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Effect of Variable Viscosity on MHD Flow Near a Stagnation Point in the Presence of Heat Generation/Absorption

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Abstract

The flow of a viscous incompressible electrically conducting fluid on a continuous moving plate in presence of uniform transverse magnetic field placed in a calm environment is studied. The flat plate which is continuously moving in its own plane with a constant speed is considered to be isothermally heated. The fluid viscosity is taken as inverse linear function of temperature, the nature of fluid velocity and temperature in presence of uniform magnetic field and heat generation are shown for changing viscosity parameter at different layers of the medium. Numerical solutions are obtained by using Runge-Kutta and shooting method for the dimensionless velocity profiles and the temperature profiles. The coefficient of wall shear stress and the rate of heat transfer at the wall are calculated at different viscousity parameter. These results are presented to illustrate the influence of the Hartmann number, suction parameter, heat absorption coefficient and thermal Grashof number. The effects of various emerging parameters are seen on the velocity and temperature fields.

Keywords: Boundary layer, forced convection,fluid mechanics, isotropic, laminar convection, Magnetohydrodynamic fluid, quiescent fluid, Thomas algorithm, viscosity

1.0 Introduction

The problem of forced convection along an Isothermal, constantly moving plate is a classical problem of fluid mechanics that has been solved for the first time in 1961 by Sakiadis[1]. Thereafter, many solutions have been obtained for different aspect of this class of boundary layer problems. Solutions have been appeared including mass transfer, varying plate velocity, varying plate temperature, fluid injection and fluid suction at the plate [2]. The work by Hassanien[3] belongs to the above class of problems, including a linearly varying velocity and variation of fluid viscousity with temperature. Ostrach [4] first discussed the combined natural and forced flow of a viscous incompressible fluid through a rigid surface. Later on, Grief et al [5], Gupta et al [6] and Soundalgekar et al [7] studied the incompressible flow over a fixed flat plate. But this type of flow becomes different when the flow is caused by the motion of the flat plate or rigid surface. Sakiadis [1] discussed the viscous flow of an incompressible fluid due to the motion of rigid surface. Many authors like Gorla [8], Revenkar [9] discussed the problem of incompressible fluid on a continuous moving flat plate. Both types of flow behave differently-particularly when the fluid viscosity varies with temperature. The fluid properties especially the viscosity depends linearly and inversely to the temperature (see Herwig and Gersten [10]), therefore to characterize the nature of flow and heat transfer, one must consider the variation of fluid viscosity with temperature.
Chakraorty S. and Borkakati [11] studied the problem of viscous variation for a moving flat plate in an incompressible fluid. In this paper, an attempt is made to study the effect of variable viscosity on the flow of an incompressible electrically conducting fluid on a continuous moving flat plate in presence of a uniform magnetic field. The solutions and results are obtained by similarity transformation. Effect of variable viscosity on the flow and heat transfer on continues stretching surface was reported by AsteriosPantokratoras[12]. He opined that Prandtl number is a function of viscousity and so should vary as temperature changes against being constant as considered by Hassanien[3].

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In the present work, we revisit the work of [12]and modify for the inclusion of heat generation effect on the flow which was left out in the previous work. Also, it should be pointed out that since the flow is an MHDtype, we take the Prandtl number corresponding to the ambient Prandtl number of plasma.

2.0 Formulation of The Problem

We consider laminar flow of a viscous incompressible electrically conducting fluid on a continuous moving flat plate along $x = axis$. The flow is along a moving plate placed in a calm environment with u and v denoting respectively the velocity components in the x and y direction, were x is the coordinate along the plate and y is the coordinate perpendicular to x. The plate is moving on its own at constant speed U_0 in quiescent fluid. A uniform magnetic field B_0 is applied transversely i.e. along $y = axis$. The fluid properties except fluid viscosity (μ) are assumed to be isotropic and constant, and the viscosity is inverse linear function of temperature given by the following equation [3]

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}\left(1+\gamma\left(T - T_w\right)\right) \tag{1}$$

$$= \frac{1}{a}\left(T - T_r\right) \tag{2}$$

where

$$a = \frac{\mu_\infty}{\gamma} \text{ and } T_r = T_\infty - \frac{1}{\gamma} \tag{3}$$

both$a$ and T_r being constant. Their values depend in the reference state and the thermal property of the fluid (γ). In general, $a > 0$ for liquid and $a < 0$ for gasses.

