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Radiative Fluid Flow over a Vertical Porous Channel Under Optically Thick Approximation in the Presence of MHD

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Abstract

Numerous non linear equations that come up in real life situations defy analytical solutions; hence numerical methods are desirable to obtain the solutions of such equations. In this study we use the Newton scheme method from Taylor series to solve fourth order non linear problem. A mathematical software MATLAB was used to solve the system of equations to obtain the unknown constants. The velocity profiles and temperature profile are studied for different physical parameters likeMagnetohydrodynamic M, Porous term P, Radiation F and thermal Grashof number Ga.The results obtained after computation taking into cognizance the parameters present shows that the Magnetic andPorous parameters increases with increasing Velocity, while the trend reverses with radiation and thermal Grashof number under optically thick approximation..

Keywords:MHD,Porous Media, Recurrence relation, vertical plate, radiation, flow rate.

1.0 Introduction

The role of thermal radiation is of major importance in some industrial applications such asglass production and furnace design and in space technology applications, such as cosmicalflight aerodynamics rocket, propulsion systems, plasma physics and space craft reentryaerothermodynamics which operate at high temperatures. Several authors have also studied thermal radiating MHD boundary layer flows with applications in astrophysical fluid dynamics. Mosa [1] discussed one of the first models for combined radiative hydromagnetic heat transfer, considering the case of free convective channel flows with an axial temperature gradient. Analytical model of MHD mixed convective Radiating fluid with viscous dissipative heat was investigated by Sahin Ahmed and Abdul Batin [2]. Anwar Beg and Ghosh [3] studied the steady and unsteady magnetohydrodynamic (MHD) free and forced convective flow of electrically, conducting, Newtonium fluid in the presence of appreciable thermal radiation heat transfer and surface temperature oscillation. In all the cases the properties of the final product depend to a great extent on the rate of cooling and the processes of stretching as explained by Karwe and Jaluria [4, 5]. By drawing such strips or filaments in an electrically conducting fluid subjected to a magnetic field, the rate of cooling can be controlled and a final product of desired characteristics can be achieved. Another interesting application of hydromagnetics to metallurgy lies in the purification of molten metals from non-metallic inclusions by the application of a magnetic field

The study of heat transfer has become important industrially for determining the quality of the final product. The dynamics of the boundary layer flow over a moving continuous solid surface originated from the pioneer work of Sakiadis [6] who developed a numerical solution for the boundary layer flow field of a stretched surface, many authors have attacked this problem to study the hydrodynamic and thermal boundary layers. Laminar mixed convection boundary layers induced by a linearly stretching permeable surface was studied by Mohamed Ali and Fahh Al-Yousef [7]. Uwanta [8] studied the oscillating free convective flow of a conducting fluid with a suspension of spherical particles.

A number of industrially important fluids such as foods, polymers, molten plastics, slurries and pulps display non-Newtonian fluid behaviour. Non-Newtonian fluids exhibit a non-linear relationship between shear stress and shear rate. Its worth mentioning here that many of the inelastic non-Newtonian fluids encountered in chemical engineering processes, are known to follow the empirical Ostwaled-de-Waele model or the so-called ‘‘power-law model’’ in which the shear stress varies

