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An Unsteady Viscous Flow model for Sub-retinal Fluid Drainage

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Abstract

We develop an unsteady two-dimensional viscous flow model for sub-retinal fluid drainage. The sub-retinal fluid and the vitreous are modelled as two distinct viscous fluids with a defined interface. An initial disturbance to the interface due to a sink located above it is imposed. Asymptotic solutions in time up to $O(t^2)$ were obtained. The results indicate a rise in the interface due to suction from the drainage needle, leading to the compression of the sub-retinal space. The implication of this scenario is that the sub-retinal fluid can be completely drained by moving the drainage needle progressively closer to the retinal tear.

Key words: Sub-retinal fluid, retinal detachment, sink, unsteady viscous flow.

1.0 Introduction

Retinal detachment refers to the separation of the inner neurosensory retina from the retinal pigment epithelium (RPE), due to retinal breaks or traction, thereby creating a space called the sub-retinal space. Fluid from the vitreous and its surroundings then flows into the sub-retinal space through the retinal breaks, or by hydrostatic and osmotic processes and active transport mechanisms [1]. The accumulation of fluid in the sub-retinal space exerts pressure on the eyeball and this eventually leads to the complete pulling away of the inner neurosensory retina from the RPE, and hence blindness, when blood supplies to the retina are cut off. The sub-retinal fluid normally accumulates in the sub-retinal space when there is a retinal detachment. This fluid is denser than the vitreous, which is the natural fluid that occupies the posterior chamber of the eye. The dynamic viscosity of the sub-retinal fluid varies between $0.012gcm^{-1}s^{-1}$ and $0.017gcm^{-1}s^{-1}$, while the vitreous has dynamic viscosity varying between $0.012gcm^{-1}s^{-1}$ and $0.028gcm^{-1}s^{-1}$ in aphakic subjects. The sub-retinal fluid has an average density of $1011.25kgm^{-3}$, that of the vitreous is $1005.3kgm^{-3}$ as reported by Quintyn *et al* [1]. It should be noted that the densities of both fluids vary with duration of retinal detachment, with the fluids getting denser as the duration gets longer.

The removal of the sub-retinal fluid is central to the successful treatment of retinal detachment through surgery. Our main objective in this study is to develop a simple mathematical model for the drainage of the sub-retinal fluid to give room for the re-attachment of the retina that gives us greater physical insights into drainage behaviour. Eye surgeons have been using various methods to remedy the situation. Some of the most commonly used methods include scleral buckling, chemical and gas injection as well as external needle drainage [2-4]. The scleral buckling procedure could lead to rupture of the eyeball in eyes with thin sclera. Gas injection on the other hand may require that the patient keeps to a particular head position for up to ten days and the gas bubble could be absorbed by the body. External needle drainage method is only effective on newly detached retinas, as accumulation of proteins with time prevents proper drainage of the sub-retinal fluid.

Some mathematical models have been developed for flow in the eye, especially motion due to the saccadic movement of the eye [5-6]. Gonzalez and Fitt [7]and Canning *et al* [8] developed mathematical models for the flow in the anterior chamber of the eye due to temperature variation in the eye. In the current study, we attempt to develop a model for the removal of sub-retinal fluid which normally accumulates in the sub-retinal space following a retinal detachment. Forbes and Hocking [9], and Tuck and Vanden-Broeck [10] have developed models for the drainage of fluids in tanks with the sink located below the free surface. The current study involves the drainage of fluid through a sink located above the interface. The flow in this case is driven by a pressure difference generated by suction due to the infusion needle.

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2.0 Mathematical Formulations

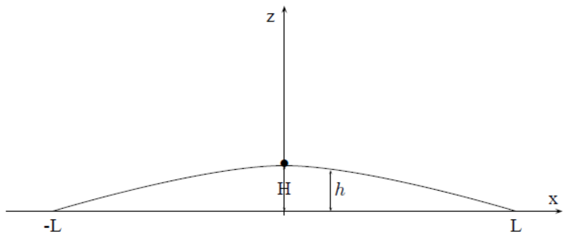

Fig. 1: Flow Geometry Showing the Retinal Tear Located at a Height H on theInterface h.

