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Population Prediction Modelling with Autoregressive Integrated Moving Average (Arima) Model

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Abstract

Human development and improvement in quality of life in a nation depend on proper planning based on population at a time and its future values, since the tremendous increase in population is a major issue in Nigeria and worldwide. Then, this research focused on the modelling of Nigeria population by obtaining forecast values.

The Box Jenkins ARIMA methodology was used for analysing and forecasting Nigeria population.

From the result obtained, Nigerian population series was stationary at the first difference. The identification stage of the model building suggests three models for Nigeria population forecast, but $ARIMA(5,2,0)$ was validated in estimation stage using AIC, SC and MSE. The model was diagnosed and results shows the model is adequate and parsimonious to forecast Nigerian population. $ARIMA(5,2,0)$ model was then used to obtain the in-sample and out- sample forecast for the next 12 years. The in-sample forecast exhibits a very close structure and pattern of the original Nigerian population series. The out-sample forecast shows a rapid growth in the Nigerian population yearly and if the trend of current growth persists, the population would be approximately 209.64 million by 2025.

In conclusion, the model obtained and used to forecast Nigerian population is adequate and parsimonious since it reflects a close pattern to the projected population of Nigeria by different bureaus.

Keywords: Population, ARIMA Model, Forecasting, Model Building and Human Development

1.0 Introduction

Nigeria has experienced a population explosion in the last 50 years due to very high fertility rates quadrupling its population during this time. The total population of Nigeria was lastly estimated to be 166.2 million people in 2012 from 45.2 million in 1960, changing 268 percent during the last 50 years. From the report in [1], the population of Nigeria represent 2.35 percent of the world's total population which arguably means that one person in every 43 people on the planet is a resident of Nigeria. Nigeria is the most populous country in Africa and its population accounts for approximately one sixth of the African population.

Due to the rapid growth of Nigeria population it is unanimously accepted that Nigeria population will hit 367 million by 2040 and this will create numbers of associated problems like food scarcity, accommodation problems, educational issues, medical and heavy traffic and so on. Furthermore, the crime rate among the societies also arises due to heavy pressure of the population. Different measures and strategies are being adopted by multicultural societies of the world to limit the size of population according to their feasibility and circumstances. Then, human development and improvement in quality of life must be of important to Nigeria government. This is to be achieved through policies and programmes aimed at promotion of both equity and excellence.

Rapid population growth has long been a concern of the government and Nigeria has a lengthy history of explicit population policy. The periodic census enumeration obtains data on the size and composition of the population at the time census was taken. But for many purposes, it is important to know the number and characteristics of the people at different dates between

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the two censuses. Then, this brings population forecasting to the for-front and in the literature on forecasting population growth, there are two standard methods that can be used to forecast future population growth. They are the cohort-component method and time series or structural equation method [2].

The cohort-component method typically projects future numbers of annual births, deaths, and migration to form a new population vector which is then used to repeat the calculations for each forecast year. Many studies and government reports undertaking population forecasting use this approach. However, this procedure only provides a general bookkeeping framework of population changes [3]. One possible reason for the use of this approach is due to its simplicity and because this approach does not require the knowledge of more advanced time series model skills.

Time series approaches have been used to obtain short to long term forecasts and are typically based on past observations of the variables to be predicted. In these models, the structure of past population status is taken into account in order to extrapolate the future course of a time series in population changes. According to [3], time series methods may perform more accurately or at least as accurately as other forecasting methods such as the cohort-component method (stochastic or non stochastic).

However, it is argued that quantitative methods are more accurate than qualitative methods since projections are inevitably uncertain and one source of uncertainty is the lack of perfect knowledge of the population at the starting point of the forecasting time interval; this is usually a minor problem in countries with an advanced system of population statistics. The most important source of uncertainty is the unpredictability of facts and events that can influence future trends in fertility, mortality and migration. It has been remarked, ‘the best demographers provide forecasts, but none would stake their reputation on the agreement of such forecasts with the subsequent realizations’ [4].

More recently, stochastic (or probabilistic) population forecasting has received great attention by researchers. The main reason for the development of stochastic forecasting methods lies in the awareness that only in this way can forecast of uncertainty be fully and coherently managed. Still, stochastic population forecasting has not yet influenced most official forecasting agencies [5] and [6], although a more probabilistic orientation has been repeatedly advocated during at least the last three decades [7]. As already observed, virtually all population forecasts, based either on the traditional scenario approach or on the stochastic approach, adopt the standard cohort–component method [8], which is characterized by two steps. In the first step, the future trajectories of the three key components of demographic change are forecast over the time horizon of interest. This first step often entails a forecast of summary indicators for these components which are then used to derive the full set of age-specific quantities (usually, rates for fertility and mortality, and absolute numbers for migration). In some cases, age-specific quantities are directly forecast. In the second step, component-specific forecasts are combined and applied to an initial population to obtain the actual population forecast, giving a full probability distribution of forecasts in the stochastic case.