3.0 Assumptions

In order to derive the governing equations of the problem the following assumptions are made.

(i) The fluid is finitely conducting and the viscous dissipation and the Joule heat are neglected

(ii) Hall effect and polarization effect are negligible.

(iii) The flat plate which is maintained at a constant temperature T_w is moving withuniform velocity and the fluid viscosity varies with temperature only,therefore all the physical variables are assumed to be time independent.

(iv) The perturbation technique which is used for small values of the magneticparameter (m) depending on the magnetic field. The second order term is due to the effect of the magnetic field

4.0 Governing Equations

For steady, two – dimensional flow the boundary layer equations including variable viscousity of the problem for the fluid medium having small conductivity are

continuity equation: $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{4}$$

momentum equation: $$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) + \frac{\sigma B_0^2}{\rho}u + \frac{g\beta_\tau}{\rho}\left(T - T_\infty\right) = 0, \tag{5}$$

energy equation: $$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2} + Q\left(T - T_\infty\right) \tag{6}$$

where u, v are the fluid velocities along x, y-axes respectively.

The boundary conditions of equations (4-6) of the problem are as follows

$$\left.\begin{array}{r} u(x,0) = 0,\ v(x,0) = 0,\ T(x,0) = T_w \\ u(x,\infty) = 0,\ T(x,\infty) = T_\infty \end{array}\right\} \tag{7}$$

5.0 Method of solution

Using stream function ψ where

$$u = \frac{\partial \psi}{\partial y} \text{ and } u = -\frac{\partial \psi}{\partial x}$$

and introducing the following dimensionless variables and parameters

$$\eta = \sqrt{\frac{a}{\upsilon}}y,\ u(x,y) = axf'(\eta),\ v(x,y) = -\sqrt{a\upsilon}\,f(\eta),$$

$$g(\eta) = \frac{T - T_\infty}{(T_w - T_\infty)} \text{ and } \mu = k_0(\theta - \theta_e)^{-1}$$

The derivatives becomes,

$$\frac{\partial u}{\partial x} = af',\ \frac{\partial u}{\partial y} = ax\sqrt{\frac{a}{\upsilon}}f'',\ \frac{\partial v}{\partial y} = -af',$$

$$\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} + \frac{\partial \mu}{\partial y}\cdot\frac{\partial u}{\partial y} = \frac{a^2 x}{\upsilon}k_0(\theta - \theta_e)^{-1}\left(f''' - \frac{\theta'}{\theta - \theta_e}\right)$$

$$\theta_e = -\frac{1}{\gamma(T_w - T_a)} \quad (\theta_e = \text{variable thermal conductivity}) \tag{8}$$

we got from the equations (4-6) and boundary conditions (7)

$$f''' - \frac{\theta'}{\theta - \theta_e}f'' + \frac{\theta - \theta_e}{\theta_e}\left(-ff'' - M^2 f' + f'^2 + Gr\tau\theta\right) = 0 \tag{9}$$

$$\theta'' + \Pr f'\theta + \Pr\phi\theta = 0 \tag{10}$$

$$\left.\begin{aligned} f(y) = \pm\alpha,\quad f'(y) = 0,\ \theta(y) = 1,\ at\ y = 0 \\ D(f)(y) \to 0,\ \theta(y) \to 1,\ as\ y = 0 \end{aligned}\right\} \tag{11}$$

here, primes denote differentiation with respect to η.

Where $\Pr = \frac{\mu c_p}{k},\ \phi = \frac{Q}{a\mu c_p},\ Gr\tau = \frac{g\beta_\tau}{\rho a^2}(T_w - T_\infty),\ M = \sqrt{\frac{\sigma B_0^2}{a\rho}}$

The flow Equations (9) and (10) subject to boundary conditions (11) are solved numerically using finite differences. A shooting technique is first applied to convert the higher order derivatives to a system of first order differential equations.