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according to a power function of the strain rate. The convection heat and mass transfer from surface embedded in Non-Newtonian fluids find several applications in geothermal engineering, petroleum recovery, filtration processes, oil extraction, solid matrix heat exchangers, thermal insulation, storage of nuclear waste materials, packed beds, porous insulation, beds of fossil fuels, nuclear waste disposal, resin transfer modelling, etc. Also, it is worth mentioning that non-Darcian forced flow boundary layers form a very important group of flows, the solution of which is of great importance in many practical applications such as in biomechanical problems, in filtration transpiration cooling and geothermal.Anwar Hossain and Wilson [9] discussed the Natural convection flow in a fluid-saturated porous medium enclosed by non-isothermal walls with heat generation. Numerical modelling of non-Newtonian fluid flow in a porous medium using a three dimensional periodic array was presented by Inoue and Nakayama [10]. Uwanta [11] analysed Mass transfer of free convective flow over a vertical plate with heat sink and jumped wall temperature. All these studies were concerned with steady flows. Pascal [12] presented similarity solutions to some unsteady flows of non-Newtonian fluids of power law behaviour. Pascal and Pascal [13] studied the non-linear effects on some unsteady non-Darcian flows through porous media. Not too long, Kok Siong Chiem and Yong Zhao [14] studied the problem of numerical study of steady/unsteady flow and heat transfer in porous media using a characteristics-based matrix-free implicit FV method on unstructured grids. The purpose of this paper is to study the problem of unsteady free convection with heat and mass transfer from an isothermal vertical flat plate to a non-Newtonian power-law fluid saturated porous medium. The Darcy-Brinkman-Forchheimer model which includes the effects of boundary and inertia forces was employed. The dimensionless non-linear partial differential equations were solved numerically using an explicit finite-difference scheme.The values of friction factor and heat transfer coefficient were determined for steady and unsteady free convection. Bestman [15] studied free convection boundary layer flow with simultaneous heat and mass transfer in a porous medium when the boundary wall moves in its own plane with suction. Hossain [16] investigated the effect of uniform transpiration rate on the heat and mass transfer characteristics in mixed convection flow of a viscous incompressible fluid along a vertical permeable plate. Anuar Ishak [17] analyzed unsteadyMHD flow and heat over a Stretching plate.Hossain et al. [18] determined the effect of radiation on natural convection flow of an optically thick viscous incompressible flow past a heated vertical porous plate with a uniform surface temperature and a uniform rate of suction where radiation is included by assuming the Rosseland diffusion approximation. Rahman and Mulolani [19] examined natural convection flow over a semi-infinite vertical plate at constant species concentration. Chamkha [20] considered the problem of steady, hydromagnetic boundary layer flow over an accelerating semi infinite porous surface in the presence of thermal radiation, buoyancy and heat generation or absorption. Hossain et al. [21] numerically investigated the effect of thermal radiation on natural convection flow along a uniformly heated vertical porous plate with variable viscosity and uniform suction velocity. Abel et al. [22] investigated numerically natural convective flows, heat and mass transfer due to the combined effect of thermal and species diffusion in viscoelastic fluid. Devi and Kandasamy [23] analyzed the effects of a chemical reaction, heat and mass transfer on an accelerating surface with a heat source and thermal stratification in the presence of suction and injection. Chamkha and Quadri [24] considered simultaneous heat and mass transfer by natural convection from a vertical semi-infinite plate embedded in a fluid saturated porous medium in the presence of wall suction or injection, heat generation or absorption effects, porous medium inertial and thermal dispersion effects. In general, the porous medium thermal dispersion effects increase the temperature of the fluid causing higher flow rates along the surface. However, this seems not to be the case in their study, as the peak values of the temperature and velocity profiles were lowered as porous medium thermal dispersion parameter increases. Saha and Hossain [25] numerically studied the problem of laminar doubly diffusive free convection flows adjacent to a vertical surface in a stable thermally stratified medium. Azizi et al. [26] investigated numerically the effects of thermal and buoyancy forces on both upward flow and downward flow of air in a vertical parallel-plates channel. Shateyi et al. [27] studied magnetohydrodynamic flow past a vertical plate with radiative heat transfer. Shateyi, S. [28] examined thermal radiation and buoyancy effects on heat and mass transfer over a semi-infinite stretching surface with suction and blowing.

The phenomenon of free or natural convection arises in fluids when temperature changes cause density variations leading to buoyancy forces acting on the fluid particles. Such flows which are driven by temperature differences abound in nature and have been studied extensively because of its applications in engineering, geophysical and astrophysical environments. Comprehensive literature on various aspects of free convection flows and its applications could be found in Ghoshdastidar [29], Nield and Bejan [30]. In particular Ghoshdastidar [29] gave various areas of applications of free convection flow such as those found in heat transfer from pipes and transmission lines as well as from electronic devices, heat dissipation from the coil of a refrigerator unit to the surrounding air, heat transfer from a heater to room air, heat transfer in nuclear fuel rods to the surrounding coolant, heated and cooled enclosures, quenching, wire – drawing and extrusion, atmospheric and oceanic circulation. Buoyancy – driven flows over porous materials enhances heat transfer. These are encountered in a wide range of thermal engineering applications such as in geothermal systems, oil extraction; ground water pollution, thermal insulation, heat exchangers, storage of nuclear wastes, packed bed catalytic reactors. Most of these application exhibit high temperature phenomena such as those found in astrophysical environments, solar power technology.Taiwo and Ogunlaran[31]studied Numerical solution of fourth order linear ordinary differentialequations by cubic spline collocation tau method.

Recently, Ibrahim*et al.* [32,33]investigated Radiative effect on MHD fluid flow in a vertical channel under optically thick approximation and studied radiation effect on a porous media under optically thick approximation using Newton schememethod from Taylor series.