In stochastic population forecasting, three main approaches have been followed to derive the probability distribution of forecasts [9]. The first approach is based on time series models, which are the most classical way to address forecasting problems. For each indicator to be forecasted, a more or less complex time series model is fitted to past data, and forecasts are derived by extrapolation based on the estimated parameters. In the literature, both classical (i.e. frequentist) and Bayesian time series methods for population forecasting have been proposed in [10 – 18].

A Bayesian approach makes it possible to incorporate information coming from any extra-sample source in the forecast. In particular, expert opinions can be used in the specification of the prior distribution to be assigned to the parameters of the model; see in particular [17]. The best-known approach using (classical) time series models is that of [11] and [19], which was originally proposed to forecast mortality, and later modified to address fertility forecasting [20] and [12]. The Lee–Carter approach is based on a log-linear model for age-specific rates of mortality, and it includes a time-dependent mortality index modeled as a random walk with constant drift. As observed in [21] and [22], the Lee–Carter method has been found to perform quite well for some developed countries, but its implementation, requiring data on age-specific rates of death, can be difficult for many developing countries [23].

This is a general problem with a time series approach for developing countries, as knowledge of the past is often limited. Moreover, systematic underestimates of life expectancy by using Lee–Carter forecasts has been found for some developed countries [24]. There are attempts to overcome the lack of available data by resorting to a Bayesian approach. In [25] Bayesian hierarchical model was used to model the dependence of age-specific rates of mortality (on a logarithmic scale) on a set of covariates and suggested a method to assign highly informative priors on the coefficients. Also in [18] a proposed approach to life expectancy forecasting for all countries in the world based on Bayesian time series models. They used a random-walk model with non-constant drift; the latter is a non-linear function of current life expectancy and aims to describe different rates of increase in life expectancy for different countries. A Bayesian hierarchical model is used, with parameters estimated by using both past data and prior information; the prior distribution is specified by using United Nations deterministic scenarios together with additional assumptions [26]. In [15], they proposed a related method to forecast total fertility rates by using a fully Bayesian time series approach. The second approach to stochastic forecasting is based on the extrapolation of empirical errors, with observed errors from historical forecasts used in the assessment of uncertainty (e.g. [27]). In particular, [28] proposed in this framework the so-called scaled model of error. Forecasts are obtained by adding shocks to fertility and mortality age-specific rates (expressed in logarithmic terms). The variance and correlation across age and time of shocks is estimated on the basis of the past forecast errors time series. The scaled model of error is implemented in the simulation program PEP ('program for error propagation'; [2]). This approach was used for deriving stochastic population forecasts within the 'Uncertain population of Europe' (UPE) project [31] for a description of its application in the UPE project). The approach has been applied to aggregate in a consistent way national level forecasts in [22], and to derive household level stochastic forecasts in [33]. Having discussed several researches on population forecast using various methods, then we wish to explore further the time series approach by forecasting Nigerian population using Box Jenkins approach of ARIMA model.

2.0 Theory and Methods

2.1 Data Exploration

The pattern and general behaviour of the rainfall series is examined from the time plot and the correlogram is as well used to reveal important information regarding the order of the autoregressive (AR) and moving average (MA) factors present in the generating process of the given time series as well as to assess stationarity. The series was examined for stationarity and the test for stationarity will be carried out using the augmented Dickey –Fuller methods. Details of the test procedures can be found in the literatures which include [34] and [35].