We develop a two-dimensional model for sub-retinal fluid drainage. A Cartesian coordinate system is located in the fluid such that the $x-$axis lies along the horizontal bottom corresponding to the retinal `floor', while the $z-$axis is directed vertically. The two fluids, the vitreous and the sub-retinal fluid occupy the plane $z>0$, as illustrated in figure 1 above. An infusion needle is introduced into the upper fluid, which will be withdrawn due to a pressure gradient created from outside at the thumb end of the infusion needle. The sub-retinal fluid and the vitreous are modelled as two distinct fluids with a defined interface. The vitreous is modelled as a viscous fluid; hence Stokes' unsteady flow model is employed. The suction from the needle therefore serves as a sink and the model is thus that of withdrawal of fluid through a sink located above the fluid interface. This is similar to the work of Forbes and Hocking [9]in potential flow. Blake [11], Blake and Chwang [12] and Pozrikidis [13] have undertaken similar work in the viscous case. For the two-dimensional unsteady Stokes' flow, we consider the field equations

$$\nabla.\mathbf{u}=0 \tag{1}$$

$$\frac{\partial \mathbf{u}}{\partial t}+\mathbf{u}.\nabla\mathbf{u}=-\frac{1}{\rho}\nabla p+\upsilon\nabla^2\mathbf{u}+F\delta(\mathbf{x}-\mathbf{x_0}) \tag{2}$$

with the dynamic boundary condition

$$p-p^0-2\mu\frac{\partial w}{\partial x}=\frac{\sigma}{R} \text{ on } z=h(x,t) \tag{3}$$

and the kinematic condition

$$\frac{\partial h}{\partial t}+u\frac{\partial h}{\partial x}=w \quad z=h(x,t) \tag{4}$$

where p is the pressure in the flow field, p^0 is the atmospheric pressure, σ is the surface tension, R is the radius of curvature of the interface, ρ is the density of the sub-retinal fluid at the interface and δ, the two-dimensional delta function is a sink or source of strength F.

We now scale the equations using the scaling parameters

$$\mathbf{x}=L\mathbf{x}^*,\ p=\frac{\mu U}{L}p^*, \mathbf{u}=U\mathbf{u}^*, t=Tt^*$$

and the field equations become

$$\nabla.\mathbf{u}^*=0$$

$$\alpha\frac{\partial \mathbf{u}^*}{\partial t^*}+\mathrm{Re}\,\mathbf{u}^*.\nabla\mathbf{u}^*=-\nabla p^*+\nabla^2\mathbf{u}^*+M\delta(\mathbf{x}-\mathbf{x}_0) \tag{5}$$

where $\alpha=\frac{\rho L^2}{\mu T}$, $\mathrm{Re}=\frac{UL}{\upsilon}$ is the Reynolds number, M is the dimensionless strength of the singularity, while L and U are the characteristic length and velocity respectively. If $\mathrm{Re}<<1$ and $\alpha\sim 1$, the convective terms can be ignored, and the equations in dropping asterisks become

$$\nabla.\mathbf{u} = 0$$

$$\alpha \frac{\partial \mathbf{u}}{\partial t} = -\nabla p + \nabla^2 \mathbf{u} + M\delta(\mathbf{x} - \mathbf{x}_0) \qquad (6)$$

Equation (6) is the forced unsteady Stokes equation which is valid for flows characterised by sudden acceleration or deceleration, which is the case in sub-retinal fluid drainage. Scaling (3) similarly, one obtains the dynamic boundary condition as

$$\Delta p - 2\frac{\partial w}{\partial z} - \Gamma\left[1 - \frac{3}{2}\left(\frac{\partial h}{\partial x}\right)^2 + \ldots\right]\frac{\partial^2 h}{\partial x^2} = 0 \quad \text{on } z = h(x,t) \qquad (7)$$

where $\Delta p = p - p^0$ and $\Gamma = \dfrac{\sigma L}{\mu Q}$ as obtained previously.