2.0 Formulation of the Problem

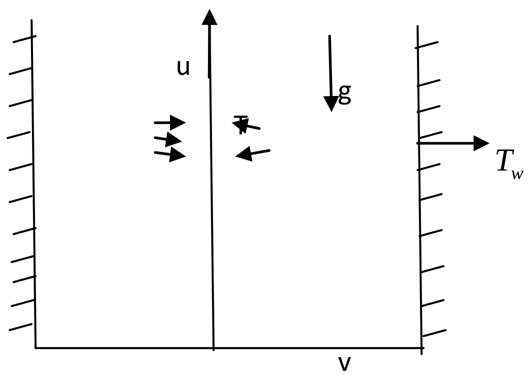

Figure 1: Physical model of the problem

Here The problem of thermal radiation effect on a porous media under optically thick approximationformulated, analysed and solved numerically. The x - axis is taken along the plate in the vertically upward direction and also the y -axis is taken normal to the plate. That the flow is fully developed, the velocity and temperature fields are symmetrical about the central line of the channel in a magnetic region. The temperature of the walls is the same and is maintained at a constant temperature. The viscosity, the thermal conductivity and specific heat are independent of temperature and the essential influence of the variation in density is included in the body force term. Steady flow equations are momemtum and energy equation.

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}-\frac{vu}{k}-\frac{\sigma\beta_0^2 u}{\rho}+g\beta\left(T_w-T\right)+v\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \quad (1)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho_0 C_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho_0 C_p}\frac{\partial q_r}{\partial y} \quad (2)$$

where u and v are the velocities of the fluid, T is the fluid temperature, β is the thermal expansion of the fluid ρ_0 is the fluid density, C_p is the specific heat capacity, v is the viscosity of the fluid, k is the thermal conductivity and the temperature of the wall T_w, K permeability of the porous medium parameter and v' the kinematic viscosity,

In this research work the mathematical formulation has Porous and Magnetic term which is not included in the work of Ibrahim [34]where α the thermal diffusivity and q_r is the radiative, using the Rosseland differential approximation.

$$\frac{\partial q_r}{\partial y}=-\frac{16}{3\alpha}\sigma T^3\frac{\partial T}{\partial y} \quad (3)$$

The boundary conditions are;

$$\left.\begin{array}{lll} T=0 & u=0 & y=b \\ T''=0 & u''=0 & y=-b \end{array}\right\} \quad (4)$$

On introducing the following non-dimensional quantities

$$\left.\begin{array}{l} y=bY,\ u=\dfrac{\alpha U}{b},\ T=-\tau\phi,\ \ P=\dfrac{b^2}{k} \\ F=bL\tau^3,\ \ G_a=\dfrac{g\beta b^3\tau}{v\alpha},\ \ M=\dfrac{\sigma b^3\beta_0^2}{\rho v} \end{array}\right\} \quad (5)$$

Substituting the non-dimensional quantities of equation (5) into (1) to (2), leads to

$$\frac{d^2U}{dY^2}-\left(M+P\right)U=G_a\phi-\gamma \tag{6}$$

$$U=F\phi^3\frac{\partial\phi}{\partial Y}-\frac{\partial^2\phi}{\partial Y^2} \tag{7}$$

Where $\gamma=-\frac{b^3}{v\alpha}\left(\frac{1}{\rho_0}\frac{\partial p}{\partial x}\right)$

Equation (6) and (7) leads to

$$\frac{\partial^4\phi}{\partial Y^4}-F\phi^3\frac{\partial^3\phi}{\partial Y^3}-\left(6F\phi^2+M+P\right)\frac{\partial^2\phi}{\partial Y^2}-\left(6F\phi-\left(M+P\right)F\phi^3\right)\frac{\partial\phi}{\partial Y}+G_a\phi=\gamma \tag{8}$$

$$\phi(1)=0=\phi''(-1),\ U(1)=0=U''(-1) \tag{9}$$

3.0 Solution to the Problem

To solve equations (8), subjected to the boundary conditions of (9), the solutions are obtained for temperature and velocity flow. using the Newton scheme method from Taylor series to solve the fourth order non linear problem.

$$G+\Delta\phi\frac{\partial G}{\partial\phi_n}+\Delta\phi'\frac{\partial G}{\partial\phi_n'}+\Delta\phi''\frac{\partial G}{\partial\phi_n''}+\Delta\phi'''\frac{\partial G}{\partial\phi_n'''}+\Delta\phi^{iv}\frac{\partial G}{\partial\phi_n^{iv}}=0 \tag{10}$$