2.2 Autoregressive Moving Average (ARMA) Model

In most case, it is best to develop a mixed autoregressive moving average model when building a stochastic model to represent a stationary time series. The order of an ARMA model is expressed in terms of both *p* and *q*. The model parameters relate to what happens in period *t* to both the past values and the random errors that occurred in past time periods. A general ARMA model can be written as follow:

$$x_t = \emptyset_1 x_{t-1} + \emptyset_2 x_{t-2} + \cdots + \emptyset_p x_{t-p} + w_t + \theta_1 w_{t-1} + \theta_2 w_{t-2} + \cdots + \theta_q w_{t-q} \qquad (1)$$

Equation (1) will be simplified by a backward shift operator B to obtain

$$\emptyset(B)x_t = \theta(B)w_t \qquad (2)$$

The ARMA model is stable, that is it has a stationary solution if all roots of $\emptyset(B) = 0$ are larger than one in absolute value. The representation is unique if all roots of $\emptyset(B) = 0$ lie outside the unit circle where $\emptyset(B) = 0$ and $\theta(B) = 0do$ not have common roots. Stable ARMA models always have an infinite order MA representation. If all roots of $\emptyset(B)$ are larger than one in absolute value, it has an infinite order AR representation. The process is invertible only when the roots of $\theta(B)$ lie outside the unit circle .Furthermore, a process is said to be causal when the roots of $\emptyset(B)$lie outside the unit circle.

2.3 Autoregressive Integrated Moving Averages (ARIMA) Model

Autoregressive integrated moving average (ARIMA) models are specific subset of univariate modeling in which a time series is expressed in terms of past values of itself (the autoregressive component) plus current and lagged values of a white noise error term (the moving average component). ARIMA models are univariate models that consist of an autoregressive polynomial, an order of integration(d), and a moving average polynomial.

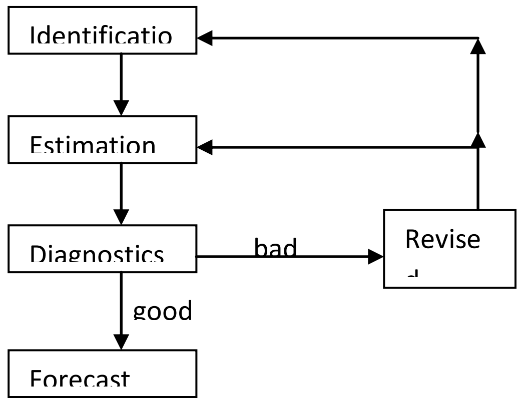
A process (x_t) is said to be an autoregressive integrated moving average process, denoted by $ARMA(p,d,q)$if it can be written as:

$$\emptyset(B)\nabla^d x_t = \theta(B)w_t \qquad (3)$$

where $\nabla^d = (1-B)^d$ with $\nabla^d x_t$and d^{th}consecutive differencing.If $E(\nabla^d x_t) = \mu$, we write the model as;

$$\emptyset(B)\nabla^d x_t = \alpha + \emptyset(B)w_t \qquad (4)$$

Where α is a parameter related to the mean of the process $\{x_t\}$, by $\alpha = \mu\{1 - \emptyset_1 - \cdots - \emptyset_p\}$ and this process is called a white noise process, that is, a sequence of uncorrelated random variables from a fixed distribution (often Gaussian) with constant mean $E\{x_t\} = \mu$,usually assumed to be "zero" and constant variance. If d = 0, it is called $ARIMA(p,q)$ model while when d = 0 and q = 0, it is referred to as autoregressive of order p model and denoted by AR *(p)*. When p = 0 and d = 0, it is called Moving Average of order q model, and is denoted by MA (q).



$\boldsymbol{Figure\ 1. Flowchart\ of\ ARIMA\ Model\ Building}$

2.4 Steps in Model Building

2.4.1 Identification

2.4.1.1 Autocorrelation function

The ACF measures the correlation between Y_t and Y_{t-k} and it is obtained using the formula below.

$$ACF(k) = \frac{\sum_{t-1+k}^{N}(Y_t - \bar{Y})(Y_{t-k} - \overline{Y})}{\sum_{t=1}^{N}(Y_t - \overline{Y})^2} = \frac{cov(Y_t Y_{t-k})}{var(Y_t)} \quad (5)$$

2.4.1.2 Partial Autocorrelation function (PACF)

The PACF measures the additional correlation between Y_t and Y_{t-k} after adjustments have been made for the intermediate values $Y_{t-1}, \dots, Y_{t-k+1}$. The PACF is closely related to ACF, their value also lies between -1 and +1. The specific computational procedures for PACFs are complicated, but these formulas do not need to be understood for us to use PACFs in the model identification phase.