Taking the divergence of (6) and using (5), the pressure satisfies the relation

$$\nabla^2 p = \nabla.[M\delta(\mathbf{x} - \mathbf{x}_0)] \qquad (8)$$

Recalling the equivalent relation for the two-dimensional delta function

$$\delta(\mathbf{x} - \mathbf{x}_0) = \frac{1}{2\pi}\nabla^2 \ln r$$

where $r = \sqrt{x^2 + (z-\lambda)^2}$, equation (10) yields the expression for the pressure in the z-direction as

$$p = \frac{M(z-\lambda)}{2\pi r^2} \qquad (9)$$

3.0 Small-time Asymptotic Solutions

In this section, we obtain approximate solutions to equation (8) with the assumptions that the pressure gradient and the flux are steady, and that $t << 1$. An asymptotic solution in t can then be obtained by expanding the velocity $\mathbf{u} = (u, w)$ and the interface elevation h as

$$u(x,z,t) = \sum_{j=0}^{n} u_j(x,z)t^j \qquad (10)$$

$$w(x,z,t) = \sum_{j=0}^{n} w_j(x,z)t^j \qquad (11)$$

and

$$h(x,t) = \sum_{j=0}^{n} h_j(x)t^j \qquad (12)$$

where $t << 1$, u_0 and w_0 satisfy the steady case of (6) while $u_1, u_2, \ldots, u_n$ and $w_1, w_2, \ldots, w_n$ all satisfy (6). We now impose the condition that $h_0(x) = 0$, which implies a switch-on at $t = 0$.

Though the movement of the interface is primarily upwards due to the suction from the needle, the velocity in the $x-$direction is substantial, and for small slopes of the interface elevation, the dynamic and kinematic boundary conditions become

$$\Delta p - 2\frac{\partial w}{\partial z} - \Gamma\frac{\partial^2 h}{\partial x^2} = 0 \quad \text{on } z = h(x,t) \qquad (13)$$

and

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial z} = w \quad \text{on } z = h(x,t) \qquad (14)$$

Now define F and f thus:

$$F = \Delta p - 2\frac{\partial w}{\partial z} - \Gamma\frac{\partial^2 h}{\partial x^2} \tag{15}$$

and

$$f = \frac{\partial h}{\partial t} + u\frac{\partial h}{\partial z} - w. \tag{16}$$

Expanding F and f in Taylor series about $z=0$ and evaluating on $z=h$, one obtains

$$\sum_{j=0}^{n}\frac{h^j}{j!}\frac{\partial^j F}{\partial z^j} = 0 \tag{17}$$

and

$$\sum_{j=0}^{n}\frac{h^j}{j!}\frac{\partial^j f}{\partial z^j} = 0 \tag{18}$$

Substituting F and f into (19) and (20), one obtains the dynamic boundary condition as

$$\left\{\Delta p - 2\frac{\partial w}{\partial z} - \Gamma\frac{\partial^2 h}{\partial x^2}\right\} + h\frac{\partial}{\partial z}\left\{\Delta p - 2\frac{\partial w}{\partial z} - \Gamma\frac{\partial^2 h}{\partial x^2}\right\} + \frac{h^2}{2}\frac{\partial^2}{\partial z^2}\left\{\Delta p - 2\frac{\partial w}{\partial z} - \Gamma\frac{\partial^2 h}{\partial x^2}\right\} + \cdots = 0 \tag{19}$$

and the kinematic condition becomes

$$\left\{\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} - w\right\} + h\frac{\partial}{\partial z}\left\{\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} - w\right\} + \frac{h^2}{2}\frac{\partial^2}{\partial z^2}\left\{\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} - w\right\} + ... = 0 \tag{20}$$

Using (10 - 13) and evaluating on $z=0$, we obtain the following sequence of equations from the dynamic boundary condition:

$O(1)$:

$$2\frac{\partial w_0}{\partial z} - \Delta p = 0 \tag{21}$$

$O(t)$:

$$\Gamma\frac{\partial^2 h_1}{\partial x^2} + 2\frac{\partial w_1}{\partial z} + 2h_1\frac{\partial^2 w_0}{\partial z^2} = 0 \tag{22a}$$

$O(t^2)$:

$$\Gamma\frac{\partial^2 h_2}{\partial x^2} + h_1^{\,2}\frac{\partial^3 w_0}{\partial z^3} + 2\frac{\partial w_2}{\partial z} + 2h_1\frac{\partial^2 w_1}{\partial z^2} + 2h_2\frac{\partial^2 w_0}{\partial z^2} = 0 \tag{22b}$$

From the kinematic condition, we have the sequence of equations

$O(1)$:

$$h_1 - w_0 = 0 \tag{23}$$

$O(t)$:

$$2h_2 + u_0\frac{\partial h_1}{\partial x} - h_1\frac{\partial w_0}{\partial z} - w_1 = 0 \tag{24}$$

$O(t^2)$:

$$3h_3 + u_0\frac{\partial h_2}{\partial x} - h_2\frac{\partial w_0}{\partial z} - \frac{h_1^{\,2}}{2}\frac{\partial^2 w_0}{\partial z^2} + h_1\frac{\partial h_1}{\partial x}\frac{\partial u_0}{\partial z} - \frac{\partial w_1}{\partial z} + w_1\frac{\partial h_1}{\partial x} - w_2 = 0 \tag{25}$$

From (21), one obtains the pressure difference as

$$\Delta p = \frac{m\left(x^2 - \lambda^2\right)}{\pi\left(x^2 + \lambda^2\right)^2} \quad (26)$$

and from (23 - 25), one obtains the solutions

$$h_1(x) = \frac{m\lambda}{2\pi\left(x^2 + \lambda^2\right)} \quad (27)$$

and

$$h_2(x) = \frac{m^2\lambda\left(\lambda^2 - 3x^2\right)}{8\pi^2\left(x^2 + \lambda^2\right)^3} \quad (28)$$

$$h_3(x) = -\frac{m^3\lambda\left(5x^4 - 10\lambda^2x^2 + \lambda^4\right)}{16\pi^3\left(x^2 + \lambda^2\right)^5} \quad (29)$$

Thus the expression for the interface elevation is given by

$$h(x,t) = -\frac{m\lambda t}{2\pi\left(x^2 + \lambda^2\right)}\left\{1 - \frac{mt\left(\lambda^2 - 3x^2\right)}{4\pi\left(x^2 + \lambda^2\right)^2} + \frac{m^2t^2\left(5x^4 - 10\lambda^2x^2 + \lambda^4\right)}{8\pi^2\left(x^2 + \lambda^2\right)^4}\right\} \quad (30)$$

Figure 2 shows the progressive movement of the drainage needle closer and closer to the retinal tear, leading to the stretching of the surface. The retinal surface begins to deform as the needle gets very close to the tear as can be seen in Figure 2(d). Figure 3 on the other shows the elevations of the interface at different times. Observe that the interface begins to deform at a height of about 0.61 unit at $t \approx 0.42$. This is clearly seen in Figure 3(b), (c) and (d).