Thus from (8) we have

$$\frac{\partial G}{\partial\phi_n}=-3F\phi_n{}^2\phi_n'''-12F\phi_n\phi_n''-\left(6F-3(M+P)F\phi_n^2\right)\phi_n'+G_a \tag{11}$$

$$\frac{\partial G}{\partial\phi_n'}=-\left(6F\phi_n-\left(M+P\right)F\phi_n^3\right) \tag{12}$$

$$\frac{\partial G}{\partial\phi_n''}=-\left(6F\phi_n^2+M+P\right) \tag{13}$$

$$\frac{\partial G}{\partial\phi_n'''}=-F\phi_n^3 \tag{14}$$

$$\frac{\partial G}{\partial\phi_n^{iv}}=1 \tag{15}$$

Substituting equations (11) to (15) into equation (10) we obtain

$$\begin{aligned}&\Delta\phi_n\left(-3F\phi_n{}^2\phi_n'''-12F\phi_n\phi_n''-\left(6F-3(M+P)F\phi_n^2\right)\phi_n'+G_a\right)\\&+\Delta\phi_n'\left(-\left(6F\phi_n-\left(M+P\right)F\phi_n^3\right)\right)+\Delta\phi_n''\left(-\left(6F\phi_n^2+M+P\right)\right)\\&+\Delta\phi_n'''\left(-F\phi_n^3\right)+\Delta\phi_n^{iv}(1)=0\end{aligned} \tag{16}$$

Expanding equations (16) where

$$\Delta\phi_n=\phi_{n+1}-\phi_n,\quad \Delta\phi_n'=\phi_{n+1}'-\phi_n',\quad \Delta\phi_n''=\phi_{n+1}''-\phi_{n,}''$$

$$\Delta\phi_n'''=\phi_{n+1}'''-\phi_n''',\quad \Delta\phi_n^{iv}=\phi_{n+1}^{iv}-\phi_n^{iv}$$

$$\left(\phi_{n+1}-\phi_n\right)\left(-3F\phi_n{}^2\phi_n'''-12F\phi_n\phi_n''-\left(6F-3(M+P)F\phi_n^2\right)\phi_n'+G_a\right)$$
$$+\left(\phi_{n+1}'-\phi_n'\right)\left(-\left(6F\phi_n-\left(M+P\right)F\phi_n^3\right)\right)+\left(\phi_{n+1}''-\phi_n''\right)\left(-\left(6F\phi_n^2+M+P\right)\right)$$
$$+\left(\phi_{n+1}'''-\phi_n'''\right)\left(-F\phi_n^3\right)+\left(\phi_{n+1}^{iv}-\phi_n^{iv}\right)(1)=0$$
$$\Rightarrow -3F\phi_n{}^2\phi_n'''\phi_{n+1}-12F\phi_n\phi_n''\phi_{n+1}-6F\phi_n'\phi_{n+1}+3MF\phi_n^2\phi_n'\phi_{n+1}+3PF\phi_n^2\phi_n'\phi_{n+1}$$
$$-G_a\phi_{n+1}+3F\phi_n{}^2\phi_n'''\phi_n+12F\phi_n\phi_n''\phi_n+6F\phi_n'\phi_n+3MF\phi_n^2\phi_n'\phi_n+3PF\phi_n^2\phi_n'\phi_n$$
$$+G_a\phi_n-6F\phi_n\phi_{n+1}'-MF\phi_n^3\phi_{n+1}'-PF\phi_n^3\phi_{n+1}'+6F\phi_n\phi_n'-MF\phi_n^3\phi_n'-PF\phi_n^3\phi_n'$$
$$-6F\phi_n^2\phi_{n+1}''-M\phi_{n+1}''+P\phi_{n+1}''+6F\phi_n^2\phi_n''+M\phi_n''+P\phi_n''$$
$$-F\phi_n^3\phi_{n+1}'''+F\phi_n^3\phi_n'''+\phi_{n+1}^{iv}-\phi_n^{iv}=0 \qquad (17)$$

Collecting the terms involving

$\phi_{n+1},\ \phi_{n+1}',\ \phi_{n+1}'',\ \phi_{n+1}'''\ and\ \phi_{n+1}^{iv}$

On the L.H.S and the terms involving

$\phi_n,\ \phi_n',\ \phi_n'',\ \phi_n'''$ neglecting ϕ_n^{iv}

On the R.H.S gives

$$\phi_{n+1}^{iv}-\left(F\phi_n^3\right)\phi_{n+1}'''-\left(6F\phi_n^2-M-P\right)\phi_{n+1}''-\left(6F\phi_n+\left(M+P\right)F\phi_n^3\right)\phi_{n+1}'$$
$$-\left(3F\phi_n{}^2\phi_n'''+12F\phi_n\phi_n''+\left(6F-3(M+P)F\phi_n^2\right)\phi_n'+G_a\right)\phi_{n+1}$$
$$=-3F\phi_n{}^2\phi_n'''-12F\phi_n\phi_n''-\left(6F-3(M+P)F\phi_n^2\right)\phi_n' \qquad (18)$$