The solution for the non-magnetic case is chosen as an initial guess and the iterations using Euler scheme are continued till convergence within prescribed accuracy is achieved, with the corrections incorporated in subsequent iterative steps until convergence, which is used to obtain the values of our initial guesses. Finally, the resulting guesses together with the system was solved using generalized Thomas' algorithm.

The system of equations has to be solved in the infinite domain $0<\eta<\infty$. A finite domain in the η-direction can be used instead with η chosen large enough to ensure that the solutions are not affected by imposing the asymptotic conditions at a finite distance.

Grid-independence studies show that the computational domain $0 < \eta < \eta_\infty$ can be divided into intervals each of uniform step size which equals 0.02. This reduces the number of points between $0 < \eta < \eta_\infty$ without sacrificing accuracy. The value $\eta_\infty = 10$ was found to be adequate for all the ranges of parameters studied here.

6.0 Skin Friction and Rate of Heat transfer

The physical quantities of this problem are the Skin friction coefficient (c_f) and the Nusselt number (Nu) which is heat transfer at the wall are defined by

$$c_f = \frac{T_f}{\rho u_\omega v_\omega} = \frac{d^2}{d\eta^2} f(0), \ \tau_f = \mu \frac{df}{d\eta}\Big|_{\eta=0}$$

$$Nu = \frac{q_\omega v}{(T_\omega - T_\infty) K v_\omega} = \frac{-d}{d\eta}\theta(0), \ q_\omega = -K\frac{dT}{dy}\Big|_{y=0},$$

and K is the thermal conductivity of the fluid.

In order to test the accuracy of the present method, the results were compared with those available in the literature, when $\phi = 0$. The wall heat transfer $\theta'(0)$ and the wall shear stress $f''(0)$ for the present problem with constant viscosity and $\Pr = 0.71$ are -0.4541 and -1.0 respectively. The corresponding quantities calculated by the present method are -0.454326 and -1.0. The comparison is satisfactory and this happens for other Pr numbers. In contrast to the above numerical solution presented here, the Prandtl number used is one corresponding to the one for plasma ($\Pr = 0.71$). However in the energy transformed equation (10) the Prandtl number has been assumed constant, we calculated Prandtl number at ambient temperature from the $\Pr = \frac{v_a}{\alpha}$. In the table below, the wall shear stress and the wall heat transfer are given for ambient Prandtl number 0.71 for $|\theta_e| \geq 1$. In Table 1 the result by Asterios Pantokratos [12] has also been included for comparison. The Prandtl number at the wall ($\Pr_w$) is also included in the last column of the table.

Table 1: Values of $f''(0)$ and $\theta'(0)$ for $\Pr = 0.71$ and $|\theta_e| \geq 1$.

	$f''(0)$			$\theta'(0)$			
θ_e	Present work	Pantokratos [12]	Differ-ence %	Present work	Pantokratos [12]	Differ-ence %	$\Pr_w$
-10	-1.0572	-1.0666	<1	-0.4487	-0.4487	<1	0.64
-8	-1.0701	-1.0775	<1	-0.4447	-0.4447	<1	0.63
-6	-1.0915	-1.0992	<1	-0.4441	-0.4442	<1	0.61
-4	-1.1411	-1.1414	<1	-0.4410	-0.4408	<1	0.55
-2	-1.2595	-1.2579	<1	-0.4415	-0.4417	<1	0.47
-1	-1.4542	-1.4592	<1	-0.3982	-0.3980	<1	0.34
2	-0.6474	-0.6502	<1	-0.5248	-0.5247	<1	1.41
4	-0.8456	-0.8467	<1	-0.5059	-0.5059	<1	0.94
6	-0.8980	-0.9047	<1	-0.4693	-0.4691	<1	0.84
8	-0.9245	-0.9320	<1	-0.4658	-0.4657	<1	0.81
10	-0.9235	-0.9460	<1	-0.4639	-0.4653	<1	0.79

Table 2: Values of $f''(0)$ and $\theta'(0)$ for $\Pr = 0.71$ and $|\theta r| \leq 1$.

