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Empirical Models for the Correlation of Global Solar Radiation from Air Temperature Data in Maiduguri, Nigeria

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Abstract

The work focussed on the evaluation of the global solar radiation using ratio of minimum to maximum temperature, alongside the measured global solar radiation, and sunshine duration data (2001-2012) for Maiduguri. Five empirical model methods, the linear, quadratic, cubic, exponential and logarithm regression models were tested by comparing their estimated global solar radiation with measured global solar radiation data obtained Maiduguri meteorological agency (NIMET) station with the aim of determining which model estimate correlates more with measured values. The analysis showed a good agreement between the measured data and the computed results. The cubic model performance was good with 93.74% of clearness index, followed by quadratic model which has 90.96% and linear model which showed 87.30% of clearness index, the least was the exponential model which has 84.86%..

1.0 Introduction

Global solar radiation is of economic importance as a renewable energy alternative, recently global solar radiation has been studied due to its importance in providing energy for earth's climate system. The solar radiation reaching the earth's surface depends upon climatic conditions of a location, which is important to the prediction and design of a solar energy system [1]. However, the facilities for global solar radiation measurement are available only in few locations in the country. Consequently there is the need to use solar radiation from some measured meteorological parameters, such as relative temperature ratio relative sunshine hours, cloud etc.

In Nigeria several researches have been carried out for estimating solar radiation at different location, using different meteorological data [2].

However, none has been found in the literature of the models developed for estimating global solar radiation using only relative temperature ratio for Maiduguri. Hence this work is carried at developing an Angstrom – type of empirical model for estimation of global solar radiation using relative temperature only for Maiduguri.

Maiduguri is located at latitude 11.8333^0N and longitude 13.1500^0E. It is the capital of Borno state, Nigeria. Maiduguri is noted for its hotness during dry season with an average temperature of about 38.9^0C. This research work will help in utilizing the solar energy potential to solve the energy problems in the state. The measured global solar radiation, sunshine hours, minimum and maximum temperature data used in this research was obtained from the Gunn–Bellani radiation integrator, Nigeria Meteorological Agency (NIMET) Maiduguri.

2.0 Methodology

Based on the aforementioned, this study was focused on the use of the Angstrom – Prescott Model in estimation of the monthly average global solar radiation. The equations used in the study are of the form indicated in equations (1) to (5).
The regression models employed in this work are as follows

2.1 Model

$Linear H_m/H_o = a + b\ (Tmin/Tmax)$ -------------------------------------- ---(1)
$Quadratic H_m/H_o = a + b(Tmin/Tmax) + C(Tmin/Tmax)^2$ ---------------(2)

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$CubicH_m/H_o = a + b(Tmin/Tmax) + C(Tmin/Tmax)^2 d(Tmin/Tmax)^3$ ---(3)
$ExponentialH_m/H_o = ae^{b(Tmin/Tmax)}$---------------------------------------(4)
$LogarithmH_m/H_o = a + blog\ (Tmin/Tmax)$ ------------------------------------(5)
These are the linear model, the actual Angstrom – Prescott model and it is the most commonly adopted model [3], the quadratic and cubic (second and third order polynomial) regression models first derived in [4] and [5] respectively, the logarithmic and the exponential models proposed in [6] and [7] respectively.
The regression model used in evaluating the correlation coefficient between the clearness index (H_m/H_0) and the relative temperature ratio$(Tmin/Tmax)$. The clearness index is the ratio of the monthly averaged daily global solar radiation (Hm) and the extra-terrestrial radiation (H_0). The relative temperature ratio is the ratio between the daily minimum temperature and daily maximum temperature. The monthly mean daily extra-terrestrial solar radiations were computed using.