Where ϕ_n is our initial solution and ϕ_{n+1} is the assumed solution.
Let

$$\phi_{N,\,n+1}\left(y\right)=\sum_{i=0}^{6}a_iy^i$$

Considering N=6

$$\phi_{n+1}(y)=a_0+a_1y+a_2y^2+a_3y^3+a_4y^4+a_5y^5+a_6y^6$$
$$\phi_{n+1}'(y)=a_1+2a_2y+3a_3y^2+4a_4y^3+5a_5y^4+6a_6y^5$$
$$\phi_{n+1}''(y)=2a_2+6a_3y+12a_4y^2+20a_5y^3+30a_6y^4$$
$$\phi_{n+1}'''(y)=6a_3+24a_4y+60a_5y^2+120a_6y^3$$
$$\phi_{n+1}^{iv}(y)=24a_4+120a_5y+360a_6y^2 \qquad (19)$$

Putting (19) into (18)

$$24a_4+120a_5y+360a_6y^2-\left(F\phi_n^3\right)\left(6a_3+24a_4y+60a_5y^2+120a_6y^3\right)$$
$$-\left(6F\phi_n^2-M-P\right)\left(2a_2+6a_3y+12a_4y^2+20a_5y^3+30a_6y^4\right)$$
$$-\left(6F\phi_n+\left(M+P\right)F\phi_n^3\right)\left(a_1+2a_2y+3a_3y^2+4a_4y^3+5a_5y^4+6a_6y^5\right)$$
$$-\left(3F\phi_n{}^2\phi_n'''+12F\phi_n\phi_n''+\left(6F-3(M+P)F\phi_n^2\right)\phi_n'+G_a\right)\left(a_0+a_1y+a_2y^2+a_3y^3+a_4y^4+a_5y^5+a_6y^6\right)$$
$$=-3F\phi_n{}^2\phi_n'''-12F\phi_n\phi_n''-\left(6F-3(M+P)F\phi_n^2\right)\phi_n' \qquad (20)$$

Simplifying by collecting terms in $a_0,\ a_1,\ a_2,\ a_3,\ a_4,\ a_5\ and\ a_6$ we obtain.

$$-a_0\left(3F\phi_n{}^2\phi_n'''+12F\phi_n\phi_n''+\left(6F-3(M+P)F\phi_n^2\right)\phi_n'+G_a\right)$$
$$-a_1\left\{\left(6F\phi_n+(M+P)F\phi^3\right)-y\left(3F\phi_n{}^2\phi_n'''+12F\phi_n\phi_n''+\left(6F-3(M+P)F\phi_n^2\right)\phi_n'+G_a\right)\right\}$$
$$-a_2\left\{\begin{matrix}\left(12F\phi_n{}^2-2M-2P\right)+\left(12F\phi_n-2(M+P)F\phi_n{}^3\right)y\\ +y^2\left(3F\phi_n{}^2\phi_n'''+12F\phi_n\phi_n''+\left(6F-3(M+P)F\phi_n^2\right)\phi_n'+G_a\right)\end{matrix}\right\}$$
$$-a_3\left\{\begin{matrix}6F\phi_n{}^3+\left(36F\phi_n{}^2-6M-6P\right)y+\left(18F\phi_n+3(M+P)F\phi_n{}^3\right)y^2+\\ y^3\left(3F\phi_n{}^2\phi_n'''+12F\phi_n\phi_n''+\left(6F-3(M+P)F\phi_n^2\right)\phi_n'+G_a\right)\end{matrix}\right\}$$
$$-a_4\left\{\begin{matrix}24yF\phi_n{}^3-24+y^2\left(72F\phi_n{}^2-12M-12P\right)+y^3\left(24F\phi_n+4(M+P)F\phi_n{}^3\right)\\ +y^4\left(3F\phi_n{}^2\phi_n'''+12F\phi_n\phi_n''+\left(6F-3(M+P)F\phi_n^2\right)\phi_n'+G_a\right)\end{matrix}\right\}$$
$$-a_5\left\{\begin{matrix}-120y+60y^2\left(F\phi_n{}^3\right)+y^3\left(120F\phi_n{}^2-20M-20P\right)+y^4\left(30F\phi_n+5(M+P)F\phi_n{}^3\right)\\ +y^5\left(3F\phi_n{}^2\phi_n'''+12F\phi_n\phi_n''+\left(6F-3(M+P)F\phi_n^2\right)\phi_n'+G_a\right)\end{matrix}\right\}$$
$$-a_6\left\{\begin{matrix}-360y^2+120y^3F\phi_n{}^3+y^4\left(180F\phi_n{}^2-30M-30P\right)+y^5\left(36F\phi_n+6(M+P)F\phi_n^3\right)\\ +y^6\left(3F\phi_n{}^2\phi_n'''+12F\phi_n\phi_n''+\left(6F-3(M+P)F\phi_n^2\right)\phi_n'+G_a\right)\end{matrix}\right\}$$
$$=-3F\phi_n{}^2\phi_n'''-12F\phi_n\phi_n''-\left(6F-3(M+P)F\phi_n^2\right)\phi_n'+\tau_1T_6(y) \qquad (21)$$