2.4.2 Diagnostics

In the model building process, if an ARIMA (p, d, q) model is chosen (based on the ACFs and PACFs), some checks on the model adequacy are required. A residual analysis is usually based on the fact that the residuals of an adequate model should be approximately white noise. Therefore, checking the significance of the residual autocorrelations and comparing with approximate two standard error bounds, i.e., $\pm 2/\sqrt{n}$ are need. Ljung-Box statistic (Q-statistics) as well is an objective diagnostic measure of white noise for a time series, assessing whether there are patterns in a group of autocorrelations.

$$Q = n(n+2)\sum_{i=1}^{K}\frac{ACF(i)^2}{N-1}\ for\ i = 1\ to\ k\ with\ (k-p-q) degree\ of\ freedom \quad (6)$$

2.4.3 Model Selection

(i) Akaike's information criterion (AIC): $AIC = Log\hat{\sigma}^2 + \frac{2(p+q)}{n}$ where $\hat{\sigma}^2$ is the estimated variance of e_t.

(ii) Schwarz's Bayesian Information criterion (SC, BIC, or SBC): $BIC = Log\hat{\sigma}^2 + \frac{2(p+q)}{n}\log(n)$

Both criteria are likelihood-based and represent a different trade-off between "fit", as measured by the log-likelihood value, and "parsimony", as measured by the number of free parameters, p + q.

2.4.4 Forecasting

The last step in time series modeling is forecasting. There are two kinds of forecasts: sample period forecasts and post-sample period forecasts. The former will be used to develop confidence in the model and the latter will be used to generate genuine desired forecasts. In forecasting, the goal is to predict future values of a time series, $x_{t+m}, m = 1,2, \dots$ based on the data collected to the present, $x = \{x_t, x_{t-1}, \dots, x_1\}$.

2.3.4.1 Forecasting Evaluation

Once forecasts are made they can be evaluated if the actual values of the series to be forecasted are observed. There are some measurements of the accuracy of forecasts. These are root mean square error (RMSE), mean absolute error (MAE) and mean absolute percentage error (MAPE).

$$MAE = \frac{1}{h+1}\sum_{t=s}^{h+s}\left(\hat{X}_t - X_t\right)^2 \qquad (7)$$

the root mean square forecast error (RMSE) is defined as:

$$RMSE = \sqrt{\frac{1}{h+1}\sum_{t=s}^{h+s}\left(\hat{X}_t - X_t\right)^2} \qquad (8)$$

and the mean absolute percentage forecast error MAPE is given as,

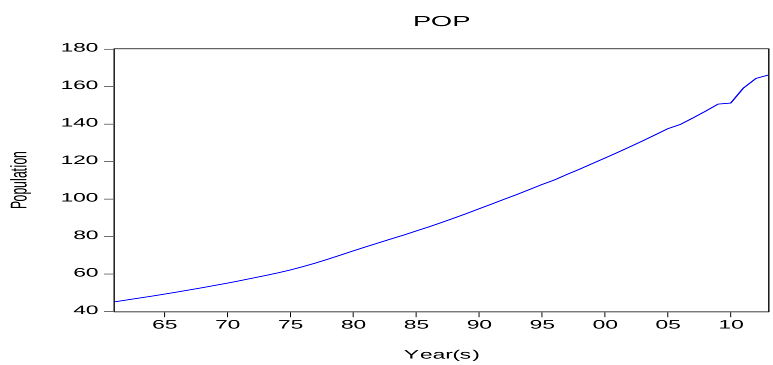
$$MAPE = \frac{100}{h+s}\sum_{t=s}^{h+s}\left|\frac{\hat{X}_t - X_t}{\hat{X}_t}\right| \qquad (9)$$

$where\ t = s, 1+s, \ldots, h+s$. The actual and predicted values for corresponding t values are denoted by $\hat{X}_t\ and\ X_t$respectively. The smaller the values of *RMSE* and *MAPE*, the better the forecasting performance of the model.

3.0 Results and Conclusion

In order to discuss the structure and pattern of Nigerian Population growth, we analysed Nigeria population data from 1961 – 2013 using the autoregressive integrated moving average (ARIMA) technique.

The examination of the time plot shows an upward increasing trend and this suggests that the given time series is non-stationary. The movement is secular in nature and expect a small shift in the movement in 2009.



$\boldsymbol{Fig. 2: Tlme\ Plot\ of\ Nigerian\ Population\ from\ 1960 - 2013}$

3.1 Augmented Dickey Fuller Unit Root Test Result

The augmented dickey fuller test show that Nigerian population is stationary at the second difference that is $I(2)$ at both 5% and 1% level of significance with $p - value = 0.01$. Since the order of integration of the differenced population series is two, then $d = 2$.