$H_o = \frac{24x60}{\pi} G_{sd} d_r \ [w_s \sin Q \sin\delta + \cos Q \cos\varphi \sin\omega_s\ ]$ ---------------------------(6)

$w_s$and δ are sunset hour angle and solar declination respectively and are defined as

$w_s = cos{-}^1\ (-tan\varphi tan\delta)$ ---(7)

$\delta = 0.409 \sin\ (\frac{2\pi j}{365}) - 1.39$ ---(8)

Where Gsc is the solar constant given by

$Gsc = 0.0820 mjm^{-2} min^{-1}$ ---(9)

And dr is the relative earth – sun distance given by

$dr = 1 + 0.033 \cos(\frac{2\pi j}{365})$ − − − − − − −--------------------------------------(10)

j is the day of year (known as the Julian day)

$N = \frac{2\pi j}{15} w_s$ − (11)

In which was represent the latitude of the station or location of measurement.

3.0 Data Analysis

SPSS computer software[8] was applied to obtain the regression constant a, b, c, and d, and the correlation coefficient r and the coefficient of determination R^2. The accuracy of the estimated values of the global solar radiation were tested by calculating the MBE (Mean Bias Error), RMSE (Root Mean Square Error) and MPE (Mean Percentage Error) for the five regression models used in this work [2].

$MBE = \sum_1^k (Hest - Hmeans)\ /K$ -----------------------------------(12)

$RMSE = \frac{1}{K}\sqrt{\sum_1^K\ (Hest - Hmeans)^2}$ ------------------------------------(13)

$MPE = \frac{1}{K}\sum_2^K \left(\frac{Hest-Hmeans}{Hmean}\right) x\ 100$ --(14)

In all three relations, K stands for the number ofobservation while Hest and Hmeas are the estimated and measured monthly averaged daily global solar radiation respectively.
The input parameters used in this analysis are presented in Table 1.

Table 1: Input parameters for estimation of monthly mean daily global solar radiation for Maiduguri, Nigeria.

Months	H_m	H_0	$\frac{H_m}{H_0}$	Tmin	Tmax	$\theta = Tmin/Tmax$
Jan	23.41	31.40	0.752	15.32	34.10	0.449
Feb	25.62	33.81	0.758	17.64	36.68	04.81
Mar	26.86	36.54	0.735	20.06	38.48	0.521
Apr	25.86	37.96	0.681	26.67	41.33	0.645
Mar	25.08	37.91	0.662	26.29	39.35	0.668
Jun	23.57	37.51	0.628	26.27	38.87	0.676
July	21.62	37.56	0.576	24.81	34.65	0.716
Aug	20.87	37.68	0.554	23.84	31.77	0.750
Sep	21.24	36.78	0.577	24.00	33.77	0.718
Oct	23.67	34.39	0.688	23.81	35.19	0.676
Nov	22.52	31.58	0.713	17.93	34.77	0.516
Dec	24.30	30.68	30.68	0.792	15.90	0.413

4.0 Results and Discussion

4.1 Regression

Regression and correlation analyzes was carried out between the clearness index and the ratio of the minimum to maximum temperature. The estimated values of the clearness index ($^H/H_0$) were compared with the actual values of the meaured clearness index ($^H/H_0$), and are reported on Table 2: Figure 1 further illustrates the comparison between the actual data and the predicted values.

Table 2: Comparison between the Actual Clearness Index and the Predicted clearness index using the five models.

H/Ho	*Predicted H/Ho Model 1*	*Predicted H/Ho Model 2*	*Predicted H/Ho Model 3*	*Predicted H/Ho Model 4*	*Predicted H/Ho Model 5*	Tmin/Tmax
0.751682	0.771870593	0.763962909	0.760879103	0.996508914	0.772587261	0.449385
0.757616	0.75221092	0.756893509	0.744504602	0.983785198	0.7488255	0.480771
0.735134	0.726777546	0.742741867	0.729503089	0.967324751	0.720288773	0.521375
0.6813	0.649239781	0.664747442	0.675208323	0.917142408	0.645308587	0.645161
0.661503	0.634913476	0.6445921	0.655630411	0.907870442	0.633047118	0.668033
0.628346	0.630038989	0.637325762	0.647813063	0.904715568	0.628970717	0.675815
0.575632	0.604858321	0.596485769	0.596312444	0.888418783	0.608633325	0.716015
0.553808	0.583411666	0.557336892	0.535044543	0.874538536	0.592192704	0.750254
0.57746	0.608149822	0.602138677	0.604204389	0.890549039	0.611225915	0.71076
0.688196	0.629645071	0.636729494	0.647153549	0.904460737	0.628643346	0.676444
0.713113	0.73025706	0.745011376	0.731311194	0.969576688	0.724058899	0.51582
0.792101	0.794516819	0.767924267	0.788325354	1.011165523	0.802107918	0.413231

