Journal of the Nigerian Association of Mathematical Physics
Volume27, (July, 2014), pp143 – 148
© J. of NAMP

MHD Oscillatory Slip Flow with Temperature Dependent Heat Source In a Channel Filled With Porous Medium

Disu Akeem Babatunde

Abstract

This study was conducted to investigate heat transfer of Magnetohydrodynamics (MHD) oscillatory slip flow of a conducting optically thin fluid through a channel filled with porous medium in the present of temperature dependent heat source. The governing equations of oscillatory fluid flow are non-dimensionlised, simplified and solved. The closed form solutions were obtained for velocity and temperature.The numerical computations were presented graphically to show the salient features of the fluid flow. The skin-friction and Nusselt number were also analyzed.

Keywords: MHD oscillatoryslip flow, Temperature Dependent Heat source, Porous Medium

1.0 Introduction

The study of oscillatory fluid in porous channel has been recieved considerable attention due to its application in soil mechanic, ground water hydrology, irrigation, water purification processes, absorption and filtration process in chemical and petroluem engineering.

In view of these applications, Makinde and Mhone [1] presented the combined effects of radiative heat tranfer and MHD oscillatory flow in a channel filled with saturated porous medium. Mehmood and Ali [2] repoted the effect of Navier boundary condition on the lower wall of the unsteady MHD oscillatory flow in a planer channel filled with porous mediun. It is assumed that the navier slip boundary condition effect depends on the shear stress of both lower and upper walls by Eegunjobi and Makinde [3]. As a result of this, Adesanya and Makinde [4] investigated the navier slip conditions on the lower (cold) and upper (heated) walls on the pulsatile flow of MHD oscillatory fluid through a channel with non-uniform wall temperature that is filled with saturated porous medium.

In all the studies mentioned above, MHD oscillatory slip flow with variable temperature in a channel filled with saturated porous medum has not recieved attention. Therefore, we present MHD oscillatory slip flow of an optically thin fluid through a channel filled with saturated porous medium in the present of temperature dependent heat source. This is presented as follows : session two presents the problem formulation, three session describes the method of solution and results, session four discussion of results and finally, session five presents the conclusion.

2.0 Problem Formulation

Consider the unsteady laminar slip flow of an electrically conductiong heat-generating, optically thin oscillatory fluid flow in a channel filled with saturated porous medium in the present of thermal radiation with temperature variation.Choose a Cartesian coordinate system (X, Y), where X-axis is taken along the flow and Y-axis is taken normal to the flow direction. A uniform transverse magnetic field of B_0 is applied in the present of thermal buoyancy effects in the drection of Y-axis. Then, assumed a Boussinesq incompressible fluid model, the equations governing the flow are :

$$\frac{\partial U}{\partial t'} = -\frac{1}{\rho}\frac{\partial P'}{\partial X} + \nu\frac{\partial^2 U}{\partial Y^2} - \frac{\nu}{K}U - \frac{\sigma B_0^2}{\rho}U + g\beta(T - T_0) \quad (1)$$

$$\frac{\partial T}{\partial t'} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial Y^2} - \frac{1}{\rho C_p}\frac{\partial q}{\partial Y} - \frac{Q(T-T_0)}{\rho C_p} \quad (2)$$

With boundary conditions

$$U = \gamma'\frac{\partial U}{\partial Y}, \quad T = T_0 + \delta'\frac{\partial T}{\partial Y}, \qquad Y = 0 \quad (3)$$

Corresponding author: Disu Akeem BabatundeE-mail: adisu@noun.edu.ng, Tel.: +2348077138381

$U = \eta' \frac{\partial U}{\partial Y}, T = T_1 + \vartheta' \frac{\partial T}{\partial Y} Y = L$

where t' time, U axial velocity , ρ fluid density, P' fluid pressure, ν kinematic viscosity, K porous permeability, σ electrical conductivity, B_0^2magnetic field intensity, g gravitational acceleration, β volumetric expansion, C_p specfic heat at constant pressure, k thermal conductivity, T fluid temperature, T_0 referenced fluid temperature ,γ' and η' are the slip parameter due to the porous medium. It is assumed that the fluid is optically thin with a relatively low density and the radiative heat flux is given by Cogley *et al.* [5] as follow:

$$\frac{\partial q}{\partial y} = 4\chi(T_0 - T_1) \quad (4)$$

where χ is the mean radiation absorption coefficient . The following dimensionless quantities are introduced:

$$x = \frac{X}{L}, \ y = \frac{Y}{L}, \ Re = \frac{U_0 L}{\nu}, P = \frac{LP'}{\rho\nu U_0}, u = \frac{U}{U_0}, \theta = \frac{T - T_0}{T_1 - T_0}, t = \frac{t' U_0}{L},$$