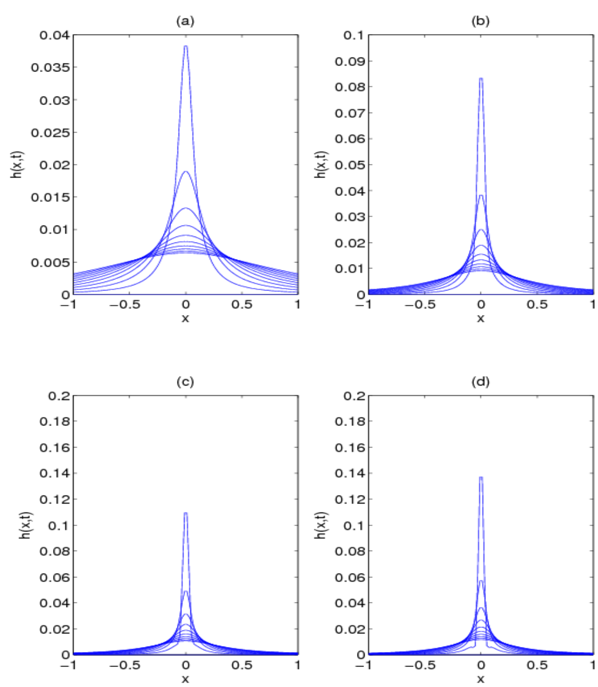

Figure 2: Surface Elevation Due to the Vertical Movement of the Drainage Needle for $t = 0:0.01:$ (a) $\lambda = 0:0.1:1$ (b) $\lambda = 0:0.05:0.5$ (c) $\lambda = 0:0.04:0.4$ (d) $\lambda = 0:0.035:0.35$.

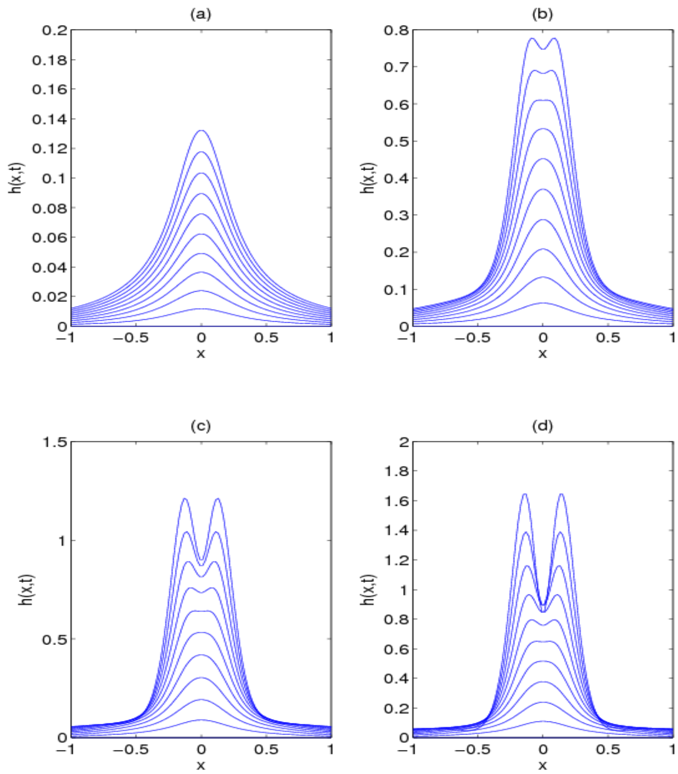

Figure 3: Surface Elevation with Time for $\lambda = 0.35$: (a) $t = 0:0.01:0.1$ (b) $t = 0:0.05:0.5$ (c) $t = 0:0.07:0.7$ (d) $t = 0:0.085:0.85$.

4.0 The Two-Fluid Flow Model

In this section, we consider a situation in which there is a flow in each region of the flow domain due to the actions of the sink through the needle and the sink at the retinal tear. The governing equations become

$$\nabla.\mathbf{u}^{v} = 0$$

$$\alpha \frac{\partial \mathbf{u}^{\mathbf{v}}}{\partial \mathbf{t}} = -\nabla p^{v} + \nabla^{2}\mathbf{u}^{v} + m^{v}\delta(\mathbf{x}-\mathbf{x}_{1}) \tag{31}$$

and

$$\nabla.\mathbf{u}^{s} = 0$$

$$\alpha \frac{\partial \mathbf{u}^{s}}{\partial t} = -\nabla p^{s} + \nabla^{2}\mathbf{u}^{s} + m^{s}\delta(\mathbf{x}-\mathbf{x}_{0}) \tag{32}$$

subject to the dynamic boundary conditions

$$\Delta p - 2\frac{\partial w^{v}}{\partial z} = 0 \quad \text{on } z = 0 \tag{33}$$

$$\Gamma\nabla^2 h = \left[p^s - p^v + 2\left(\kappa \frac{\partial w^v}{\partial z} - \frac{\partial w^s}{\partial z} \right) \right]_{z=0} \tag{34}$$

and the kinematic condition

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} - w = 0 \quad \text{on } z = 0 \tag{35}$$