Table 1: Unit Root Test Using Augmented Dickey-Fuller (ADF)

			t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic			-3.524976	0.0110
Test critical values:	1% level		-3.588509	
	5% level		-2.929734	

3.2 Model Building for Nigerian Population

3.2.1 Model Identification

Since the degree of differencing is two, then the autoregressive and moving-average orders are selected by examining the correlogram containing the sample autocorrelations and sample partial autocorrelations in table 2 below.

A close look at the correlogram shows that the ACF and PACF of the population series revealed that the ACF dies off smoothly at a geometric rate with spikes at one and five, this suggest that $p = 1$ and $q = 5$. The partial autocorrelations were zero after lag one and this suggest $p = 1$ and $q = 0$. Since the ACF tailed-off at lag five and the PACF cut-off after lag 1, we identify p = 5 and q = 1. Fitting the models suggested by these observations, we have $ARIMA\ (1,2,5)$, $ARIMA\ (5,2,0)$ and $ARIMA\ (1,2,1)$.

Table 2: Correlogram of Nigerian Population Series

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
. |*******	. |*******	1	0.942	0.942	49.710	0.000
. |******|	. | . |	2	0.882	-0.046	94.129	0.000
. |******|	. | . |	3	0.823	-0.015	133.65	0.000
. |******|	. | . |	4	0.770	0.012	168.94	0.000
. |***** |	. | . |	5	0.714	-0.056	199.91	0.000
. |***** |	. | . |	6	0.659	-0.024	226.84	0.000
. |**** |	. | . |	7	0.604	-0.026	249.98	0.000
. |**** |	. | . |	8	0.551	-0.030	269.62	0.000
. |**** |	. | . |	9	0.496	-0.038	285.95	0.000
. |*** |	. | . |	10	0.443	-0.029	299.24	0.000
. |*** |	. | . |	11	0.390	-0.030	309.80	0.000
. |** |	. | . |	12	0.338	-0.031	317.93	0.000
. |** |	. | . |	13	0.287	-0.030	323.94	0.000
. |** |	. | . |	14	0.237	-0.031	328.15	0.000

3.3.2 Estimation

The summary of results obtained for the three suggested model in table 3, 4 and 5 are as follows;
For $ARIMA$ (1,2,5) model, the fitted model is given as;

$$y_t = 8.76 + u_t$$
$$\boldsymbol{(1 - 1.03L)u_t = (1 - 0.72L)(1 + 0.83L^2)(1 - 1.22L^3)(1 - 1.05L^4)(1 + 1.11L^5)\varepsilon_t}$$

where $R^2 = O.99992, AIC = 0.748224, SIC = 1.054148$ and $\sigma_e^2 = 4.49232$
For $ARIMA$ (5,2,0) model, the fitted model is obtained as;

$$y_t = -8.34 + u_t$$
$$\boldsymbol{(1 + 2.07L)(1 + 3.18L^2)(1 - 5.06L^3)(1 + 4.28L^4)(1 - 1.35L^5)u_t}$$

where $R^2 = O.99994, AIC = 0.330941, SIC = 0.609213$ and $\sigma_e^2 = 2.76587$

While for $ARIMA$ (1,2,1) model, the fitted model is given as;

$$y_t = -4.98 + u_t$$
$$\boldsymbol{(1 - 0.17L)u_t = (1 + 0.38L)\varepsilon_t}$$

where $R^2 = O.99996, AIC = 2.355436,\ SIC = 2.508398$ and $\sigma_e^2 = 26.30034$
The three model as a good fit since at least 99% of variation in the dependent variable is explained. To select the best model to analyse Nigerian population, we compare the values of the AIC, SBI and σ_e^2 obtained for each model, where the model with the smaller values of information criteria and σ_e^2 will be chosen for the analysis. Since $ARIMA$ (5,2,0) model has the lowest AIC, SBC and σ_e^2, then this model is believe to estimate Nigeria population better.

Table 3:ARIMA (1, 2, 5) Model Estimation

	Coefficient	Std. Error	t-Statistic	Prob.
C	8.754687	2.222197	3.939653	0.0003
D(POP,2)	0.245263	0.026213	9.356523	0.0000
AR(1)	1.027506	0.001612	637.2553	0.0000
MA(1)	0.720500	0.130147	5.536031	0.0000
MA(2)	-0.828941	0.181263	-4.573142	0.0000
MA(3)	1.218944	0.135366	9.004809	0.0000
MA(4)	1.045954	0.184122	5.680765	0.0000
MA(5)	-1.104997	0.234255	-4.717071	0.0000
R-squared	0.999927	Mean dependent var		96.22248
Adjusted R-squared	0.999915	S.D. dependent var		35.38514
S.E. of regression	0.327048	Akaike info criterion		0.748224
Sum squared resid	4.492324	Schwarz criterion		1.054148
Log likelihood	-10.70561	Hannan-Quinn criter.		0.864722
F-statistic	81938.18	Durbin-Watson stat		1.875675
Prob(F-statistic)	0.000000			