We now collate equations (21) at point $y=y_i$, where

$$y_i=a+\frac{(b-a)i}{N},\ i=1,2,........,N-1$$

In the problem under consideration, N=6, a =-1 and b=+1.
The boundary conditions are

$$\phi(1)=0=\phi''(-1),\ U(1)=0=U''(-1)$$

Thus, we obtain five equations from (21).

Let ϕ_n be our initial solution

$$\phi_n=\frac{\gamma}{G_a}\left[1-\frac{b_2^2\cosh b_1y}{b_2^2-b_1^2\cosh b_1}+\frac{b_1^2\cosh b_2y}{b_2^2-b_1^2\cosh b_2}\right]$$
$$\phi_n'=\frac{\gamma}{G_a}\left[-\frac{b_1b_2^2\sinh b_1y}{b_2^2-b_1^2\cosh b_1}+\frac{b_2b_1^2\sinh b_2y}{b_2^2-b_1^2\cosh b_2}\right]$$
$$\phi_n''=\frac{\gamma}{G_a}\left[-\frac{b_1^2b_2^2\cosh b_1y}{b_2^2-b_1^2\cosh b_1}+\frac{b_2^2b_1^2\cosh b_2y}{b_2^2-b_1^2\cosh b_2}\right]$$
$$\phi_n'''=\frac{\gamma}{G_a}\left[-\frac{b_1^3b_2^2\sinh b_1y}{b_2^2-b_1^2\cosh b_1}+\frac{b_2^3b_1^2\sinh b_2y}{b_2^2-b_1^2\cosh b_2}\right]$$

We now substitute for $\phi_n,\ \phi_n',\ \phi_n'',\ \phi_n'''$ into (21) and then solve the equation simultaneously to obtain the unknowns $a_0,\ a_1,\ a_2,\ a_3,\ a_4,\ a_5,a_6\ and\ \tau_2$ using Gaussian elimination method with partial pivoting using MATLAB software to obtain the unknown constants

$$\phi_6\left(y\right)=\sum_{1=0}^{6}a_i y^i+\tau_2 e^y$$

But

$$\phi_6(y)=a_0+a_1y+a_2y^2+a_3y^3+a_4y^4+a_5y^5+a_6y^6+\tau_2e^y$$

$$\phi_6(1)=a_0+a_1+a_2+a_3+a_4+a_5+a_6+\tau_2e^1=0$$

$$\phi_6(-1)=a_0-a_1+a_2-a_3+a_4-a_5+a_6+\tau_2e^{-1}=0$$

$$\phi_6''(1)=2a_2+6a_3+12a_4+20a_5+30a_6+\tau_2e^1=0$$

$$\phi_6''(-1)=2a_2-6a_3+12a_4-20a_5+30a_6+\tau_2e^{-1}=0$$

$$\begin{pmatrix} 6.8999 & 7.4433 & 18.4310 & -8.1498 & 60.1085 & -109.3592 & 394.5668 & 0\\ -86.8888 & -0.4444 & 16.8997 & -9.0435 & 30.0021 & -92.1935 & 67.8888 & 0\\ 48.3090 & 2.9677 & 0.4555 & -35.8996 & 49.3447 & -201.5656 & 701.7787 & 0\\ -24.5000 & -8.4444 & -23.9995 & -4.4454 & 14.0000 & 55.7799 & 49.7765 & 0\\ -28.5365 & -17.6767 & 4.5557 & 2.5567 & 44.5375 & 77.8888 & 202.8867 & 0\\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2.7183\\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & 0.3679\\ 0 & 0 & 2 & 6 & 12 & 20 & 30 & 2.7183\\ 0 & 0 & 2 & -6 & 12 & -20 & 30 & 0.3679 \end{pmatrix}$$

$$\times\begin{pmatrix} a_0\\ a_1\\ a_2\\ a_3\\ a_4\\ a_5\\ a_6\\ \tau_2 \end{pmatrix}=\begin{pmatrix} 0.6700\\ 0.3333\\ -103.2648\\ -0.3333\\ -0.6667\\ 0\\ 0\\ 0\\ 0 \end{pmatrix} \tag{22}$$

$$a_0=-0.7496,\quad a_1=9.0077,\quad a_2=2.3417\quad a_3=-4.6900$$

$$a_4=-0.0629\quad a_5=1.5905,\quad a_6=0.0867,\quad \tau_2=-3.1686$$

Determination of the Temperature profile, to obtain the solution for the temperature

$$\phi_6\left(y\right)=\sum_{1=0}^{6}a_i y^i+\tau_2 e^y$$

in the interval $-1\le y\le 1$ since the various values of a_i and τ_2 are known from the MatLab Program