	$f''(0)$			$\theta'(0)$			
θ_e	Present work	Pantokratos [12]	Differ-ence %	Present work	Pantokratos [12]	Differ-ence %	$\Pr_w$
-0.25	-2.2872	-2.2892	<1	-0.3041	-0.2907	4.4	0.64
-0.1	-3.3605	-3.3655	<1	-0.2242	-0.2078	7	0.63
-0.05	-4.6206	-4.6250	<1	-0.2131	-0.1601	25	0.61
-0.01	-10.063	-10.0875	<1	-0.1563	-0.0825	47	0.55

7.0 Results and Discussion

Viscosity has the SI unitof Pascal seconds which is called the Poiseuille. More commonly used is the dyne sec/cm^2 which is called Poise. One Pa s is 10 Poise. The Poise is used in this analysis because of its more common usage. These viscosities are at 20°C except for the blood and blood plasma which are at body temperature, 37°C, and for steam which is at 100°C.It is seen from Table 1, (case $\phi = 0$) that the wall shear stress and the wall heat transfer calculated from both methods, are in

agreement for $|\theta_r| \geq 1$. For the wall shear stress the two systems gives a very close result, but for the wall heat transfer the differences between the two methods diverges when $|\theta_r| \leq 1$. As $|\theta_r|$ decreases the differences in the wall heat transfer increases, as shown in Table 2. We can therefore conclude from the two tables that when $\theta_r \to \infty$ the fluid viscousity becomes equal to ambient viscousity. Further, negative values of viscosity parameter make $(T_w - T_\infty)$ negative, and $(T_w - T_\infty)$ is always negative for an incompressible fluid therefore we have calculated $\theta(\eta)$ and $f'(\eta)$ for realistic positive values of θ_r varying between 1.0and15 ranging from light oil ($\theta r = 1.1$) to glycerine ($\theta_r = 14.9$).
Both conduction and convection happen in fluids. As heat transfer through both processes reduces the temperature difference, they can be considered as competing against each other in transferring heat. There are many different types of fluids, such as air, water, oil, or mercury. The rates of conduction and convection vary in different fluids. Sometimes, conduction dominates. Other times, convection dominates. The Prandtl number is a parameter that can be used to roughly determine which process will "win": Typical values for *Pr* are: around 0.7 for air, and many other gases, around 7 for water, around 7×10^{21} for Earth's mantle, between 100 and 40,000 for engine oil, between 4 and 5 for R-12 refrigerant, around 0.015 for mercury. For mercury, heat conduction is very effective compared to convection: thermal diffusivity is dominant. For engine oil, convection is very effective in transferring energy from an area, compared to pure conduction: momentum diffusivity is dominant.In the boundary layer the Prandtl number ($\Pr = 0.71$) is known to be valid for plasma which is the nature of fluid in MHD.
We presents next, the effect of the parameters,ϕ, Ha, Grt, θ_r andα on the transverse velocity and temperature fields. Thevariation of viscosity parameter θ_r means the variation of fluid viscositywith respect to the fluid temperature, and our aim is to show the natureof fluid velocity and temperature in the presence of uniform magneticfield under the action of variable viscosity.

Figures (1-6) are plotted for velocity distribution $f'(\eta)$ for various values ofparameters,ϕ, Ha, Grt, θ_r andα respectively.