Where u^s, w^s, p^s and u^v, w^v, p^v are velocity and pressure fields in the sub-retinal and vitreous regions respectively, and $\kappa = \frac{\mu^v}{\mu^s}, u = \frac{1}{2}\left(u^s + u^v\right), w = \frac{1}{2}\left(w^s + w^v\right)$, $\mathbf{x}_0 = (0,\lambda)$, $\mathbf{x}_1 = (0,\lambda+\alpha)$, α is the distance of the needle tip from the retinal tear, and Γ is as previously defined. Thus we obtain the expression for the pressures in the vitreous and sub-retinal space as

$$p^v = \frac{m^v(z-\lambda-\alpha)}{2\pi r_1^2} \text{ and } p^s = \frac{m^s(z-\lambda)}{2\pi r^2}$$

where $r_1 = \sqrt{x^2 + (z-\lambda-\alpha)^2}$.

4.1 Asymptotic Solution

An asymptotic solution in time of the form

$$u^v(x,z,t) = \sum_{j=0}^{n} u_j^v(x,z)t^j \tag{36}$$

$$u^s(x,z,t) = \sum_{j=0}^{n} u_j^s(x,z)t^j \tag{37}$$

$$w^v(x,z,t) = \sum_{j=0}^{n} w_j^v(x,z)t^j \tag{38}$$

$$w^s(x,z,t) = \sum_{j=0}^{n} w_j^s(x,z)t^j \tag{39}$$

and

$$h(x,t) = \sum_{j=0}^{n} h_j(x)t^j \tag{40}$$

Using equations (33)-(35), we define F_1, F_2 F_1, F_2 and F_3 as

$$F_1 = \Delta p^v - 2\frac{\partial w^v}{\partial z} \tag{41}$$

$$F_2 = \Gamma\nabla^2 h - \left[p^s - p^v + 2\left(\kappa \frac{\partial w^v}{\partial z} - \frac{\partial w^s}{\partial z} \right) \right] \tag{42}$$

$$F_3 = \frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} - w \tag{43}$$

The Taylor's series expansions of (41), (42) and (43), evaluated on $z = h$ yield

$$\sum \frac{h^j}{j!}\frac{\partial^j F_1}{\partial z^j} = 0 \tag{44}$$

$$\sum \frac{h^j}{j!}\frac{\partial^j F_2}{\partial z^j} = 0 \tag{45}$$

and

$$\sum \frac{h^j}{j!}\frac{\partial^j F_3}{\partial z^j} = 0 \tag{46}$$

Using equations (36)-(39), one obtains the following sequence of equations:

$O(1)$:

$$2\frac{\partial w_0^v}{\partial z} - \Delta p = 0 \tag{47}$$

$O(t)$:

$$\frac{\partial w_1^v}{\partial z} + h_1 \frac{\partial^2 w_0^v}{\partial z^2} = 0 \tag{48}$$

$O(t^2)$:

$$\frac{\partial w_2^v}{\partial z} + h_1 \frac{\partial^2 w_1^v}{\partial z^2} + h_2 \frac{\partial^2 w_0^v}{\partial z^2} + \frac{h_1^2}{2}\frac{\partial^3 w_0^v}{\partial z^3} = 0 \tag{49}$$

and

$O(1)$:

$$\Delta P - 2\left(\kappa \frac{\partial w_0^v}{\partial z} - \frac{\partial w_0^s}{\partial z}\right) = 0 \tag{50}$$

$O(t)$:

$$\Gamma \frac{\partial^2 h_1}{\partial x^2} - \left(\kappa \frac{\partial w_1^v}{\partial z} - \frac{\partial w_1^s}{\partial z}\right) - 2h_1\left(\kappa \frac{\partial w_0^v}{\partial z} - \frac{\partial w_0^s}{\partial z}\right) = 0 \tag{51}$$