Determination of the Velocity profile

The velocity distribution of the flow can be obtained from equation (6)

$$\frac{d^2U}{dY^2}-\left(M+P\right)U=G_a\phi-\gamma$$

$$U=-\left(\frac{\left(G_a\phi+\gamma\right)e^{-\sqrt{M+P}}}{M+P\left(e^{2\sqrt{M+P}}+e^{-2\sqrt{M+P}}\right)}\right)e^{y\sqrt{M+P}}$$

$$+\left(\frac{\left(G_a\phi+\gamma\right)e^{-\sqrt{M+P}}}{M+P\left(e^{2\sqrt{M+P}}+e^{-2\sqrt{M+P}}\right)}\right)e^{-y\sqrt{M+P}}-\frac{1}{M+P}\left(G_a\phi+\gamma\right) \tag{23}$$

$$U=-\left(\frac{\left(G_a\left(a_0+a_1y+a_2y^2+a_3y^3+a_4y^4+a_5y^5+a_6y^6+\tau_2e^y\right)+\gamma\right)e^{-\sqrt{M+P}}}{M+P\left(e^{2\sqrt{M+P}}+e^{-2\sqrt{M+P}}\right)}\right)e^{y\sqrt{M+P}}$$
$$+\left(\frac{\left(G_a\left(a_0+a_1y+a_2y^2+a_3y^3+a_4y^4+a_5y^5+a_6y^6+\tau_2e^y\right)+\gamma\right)e^{-\sqrt{M+P}}}{M+P\left(e^{2\sqrt{M+P}}+e^{-2\sqrt{M+P}}\right)}\right)e^{-y\sqrt{M+P}}$$
$$-\frac{1}{M+P}\left(G_a\left(a_0+a_1y+a_2y^2+a_3y^3+a_4y^4+a_5y^5+a_6y^6+\tau_2e^y\right)+\gamma\right) \qquad (24)$$

We have our assumed solution to be

$$\phi(y)=a_0+a_1y+a_2y^2+a_3y^3+a_4y^4+a_5y^5+a_6y^6+\tau_2e^y$$
$$\phi'(y)=a_1+2a_2y+3a_3y^2+4a_4y^3+5a_5y^4+6a_6y^5+\tau_2e^y$$
$$\phi''(y)=2a_2+6a_3y+12a_4y^2+20a_5y^3+30a_6y^4+\tau_2e^y$$

Substituting the expression $\phi(y),\ \phi'(y)\ and\ \phi''(y)$ into (6) gives

$$U=F\left(a_0+a_1y+a_2y^2+a_3y^3+a_4y^4+a_5y^5+a_6y^6+\tau_2e^y\right)^3$$
$$\left(a_1+2a_2y+3a_3y^2+4a_4y^3+5a_5y^4+6a_6y^5+\tau_2e^y\right)$$
$$-\left(2a_2+6a_3y+12a_4y^2+20a_5y^3+30a_6y^4+\tau_2e^y\right) \qquad (25)$$

Evaluating equation (6) in the interval $-1\le y\le 1$ will enable us to determine the effect of different radiation parameter

Determination of the Non-dimensional flow rate

The non-dimensional flow rate through the channel per unit width is given by

$$2\int_0^1\frac{U}{\gamma}dy=\frac{2}{\gamma}\int_0^1 \begin{array}{l}[F\left(a_0+a_1y+a_2y^2+a_3y^3+a_4y^4+a_5y^5+a_6y^6+\tau_2e^y\right)^3\\ \left(a_1+2a_2y+3a_3y^2+4a_4y^3+5a_5y^4+6a_6y^5+\tau_2e^y\right)\\ -\left(2a_2+6a_3y+12a_4y^2+20a_5y^3+30a_6y^4+\tau_2e^y\right)]dy\end{array}$$
$$=4.0567\ \ per\,unit \qquad (26)$$

Using the equation (6) and evaluating the integral will enable us to determine the flow rate.
Determination of Heat Transfer coefficient
Thus heat transfer coefficient due to thermal conduction is given by

$$h=-\left(\frac{d\phi}{dy}\right)$$
$$h=-a_1-2a_2y-3a_3y^2-4a_4y^3-5a_5y^4-6a_6y^5-\tau_2e^y$$
$$h=-9.0077-2\left(2.3417\right)y+3\left(4.6900\right)y^2+4\left(0.0629\right)y^3-5\left(1.5905\right)y^4$$
$$-6\left(0.0867\right)y^5+\left(3.1686\right)e^y \qquad (27)$$