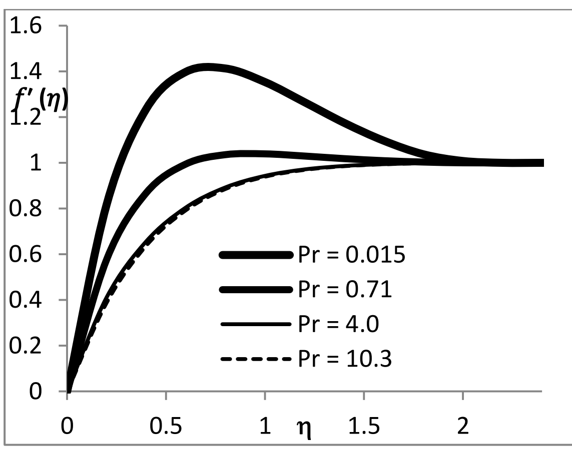

Figure 1: Transverse velocity $f'(\eta)$profile for different Values of Prandtl number

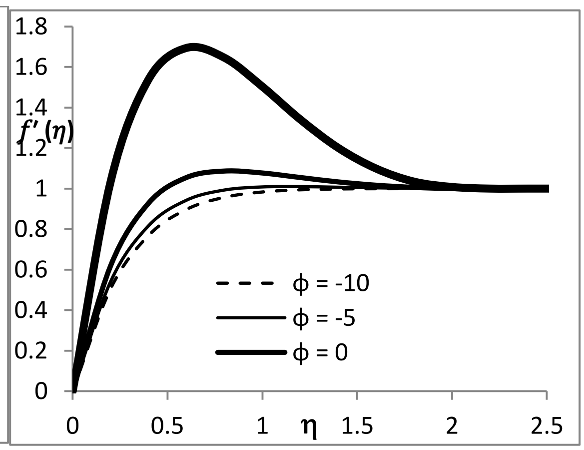

Figure 2: Transverse velocity $f'(\eta)$profile for different Values of Heat Parameter

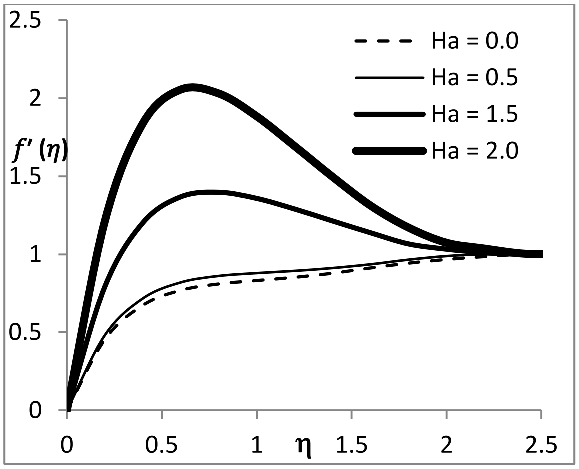

Figure 3: Transverse velocity $f'(\eta)$profile for ofHartmann Number

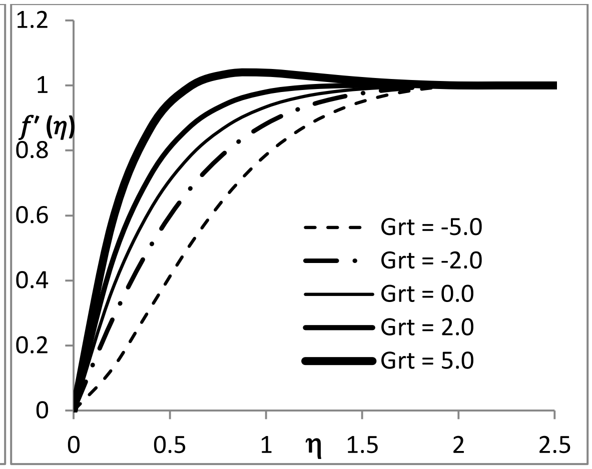

Figure 4: Transverse velocity $f'(\eta)$profile for different values different Values of Grashof Number