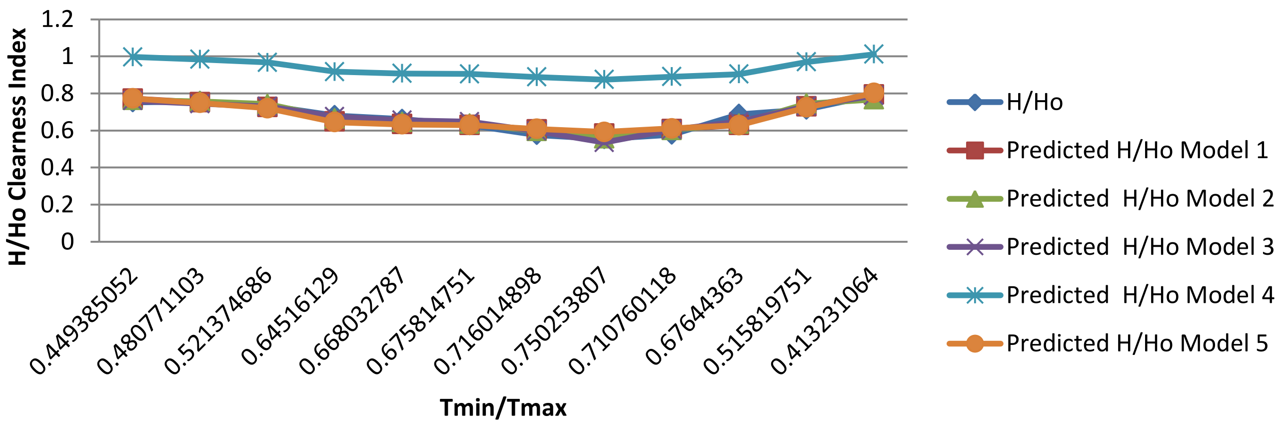

Figure 1: Variation of the Measured $^H/_{Ho}$ and ($^{Tmin}/T_{max}$)

Table 3: The Angstrom Coefficient of the Regression Models

Models	a	b	c	D	r	R^2	Adjusted R
Linear	1.05357351	-0.626382513	-	-	0.934389781	0.873084263	0.860392689
Quadratic	0.495166249	1.367774076	-1.712631002		0.953733498	0.909607585	0.889520381
Cubic	2.911132115	-11.56787306	20.91405424	-12.95126977	0.968196263	0.937404003	0.913930504
Exponential	1.178686903	-0.405393966	-	-	0.921742182	0.84960865	0.83569515
Logarithmic	0.491056975	-0.810435746	-	-	0.921220188	0.848646635	0.833511299

Table 4 contains summaries of various Angstrom regression models analyses obtained from the application of equations (1) to (5). It is clear that the correlation coefficient r and correlation of determination R^2 vary from one model to another model. Generally, correlation coefficients (0.9344-0.9212) are high for all the models. This implies that, there are statistically significant relationships between the clearness index and ratio of minimum to maximum daily temperature. This is further demonstrated by high values of coefficient of determination R^2 (0.8731-0.8486) across the five models.

4.2 Linear Regression Model

The correlation coefficient of 0.9344 exists between the clearness index and ratio of minimum to maximum daily temperature, also coefficient of determination of 0.8731 implies 87.31% model. H/H_0 = at b ($^{Tmin}/T_{max}$)

4.3 Quadratic Regression Model:

The correlation coefficient 0.9537 exists between the clearness index and the ratio of minimum to maximum daily temperature, also coefficient of determination of 0.9096 implies 90.96% of clearness index can be accounted by using Quadratic regression model.