Table 4: ARIMA (5, 2, 0) Model Estimation

	Coefficient	Std. Error	t-Statistic	Prob.
C	-8.343327	11.85368	-0.703860	0.4857
D(POP,2)	0.347378	0.006392	54.34552	0.0000
AR(1)	2.067174	0.134994	15.31306	0.0000
AR(2)	-3.182106	0.238670	-13.33264	0.0000
AR(3)	5.055394	0.380432	13.28857	0.0000
AR(4)	-4.276437	0.583927	-7.323580	0.0000
AR(5)	1.347724	0.328504	4.102617	0.0002
R-squared	0.999947	Mean dependent var		100.2528
Adjusted R-squared	0.999939	S.D. dependent var		33.99579
S.E. of regression	0.266308	Akaike info criterion		0.330941
Sum squared resid	2.765877	Schwarz criterion		0.609213
Log likelihood	-0.611647	Hannan-Quinn criter.		0.435183
F-statistic	122213.8	Durbin-Watson stat		2.395918
Prob(F-statistic)	0.000000			

Table 5:ARIMA (1, 2, 1) Model Estimation

	Coefficient	Std. Error	t-Statistic	Prob.
C	-4.979709	8.665787	-0.574640	0.5683
D(POP,2)	0.170413	0.072901	2.337604	0.0238
AR(1)	1.024310	0.002030	504.6724	0.0000
MA(1)	-0.382816	0.165926	-2.307151	0.0256
R-squared	0.999571	Mean dependent var		96.22248
Adjusted R-squared	0.999543	S.D. dependent var		35.38514
S.E. of regression	0.756139	Akaike info criterion		2.355436
Sum squared resid	26.30034	Schwarz criterion		2.508398
Log likelihood	-54.88590	Hannan-Quinn criter.		2.413685
F-statistic	35754.20	Durbin-Watson stat		1.824470
Prob(F-statistic)	0.000000			

3.3.3 Diagnostic Checking

Inspection of the time plot of the standardized residuals for Nigerian population in Figure 3 in shows no clear patterns. Under the assumption that residuals follow a white noise process, the standard errors of the residual ACF, in our case are approximately equal to $1/\sqrt{360}$. Thus, under the test that residual follows a white noise process; roughly 95% of the residual autocorrelations should fall within the range of $\pm 1.96/\sqrt{360}$. It is clear, as shown in table 6 that there is no pattern in the residuals autocorrelation plot for the selected model, which means there is no autocorrelation coefficient which lies outside the two standard errors significantly for the fitted models. Therefore, this indicates that residual for the fitted models are not significantly different from a white noise. In addition to these tests, Figure 4 shows the histograms of the residuals. As expected, the curves significantly reflect a normal distribution. Test statistics values of Breusch and Pagan (B-P) for the homoscedasticity of the residuals are also presented in Table 7. All calculated values are found to be smaller than the respective critical values, which indicating that the residual variance is constant. Therefore, the hypothesis that the residuals are white noise cannot be rejected indicating that the fitted model is adequate. That is, $ARIMA$ $(5,2,0)$ model is adequate for modeling Nigerian population.

Table 6: Correlogram of Residuals

Autocorrelation	**Partial Correlation**		**AC**	**PAC**	**Q-Stat**	**Prob**
.*| . |	.*| . |	1	-0.201	-0.201	1.9813	
. |** |	. |** |	2	0.284	0.254	6.0253	
. | . |	. |*. |	3	0.037	0.145	6.0953	
**| . |	***| . |	4	-0.266	-0.350	9.8091	
. |*. |	. | . |	5	0.181	0.049	11.575	
***| . |	.*| . |	6	-0.348	-0.160	18.252	0.000
. | . |	. | . |	7	0.072	-0.046	18.544	0.000
.*| . |	. | . |	8	-0.111	-0.057	19.264	0.000
. | . |	. |*. |	9	0.065	0.170	19.515	0.001
. | . |	.*| . |	10	-0.039	-0.166	19.607	0.001
. | . |	.*| . |	11	-0.040	-0.078	19.710	0.003
. |*. |	. | . |	12	0.081	0.035	20.135	0.005
. | . |	. |*. |	13	-0.016	0.126	20.152	0.010
. | . |	.*| . |	14	0.004	-0.177	20.153	0.017