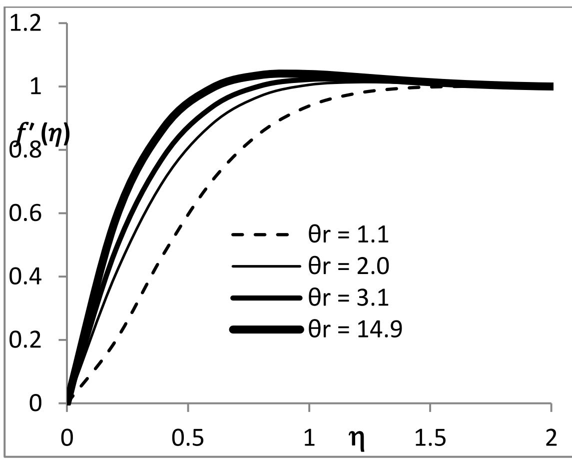

Figure 5: Transverse velocity $f'(\eta)$profilefor ofViscosity parameter

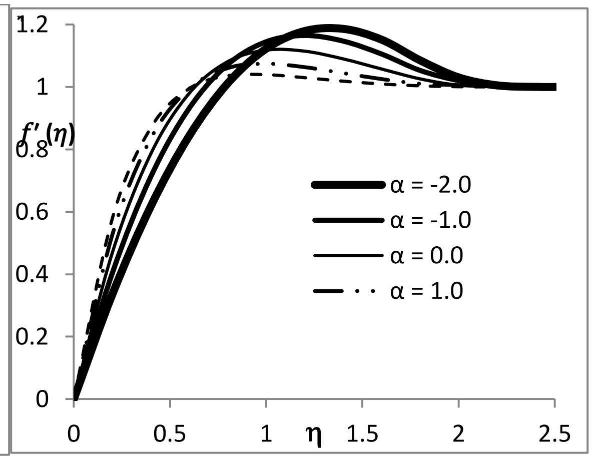

Figure 6: Transverse velocity profile for different values different values of injection/suctionparameter

Figures (7 and 8) are plotted for temperature distribution $\theta(\eta)$ various values of parametersϕ andα respectively.

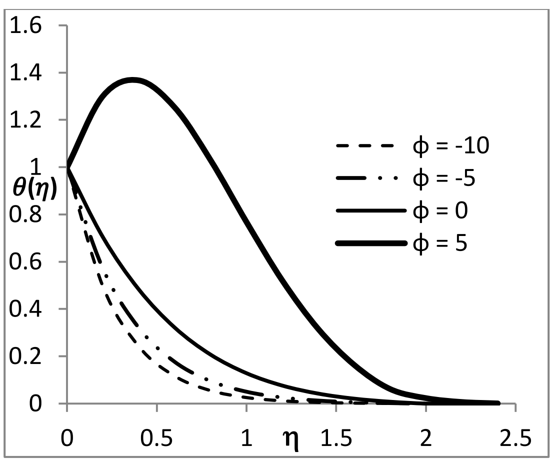

Figure 7: Temperature distribution(η) for different values ofheat source/sink

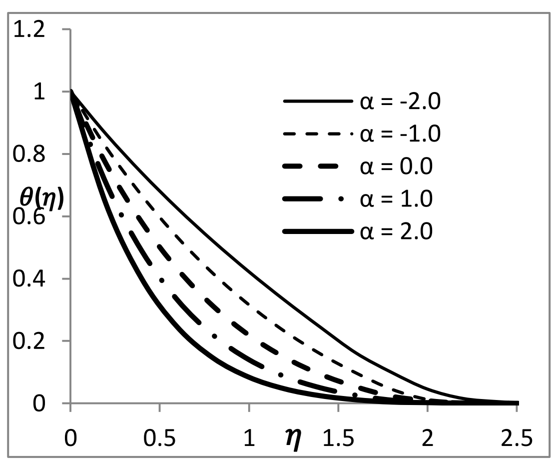

Figure 8: Temperature distribution(η) for different values of injection/suction parameter

Following are the deductions from the figures:

1.Figure 1 shows the variation of transverse velocity against positionfor differentvalues Prandtl number Pr. For lower values of Prandtl number, as in the case of mercury, heat conduction is very effective compared to convection, thermal diffusivity is dominant. And for higher values of Prandtl number, convection is very effective in transferring energy from an area, compared to pure conduction, momentum diffusivity is dominant. Thus velocity reduces as convection takes over the mode of heat transfer. Hence velocity is higher in conduction compare to convection as could be seen in Fig. 1.It is observed that the change of $f(\eta)$ with theincrease of θ_e from 5 to 30 is significant withinthe boundary layers (η ∈ (0, 2) approximately)the velocity increases as θ_e increases, and the transverse velocity boundary layer becomes thicker as θ_e increases. The same is observed in the temperature field (Figure 2) as well, the temperature boundary layer increases as viscousity parameter increases. In Figure 3, variation of $f'(\eta)$ for various values of viscousityparameter θ_e. It could be seen that axial velocityreduces as viscosity parameter increases. The boundary layer becomes thickened as viscosity increases.

