\partial r} \tag{5}$$

where, $\hat{r}$ = unit vector along the radial direction
Substituting (2) into (5) and differentiating with respect to r we obtain

$$\underline{a} = -\frac{k}{r^2} + \frac{k^2}{c^2 R r^2} - \frac{2k^2}{c^2 r^3} \tag{6}$$

Equation (6) is the generalized Newtonian acceleration vector with generalized dynamical gravitational scalar potential. The leading term $-\frac{k}{r^2}$ on the right hand side of equation (6) is the well known Newtonian term while $\frac{k^2}{c^2Rr^2} - \frac{2k^2}{c^2r^3}$ is the contribution introduced by this article. This result therefore contains Post – Newtonian and Post - Einstein correction terms of all order of c^{-2}which are for theoretical development and applications.
Consequently for a planet or comet of rest mass M (regarded as a particle) in the gravitational field of the Sun, Newton's equation of motion in spherical polar coordinates (r, θ, ϕ) is given by

$$\ddot{r} + r\dot{\theta}^2 + r\dot{\phi}^2 \sin^2\theta = -\frac{k}{r^2} + \frac{k^2}{c^2 R r^2} - \frac{2k^2}{c^2 r^3} \tag{7}$$

$$\ddot{\theta} + 2t\dot{r}\dot{\theta} - r\dot{\phi}^2 \sin\theta\cos\theta = 0 \tag{8}$$

$$r\ddot{\phi}\sin\theta + 2\dot{r}\dot{\phi}\sin\theta + 2r\dot{\theta}\dot{\phi}\cos\theta = 0 \tag{9}$$

where, a dot denotes one - time differentiation.
Equations (7– 9) are the generalized equations of motion of a particle according to the natural generalization of Newton's theory of gravitation. It is most interesting and instructive to note that these equations contain $\frac{k^2}{c^2Rr^2} - \frac{2k^2}{c^2r^3}$which is not found in Newton's dynamical gravitational Field equation and Einstein's geometrical gravitational field equation. This result therefore contains Post – Newton and Post – Einstein correction terms to all order ofc^{-2} which are henceforth opened for theoretical development, experimental verification and applications.

3.0 Motion in the Equatorial Plane (Anomalous Orbital Precession in the Solar System)

Consider the motion of a particle whose motion is confined to the equatorial plane of the Sun, such as a planet or comet or asteroid, in the solar system.
Then,

$$\theta = \frac{\pi}{2}$$

Hence, Newton's equations of motion (7 – 9) reduces to

$$\ddot{r} - r\dot{\phi}^2 = -\frac{k}{r^2} + \frac{k^2}{c^2 R r^2} - \frac{2k^2}{c^2 r^3} \tag{10}$$

And the corresponding ϕ – component (8) reduces to

$$r\ddot{\phi} + 2\dot{r}\dot{\phi} = 0 \tag{11}$$

The exact solution of the angular equation (11) is given by

$$\dot{\phi} = \frac{l}{r^2} \tag{12}$$

where, l is the constant of motion corresponding to the angular momentum per unit rest mass.
The first integral of the radial equation of motion (10) subject to (12) yields

$$\dot{r}^2(r) = 2k\left(\frac{1}{r} - \frac{1}{r_i}\right) - l^2\left(\frac{1}{r^2} - \frac{1}{r_i^2}\right) - \frac{2k^2}{c^2R}\left(\frac{1}{r} - \frac{1}{r_i}\right) - \frac{2k^2}{c^2}\left(\frac{1}{r} - \frac{1}{r_i}\right) \qquad (13)$$

where, r_i is any apsidal distance. This is the exact Newtonian radial speed of a planet. It follows from (13) subject to (12) that

$$\ddot{r} = -\frac{k}{r^2} + \frac{k^2}{c^2r^2R} + \frac{l^2}{r^3} - \frac{2k^2}{c^2r^3} \qquad (14)$$

This is the exact generalized Newtonian radial acceleration of the planet with generalized gravitational scalar potential in terms of radial coordinate.
Using the transformation

$$r(\phi) = \frac{1}{v(\phi)} \qquad (15)$$

It follows that,

$$\ddot{r} = -l^2v^2\frac{d^2v}{d\phi^2} \qquad (16)$$

Substituting (16) into (14) and dividing both sides by v^2l^2 gives

$$\frac{d^2v}{d\phi^2} = \frac{k}{l^2}\left(1 - \frac{k}{c^2R}\right) - \left(1 + \frac{2k^2}{c^2l^2}\right)v \qquad (17)$$

Equation (17) is the generalized Newtonian dynamical planetary equation of motion in the equatorial plane. This equation contains $\left(1 - \frac{k}{c^2R}\right) - \left(1 + \frac{2k^2}{c^2l^2}\right)$ which is not found in Newton's dynamical planetary equation of motion and Einstein's planetary equation of motion. The consequences / implications are that it predicts correction terms to the planetary parameters.

4.0 Remarks and Conclusion

We have in this paper shown how to formulate a generalized Newtonian acceleration vector in the spherical polar coordinates to derive the generalized Newton's dynamical equations of motion and the planetary equation of motion. The generalized Newtonian acceleration vector, generalized equations of motion in (r, θ, ϕ) components and the generalized Newton's dynamical planetary equation of motion are found to be equations (6), (7), (8), (9) and (17) respectively.The post - Newtonian correction terms $\left(1 - \frac{k}{c^2R}\right) - \left(1 + \frac{2k^2}{c^2l^2}\right)$ in the generalized Newton's dynamical planetary equation of motion (17) can be used to explain the planetary parameters as well as the anomalous orbital precession of the orbit of the planets. The pace is therefore set for the application of the generalized dynamical planetary equation of motion to derive the planetary parameters.

5.0 References

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