Table 7: Breusch-Godfrey Serial Correlation LM Test

F-statistic	3.147562	Prob. F(2,37)		0.0547
Obs*R-squared	6.688414	Prob. Chi-Square(2)		0.0353
	Coefficient	Std. Error	t-Statistic	Prob.
C	2.753577	11.33066	0.243020	0.8093
D(POP,2)	-0.003457	0.006250	-0.553070	0.5835
AR(1)	0.358639	0.392624	0.913440	0.3669
AR(2)	-0.542375	0.575537	-0.942382	0.3521
AR(3)	0.832399	0.898531	0.926401	0.3602
AR(4)	-1.418695	1.460217	-0.971564	0.3376
AR(5)	0.768293	0.749543	1.025015	0.3120
RESID(-1)	-0.537529	0.465603	-1.154479	0.2557
RESID(-2)	0.131697	0.297286	0.442999	0.6603
R-squared	0.145400	Mean dependent var		-6.21E-14
Adjusted R-squared	-0.039378	S.D. dependent var		0.247919
S.E. of regression	0.252753	Akaike info criterion		0.260776
Sum squared resid	2.363717	Schwarz criterion		0.618553
Log likelihood	3.002162	Hannan-Quinn criter.		0.394801
F-statistic	0.786890	Durbin-Watson stat		2.015168
Prob(F-statistic)	0.617142			

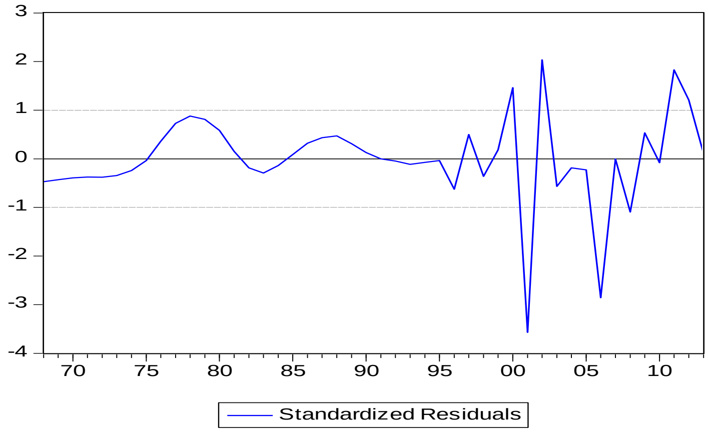


Figure 3. Time plot for Residual

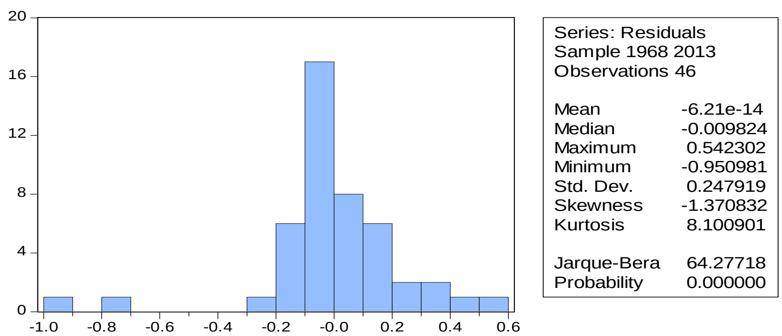


Figure 4. Histogram – Normality Test

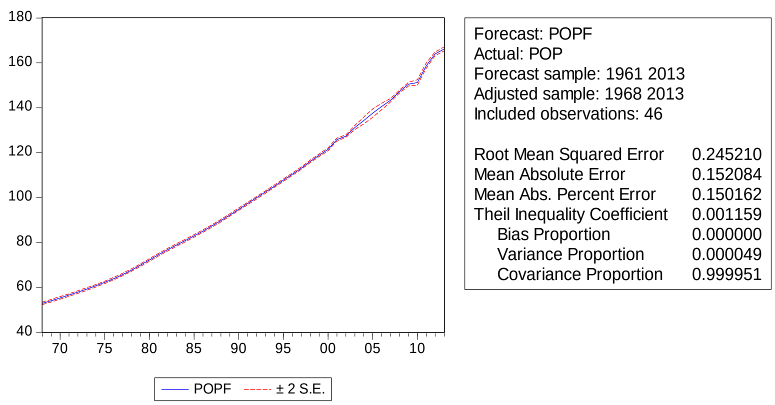
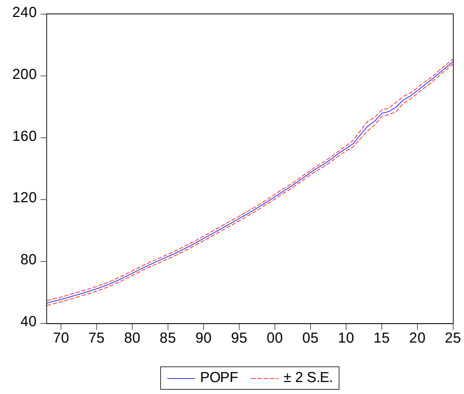