4.0 Results and Discussion

Radiative fluid flow over a vertical porous channel under optically thick approximation in the presence of MHD was formulated, analysed and solved numerically. In order to point out the effects of physical parameters namely: thermal Grash of number Ga, Magnetic parameter M, radiation parameter F, Porous term P. on the flow patterns, the computation of the flow fields are carried out. The values of velocity and temperature are obtained for the physical parameters as mentioned.
The velocity profiles have been studied and presented in Figures 2 to 5 The effect of velocity for different values of Magnetic parameter (M = 3, 5, 10) is presented in Figure 2. The trend shows that the velocity increases with increasing Magnetic parameter. The effect of velocity for different values of radiation (F = 5, 10, 15) is also presented in Figure 3. It is then

observed that the velocity decreases with increasing values of radiation. The effect of velocity for different values of thermal Grashof number (Ga= 3, 4, 5) is also presented in Figure 4. It is then observed that the velocity decreases with increasing values of thermal Grashof number.The effect of velocity for different values of Porous term (P = 3, 5, 10) is presented in Figure 5. The trend shows that the velocity increases with increasing Porous parameter.

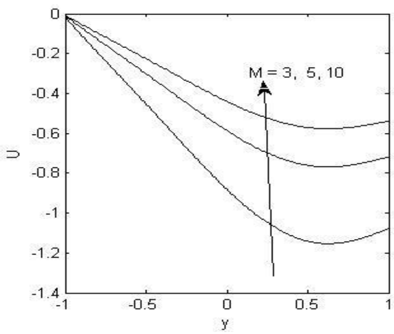

Figure 2. Velocity profiles for different values of M

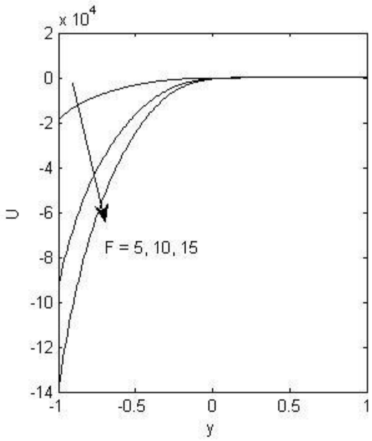

Figure 3. Velocity profiles for different values of F

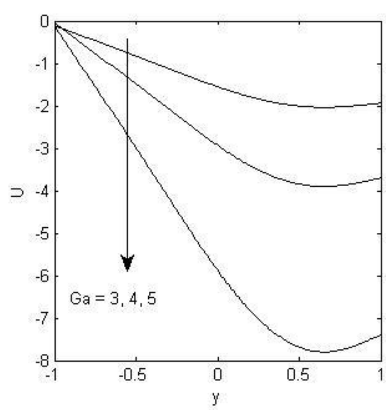

Figure 4. velocity profiles for different values of Ga

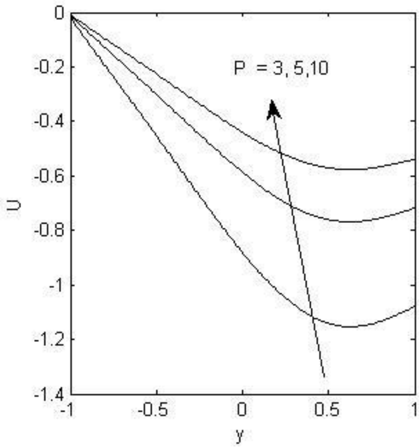

Figure 5. velocity profiles for different values of P

In Table 1, It is observed that temperature rise with increasing values of F, M and P, and the trend of heat transfer coefficient due to thermal conduction h increases with the rate of change of ϕ

Table 1: Thermal Conduction

F	M	P	$\phi(y)$	h
3	3	3	0.0573	-0.670
5	5	5	2.8883	-0.989
10	10	10	5.0023	-1.930
12	12	12	6.2207	-2.400
15	15	15	9.5532	-3.598
20	20	20	12.986	-5.923

5.0 Conclusion

Radiative fluid flow over a vertical porous channel under optically thick approximation in the presence of MHD has been studied. .In order to point out the effects of physical parameters namely: thermal Grashof number Ga, radiation parameter F, Magnetic parameter M and Porous term P are presented graphically. It is observed that velocity increases with increasing values of M and P, while F and Ga values decreases with increasing velocity. It is also observed that temperature rises with increasing values of F, M and P, and trend of heat transfer coefficient due to thermal conduction h increases with the rate of change of ϕ

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