$H/H_0 = a+b(\frac{Tmin}{Tmax}) + c(\frac{Tmin}{Tmax})^2$

4.4 Cubic Regression Model

The correlation coefficient of 0.9682 exists between the clearness index and the ratio of minimum to maximum daily temperature also coefficient of determination of 0.9374 implies 93.74% of clearness index can be obtained using the cubic regression model

$H/H_0 = a+b\ (^{Tmin}/T_{max}) + (\frac{Tmin}{Tmax})^2 + d(\frac{Tmin}{Tmax})^3$

4.5 Exponential Regression Model

The correlation coefficient of 0.9217 exist between the clearness index and the ratio of minimum to maximum daily temperature, also coefficient of determination of 0.8496 implies 84.96% of clearness index can be accounted using this model.

$H/H_0 = ae^b(T_{min}/T_{max})$

4.6 Logarithmic Regression Model

The correlation coefficient of 0.9212 exists between the clearness index and the ratio of minimum to maximum daily temperature, also coefficient of determination of 0.8486 implies 84.86% of clearness index can be accounted using the model.

$H/H_0 = a + b\log(T_{min}/T_{max})$

In all the five models discussed above, the cubic regression model presented the highest value of the coefficient of 0.9682 and 93.77% of clearness index while the logarithmic model showed the lowest of 0.9212 and 84.86% of clearness index as indicated in Table 4.

The values of coefficient of determination (R^2) are within the range of values obtained in similar studies involving different models. In work employing BC model, CD model, DB model and the molecular DCBB carried out by Suleman [9] reported values between 0.7529 to 0.9870 for Samaru (Zaria)

4.7 Model Performance Estimation

In order to determine the best model of the equation that adequately fits the data set for Maiduguri, it was necessary to determine the level of performance of the models. This was evaluated using equations (12 – 14) and presented in Table 4.

Table 4 shows that the Quadratic models have the lowest RMSE, followed by the logarithmic and the linear model, the exponential has the largest RMSE. Based on the values of MBE and MPE, Table 4 indicates that cubic, quadratic, logarithmic and linear models have the lower value which shows better performance than the exponential model.

Table 4: Performance of the regression models as measured by the statistical measures selected.

Model	MBE	MPE	RMSE
Linear	-0.20853	-0.39156	0.531231
Quadratic	o.001980	0.124386	0.000159
Cubic	0.00119	0.082566	0.193013
Exponential	9.168705	39.28059	2.695504
Logarithmic	-0.00684	0.231069	0.305251

All the regression equations give satisfactory results except exponential which exhibit over estimation of the predicted values, the variation observed in the error do not significantly affect the predictability of the models. But the low values of the MBE, MBE, MPE and RMSE shown by the quadratic and cubic model implies that it is the best model.

Observation from Figure 2 and Table 5 show that both the estimated and measured values from the five models studied of the monthly average daily global solar radiation correspond, with the exception of the exponential model which shows higher differences, the remaining models performed very well with best performance by the cubic model and closely followed by the quadratic and the logarithmic models. This shows that the cubic and the Quadratic models can be employed in the study area and site with similar climatic conditions with minimal error.

Table 5: Measured and Estimated Global Solar Radiation Using Five Regression Models

MONTHS	H Measured	Linear Regression Model	Quadratic Regression Model	Cubic Regression Model	Exponential Regression Model	Logarithmic Regression Model
January	23.41	23.42659249	23.79247601	23.69643546	31.03477168	24.06106849
February	25.62	24.57726862	25.59556011	25.1766094	33.26826417	25.32272756
March	26.86	23.72179476	27.1379845	26.65427162	35.34369787	26.31760301
April	25.86	24.099464	25.23170954	25.62877135	34.81182387	24.49387211
May	25.08	23.88723897	24.43884261	24.85734519	34.4206869	24.00113016
June	23.57	22.68910848	23.90684037	24.30023124	33.93695133	23.59343278
July	21.62	21.91237143	22.40325472	22.3967447	33.36789135	22.85950164
August	20.87	22.91801302	21.00299021	20.16291292	32.95659164	22.31651586
September	21.24	23.15960811	22.14772115	22.22370154	32.75596234	22.48196579
October	23.67	25.11680374	21.89985986	22.25838772	31.10828633	21.62174254
November	22.52	25.09082815	23.5273432	23.09469351	30.61908082	22.86566728
December	24.3	21.51853775	23.5583165	24.1841793	31.02045147	24.60699973