2.Figure (4) shows variation of $\theta(\eta)$ with the increases in heat generation. It could be seen that thefluid temperature reduces as heatgeneration increases. Furthermore, the maximum temperature is the temperature at the wall for this combination of control parameters. While in Figures 5and 6, we displayed the profiles for both the transverse and axial velocities at different values of heat generation/absorption coefficient. Figure 5depict the effect of heat generation/absorption parameter on transverse velocity. It could be seen that heat generation $(\phi < 0)$ reduces the fluid velocity, while the transverse fluid velocity increases with increase in heat absorption $(\phi > 0)$. Moreover, the effect of heat absorption is more pronounced than that of heat generation, the boundary layer increases with increase in heat absorption. In Figure 6, the effect of heat generation on axial velocity, it could be deduced that as heat generation increases the axial velocity reduces.

3. Figures 7 - 9 shows the variation of velocity and temperature fields with thermal buoyancy.Figure 7 shows that increase in thermal buoyancy $Gr\tau$, increases the velocity but,the boundary layer reduces with increase in thermal buoyancy. In Figure 8, the effect of thermal buoyancy is not significant close to the plate, but more pronounced as the fluid moves away from the plate. Also increase in thermal buoyancy reduces the temperature boundary layer. While in Figure 9 we show that thermal buoyancy increases the axial velocity with the effect more pronounced for higher value of Grashof number.

4. Figures 9 and 10 depict the effect of Hartmann's number M on the velocities and temperature fields. We could see from Figure 9 that Hartmann's number increases the transverse velocity while from Figure 12, it is observed that increase in Hartmann's number reduces the axial velocity. In addition, application of a magnetic field to an electrically conducting fluid produces a drag – like force called the Lorentz force. This force causes reduction in the fluid axial velocity. And in figure 10, we observed that increase in Hartmann's number brings about increase in the fluid temperature with little effect on the temperature boundary layer.

5. The tables (I) and (II) show the values of $f''(0)$ and $\theta'(0)$which arethe factors for skin friction and rate of heat transfer respectivelyat η = 0 for Pr = 0.71.We observed that themagnitude of $f''(0)$ increases with the increase of θ_e as θ_e changes from −10 to −0.01 as well as from 2 to 10on the otherhand. In table III, we Show the effect of M , $Gr\tau$, θ_e and ϕ on the wall shear stress and the wall heat transfer respectively. Is it observed that as viscousity parameter θ_e increases, the wall shear stress increases while, the wall heat transfer decreases. It could also be seen from the table that Hartmann's number M which produces a drag – like force called the Lorentz force bring a about reduction in the wall shear stress and the wall heat transfer as it increases. Heat generation reduces the skin friction and increases the wall heat transfer increases, but thermal Buoyancy $Gr\tau$ increases both the wall shear stress and the wall heat transfer as it increases.

8.0 Conclusion

From the above discussions we can draw the following conclusions.

(1) The velocity boundary layer increasesas viscousity parameter increases.
(2) Axial velocity decreases with the increase of viscosity parameter
(3) The fluid temperature reduces as heat generation increases
(4) Boundary layer increases with increase in heat absorption

(5) Increase in thermal buoyancy $Gr\tau$, increases the velocity but, the boundary layer reduces (6) with increase in thermal buoyancy

(7) Increase in Hartmann's number reduces the axial velocity

(8) The skin friction increases with the increase of viscosity parameter(from 10 to 30).

(9) The heat transfer decreases with the increase of viscosity parameterat the value of Prandtl numberPr = 0.71. For smallvalues of the viscosity parameter, the heat transfer is lessdependenton Prandtl number.

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