$O(t^2)$:

$$\Gamma \frac{\partial^2 h_2}{\partial x^2} - 2\left(\kappa \frac{\partial w_2^v}{\partial z} - \frac{\partial w_2^s}{\partial z}\right) - 2h_1\left(\kappa \frac{\partial^2 w_1^v}{\partial z^2} - \frac{\partial^2 w_1^s}{\partial z^2}\right) - 2h_2\left(\kappa \frac{\partial^2 w_0^v}{\partial z^2} - \frac{\partial^2 w_0^s}{\partial z^2}\right) - h_1^2\left(\kappa \frac{\partial^3 w_0^v}{\partial z^3} - \frac{\partial^3 w_0^s}{\partial z^3}\right) = 0 \tag{52}$$

where $\Delta P = p^s - p^v$ and the superscripts s and v denote flow fields in the sub-retinal and vitreous regions respectively. Here, $u_0^v, u_0^s, w_0^v, w_0^s, p^v$ and p^s are obtained from the steady state singularity solutions of equations (31)-(32) as

$$u_0^s = \frac{m^s}{2\pi}\left[\frac{x}{x^2 + (z-\lambda)^2} - \frac{x}{x^2 + (z+\lambda)^2} + \frac{4xz(z+\lambda)}{\left[x^2 + (z+\lambda)^2\right]^2}\right]$$

$$w_0^s = \frac{m^s}{2\pi}\left[\frac{z-\lambda}{x^2 + (z-\lambda)^2} - \frac{z-\lambda}{x^2 + (z+\lambda)^2} + \frac{4xz(z+\lambda)^2}{\left[x^2 + (z+\lambda)^2\right]^2}\right]$$

$$u_0^v = \frac{m^v}{2\pi}\frac{x}{x^2 + (z-\lambda-\alpha)^2}$$

$$w_0^v = \frac{m^v}{2\pi}\frac{z-\lambda-\alpha}{x^2+(z-\lambda-\alpha)^2}$$

while p^s and p^v are given by

$$p^s = \frac{m^s}{4\pi}\ln\left[x^2+(z-\lambda)^2\right]$$

$$p^v = \frac{m^v}{4\pi}\ln\left[x^2+(z-\lambda-\alpha)^2\right]$$

The corresponding sequence of equations resulting from the kinematic boundary condition remains the same as equations (23)-(25).

The expressions for the interface elevations at various orders are then obtained as

$$h_1(x) = \frac{m^v}{4\pi}\frac{\lambda+\alpha}{x^2+(\lambda+\alpha)^2} \quad (53)$$

$$h_2(x) = \frac{1}{2}\left[w_1 + h_1\frac{\partial w_0}{\partial z} - u_0\frac{\partial h_0}{\partial x}\right]_{z=0} \quad (54)$$

where

$$w_1 = \frac{1}{2}\left[w_1^s + w_1^v\right],$$

$$w_1^v = -h_1\int\frac{\partial^2 w_0^v}{\partial z^2}dz$$

and

$$w_1^s = \int\left[\kappa\frac{\partial w_1^v}{\partial z} - \Gamma\frac{\partial^2 h_1}{\partial x^2} + \left(\kappa\frac{\partial w_0^v}{\partial z} - \frac{\partial w_0^s}{\partial z}\right)\right]dz$$

and hence the expression for the interface elevation

$$h(x,t) = h_1(x)t + h_2(x)t^2 \quad (55)$$

The first order interface elevation is plotted below for various parameter values and time intervals.