Figure 5. Time Plot of In – Sample Forecast Using ARIMA (5, 2, 0) and Its Forecast Evaluation



Forecast: POPF
Actual: POP
Forecast sample: 1961 2025
Adjusted sample: 1968 2025
Included observations: 58

Root Mean Squared Error	0.691198
Mean Absolute Error	0.354375
Mean Abs. Percent Error	0.275105
Theil Inequality Coefficient	0.002706
Bias Proportion	0.000012
Variance Proportion	0.019540
Covariance Proportion	0.980448

Figure 6. Time Plot of Out − Sample Forecast Using ARIMA (5, 2, 0) and Its Forecast Evaluation

3.3.4 Forecasting

Since the model diagnostic tests show that all the parameter estimates are significant and the residual series is white noise, the estimation and diagnostic checking stages of the modeling process is complete. We can now proceed to forecasting the Nigerian population with fitted $ARIMA$ (5,2,0) model. Forecasting refers to the process of predicting future population values from a known time series. In this study, forecasting is performed using;

$$y_{t+1} = (1 - \emptyset_1 L)(1 - \emptyset_2 L^2)(1 - \emptyset_3 L^3)(1 - \emptyset_4 L^4)(1 - \emptyset_5 L^5)\Delta^2 u_t \qquad (10)$$

This model will be used to forecast in-sample and out-sample forecast. Then, figure 5 shows the in-sample forecast and its forecast evaluations. Figure 5 revealed that the in-sample forecast of Nigerian population exhibits a very similar pattern to the time plot of the original series. The forecast evaluations are low and this indicates the forecast values are precise and accurate. The out-sample forecast for Nigerian population in figure 6 using the model above established a rapid upward movement for the 12 years forecasted. Table 8 below shows the numerical values of the Nigerian population projection and the pattern exhibited is a continuous upward movement yearly. The forecast evaluation in figure 6 are as well low and this indicate the accuracy of the population projection.

Table 8: Nigeria population Projection using ARIMA (5, 2, 0) from 2014 – 2025

Year(s)	Nigerian Population in Millions
2014	170.45
2015	173.65
2016	176.92
2017	180.67
2018	183.67
2019	187.15
2020	190.70
2021	194.33
2022	198.04
2023	201.82
2024	205.69
2025	209.64

4.0 Conclusion

The main objective of this work is to build an ARIMA models for Nigerian population from 1961 – 2013. We review the Autoregressive integrated moving average model and several previous works were coated in the literature. The time plot established that Nigerian population is rapidly growing and the pattern of movement is secular over the years. From the analysis, the augmented dickey fuller unit root test shows that Nigerian population was stationary at the second difference that is $I(2)$and this indicate the value of $d = 2$.

In the identification stage of model building using the ACF and the PACF, we observed the following models ARIMA (5, 2, 0), ARIMA (1, 2, 5) and ARIMA (1, 2, 1). ARIMA (5, 2, 0) was selected since its has the lowest AIC, SBI and σ_e^2.

The model selected was diagnosed and it was found to be adequately to forecast Nigerian population. In-sample and out-sample forecast was obtained using ARIMA (5, 2, 0), the in-sample forecast exhibits a very close structure and pattern of the original series over the years considered and the forecast evaluation shows the in-sample forecast are precise and accurate. The out-sample forecast established that Nigerian population is rapidly rising years after years and this can be attributed to uncontrolled birth, illiteracy and so on. The forecast evaluation of the out-sample forecast are low and this implies the forecast is precise and accurate.

Conclusively, Nigerian population was discussed and analysed using the classical time series approach where an ARIMA (5, 2, 0) was obtained to be the appropriate model for Nigerian population projection. The in-sample forecast exhibits a close structure and pattern of the original population series while the out-sample forecast shows a rapid rise in the Nigerian population projection from 2014 – 2025 and this can be attributed mostly on uncontrolled birth in Nigeria.

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