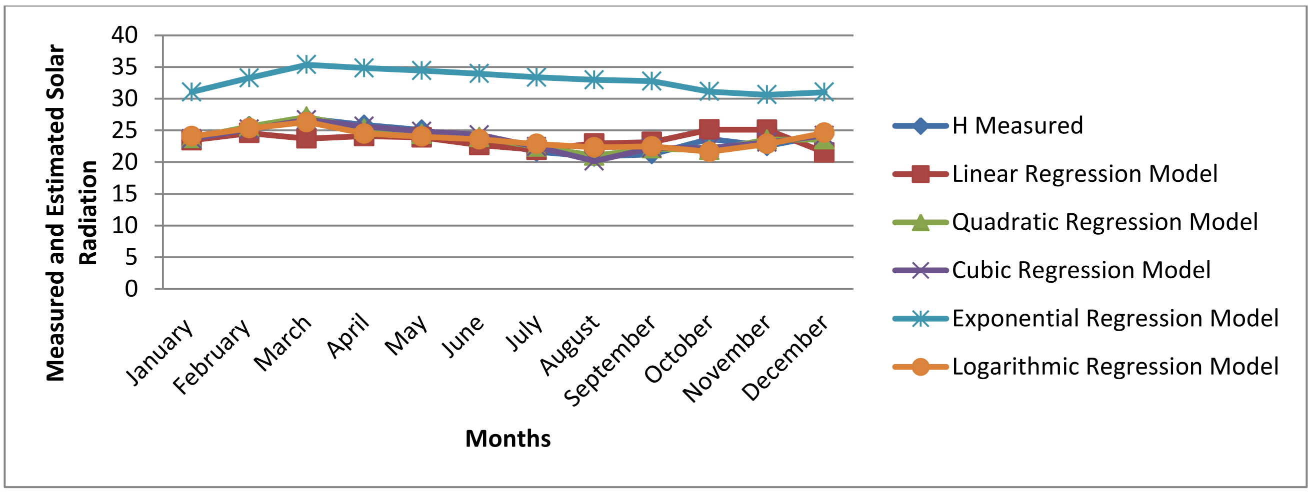

Figure 2: Comparison between observed and estimated values of the monthly average daily solar radiation (MJM^{-2}/Day) by the regression models.

5.0 Conclusion

The monthly average global solar radiation and the ratio of minimum to maximum temperature have been employed in this study to estimate the global solar radiation. Five Angstrom type regression model have been tested for the estimation. It was observed that the cubic model has the highest value of correlation coefficient and correlation of determination, which gives good results when considering statistical indicators, that is, MBE, MPE and RMSE.

6.0 Acknowledgement

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7.0 References

[1] F.W. Burari and A.S Sambo, "Model for the Prediction of Global Solar Radiation for Bauchi, using Meteorological Data "Nigeria Journal of Renewable Energy Vol. 91 Pp. 30-33 (2001)
[2] Skeiker Kamal. Correlation of Global Solar Radiation with Common Geographical and Meteorological Parameters for Damascus Province. Syria Energy Convers Manage 2006: 47: 331-45
[3] Angstrom A., Quart J. Roy met soc . 50, pp121-125 (1924)
[4] Akinoglu B . G and Ecevit A.A. , sol energy, 15, pp865-872(1987)

[5] Bahel V., Bakhsh H. And Srinivasan R, Energy, 12, pp131-135 (1987
[6] Elagib N .A and Mansell M.G. , Energy convers manage, 41, pp419-434 (2000
[7] Ampratwum D.B. and Dorvlo A. S. S. , Appl. Energy ,pp161-167(1999
[8] Marcel B. Finan: Math 1203: Trigonometry Arkansas Tech University http://mathbit.com.muthBits/TINsection/Statistics2/Exponential Mod. (2014).
[9] RBO Suleiman, K.O Obuwasemire, DM Kulla "A Method of estimating solar Radiation from air temperature Data in Samaru, Northern Guinea Savanna of Nigeria)