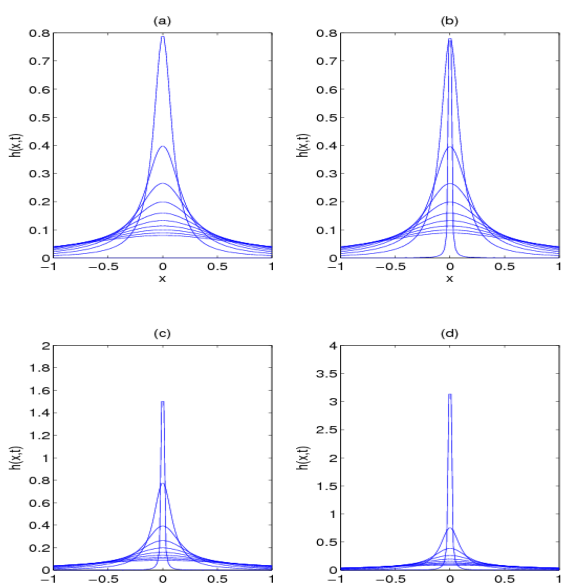

Figure 4: Surface Elevation for $t = 0.01$: (a) $\lambda = 0:0.1:1$ (b) $\lambda = 0.001:0.1:1$ (c) $\lambda = 0.002:0.1:1$ (d) $\lambda = 0.005:0.1:1$.

5.0 Conclusion

The forced Stokes' unsteady flow model was analysed using the small-time asymptotic expansion and the approximate expressions for the interface evolution were obtained. The interface evolution as depicted on figures 2 and 4 indicates a gradual compression of the sub-retinal space as the drainage needle approaches the retinal tear. Figure 3 indicates that the interface deforms with time, which is expected when the pressure in the sub-retinal region drops. It is therefore predicted that the sub-retinal fluid can be completely drained by moving the drainage needle progressively closer to the retinal tear.

References

[1] J.-C. Quintyn and G. Brasseur, Subretinal Fluid in Primary Rhegmategenous Retinal Detachment: Physiology and Composition, Survey of Ophthalmology, 49 (2004), 96-108.
[2] Steve Charles, An Engineering Approach to Vitreoretinal Surgery, Retina, The Journal of Retinal and Vitreous Diseases, 24 (2004), 435-444.
[3] A.H. Chignell, Retinal Detachment Surgery, Springer-Verlag, Berlin Heidelberg, 1988.
[4] Glenn J. Jaffe, Robert Brownlow and John Hines, Modified External Needle Drainage Procedure for Rhegmategenous Retinal Detachment, Retina, The Journal of Retinal and Vitreous Diseases, 23 (2003), 80-85.
[5] Tim David, Steve Smye, Tim Dabbs and Teifi James, A model for the fluid motion of vitreous humour of the human eye during saccadic movement, Phys. Med. Biol., 43 (1998), 1385-1399.
[6] R. Repetto, I. Ghigo, G. Saminara and C. Ciurlo, A simple hydro-elastic model of thedynamics of a vitreous membrane, Journal of Fluid Mechanics, 503 (2004), 1-14.
[7] Gabriela Gonzalez and Alastair Fitt, Mathematical modelling of human eyes-a PhD study, Mathematics Today, 39 (2003), 20-25.
[8] C.R. Canning, M.J. Greaney, J.N. Dewynne and A.D. Fitt, Fluid flow in the anterior chamber of a human eye, Mathematical Medicine and Biology, A Jour. of the IMA, 19 (2002), 31-60.
[9] L. K. Forbes and G. C. Hocking, Unsteady draining flows from a rectangular tank, Physics of Fuids, 19 (2007).
[10] E. O. Tuck and J.-M Vanden-Broeck, A cusp-like free surface flow due to a submerged source or sink, J. Austral. Math. Soc. B, 25 (1984), 443-450.
[11] J.R. Blake, A note on the image system for a stokeslet in a no-slip boundary, Proc. Camb. Phil. Soc., 70 (1971), 303-310.
[12] J.R. Blake and A.T. Chwang, Fundamental Singularities of Viscous Flow. part1: The image systems in the vicinity of a stationary no-slip boundary, Journal of Engineering Mathematics, 8 (1974), 23-29.
[13] C. Pozrikidis, Boundary integral and singularity methods for linearized viscous flow, Cambridge University Press, New York, 1992.