$$H = \frac{L^2 \sigma B_0^2}{\rho\nu}, Da = \frac{K}{L^2}, Gr = \frac{g\beta(T_1 - T_0)}{\nu U_0}, Pe = \frac{U_0 \rho C_p}{k}, N = \frac{4\chi L^2}{k}, \quad (5)$$

$$\alpha = \frac{QL^2}{k}, \gamma = \frac{\gamma'}{L}, \eta = \frac{\eta'}{L}, \delta = \frac{\delta'}{L}, \vartheta = \frac{\vartheta'}{L}$$

where U_0 is the flow velocity. The dimensionless governing equations together with the appropriate boundary conditions are as follow:

$$Re \frac{\partial u}{\partial t} = -\frac{\partial P}{\partial x} + \frac{\partial^2 u}{\partial y^2} - (s + H)u + Gr\theta \quad (6)$$

$$Pe \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} - (N + \alpha)\theta \quad (7)$$

with boundary conditions b

$$u = \gamma \frac{\partial u}{\partial y}, \ \theta = \delta \frac{\partial \theta}{\partial y}, \qquad y = 0$$

$$u = \eta \frac{\partial u}{\partial y}, \ \theta = 1 + \vartheta \frac{\partial \theta}{\partial y}, \qquad y = 1 \quad (8)$$

where $Gr, H, N, Pe, Da, s = \frac{1}{Da}, \alpha$ are Grashof number, Hartmann number, Radiation parameter, Peclet number, Reynolds, Darcy number, porous medium shape factor parameter and Heat source parameter.

3.0 Method of Solution

In order to solve equations (6) and (7) with boundary conditions (8) for purely oscillatory flow, let

$$-\frac{\partial y}{\partial x} = \lambda e^{i\omega t}, \quad u(y,t) = u_0 e^{i\omega t}, \ \theta(y,t) = \theta_0 e^{i\omega t} \ (9)$$

where λ is a constant and ω is the frequency of the oscillation. Substituting (9) in equations (6)-(8), we obtain :

$$\frac{d^2\theta_0}{dy^2} + m_1^2 \theta_0 = 0 \quad (10)$$

$$\frac{d^2 u_0}{dy^2} + m_2^2 u_0 = -\lambda - Gr\theta_0 \quad (11)$$

with boundary conditions

$$\left. \begin{aligned} u_0 = \gamma \frac{du_0}{dy}, \ \theta_0 = \delta \frac{du_0}{dy}, \qquad y = 0 \\ u_0 = \eta \frac{du_0}{dy}, \ \theta_0 = 1 + \vartheta \frac{d\theta_0}{dy}, \qquad y = 1 \end{aligned} \right\} \quad (12)$$

where $m_1 = \sqrt{N + \alpha - i\omega Pe}$ and $m_2 = \sqrt{s + H + i\omega Re}$.

Equations (10) and (11) are solved with boundary (12) and we obtain solution for fluid velociy and temperature as follow:

$$\theta(y,t) = \frac{A_1 A_2 \sin(m_1 y) + \delta m_1 \cos(m_1 y)}{A_2} e^{i\omega t} \quad (13)$$

$$u(y,t) = \left\{ -\frac{A_3(cosh(m_2 y) - sinh(m_2 y))}{A_4} + \frac{A_5(cosh(m_2 y) + sinh(m_2 y))}{A_6} + \frac{Gr\delta m_1 m_2^2 cos(m_1 y) + A_2(GrA_1 m_2^2 sin(m_1 y) + \lambda(m_2^2 + m_1^2))}{A_2 m_2^2 (m_2^2 + m_1^2)} \right\} e^{i\omega t} \quad (14)$$

4.0 Discussion of Results

We obtained the closed form solutions for equations (11) and (12) subject to boundary conditions (13). As an accuracy mathematical check, setting $\alpha = 0 \delta = 0$, and$\vartheta = 0$, the work of Adesanya and Makinde [4] is fully recovered. Numerical computations were performed on the closed form solutions and the results were illustrated graphically in Figure 1-7 shown the interesting features of significant parameters on velocity and temperature distributions. Throughout the computations , we employed default values $t = 0, \lambda = 1, N = 2, H = 1, s = 1,$

$Gr = 1, Pe = 1, \gamma = 0.1, \eta = 0.1, \delta = 0.1, \vartheta = 0.1, Re = 1, \omega = 1$ and $\alpha = 1$ unless otherwise stated.
Figure 1depicts the fluid velocity increases with an increase in the cold wall slip parameter thus enhancing fluid flow. Figure 2 illustrates that the presence of transverse magnetic field produces a resistive force onthe fluid flow. This force is called Lorentz force, which slows down the motion of the fluid. It is observed that the increases in porous permeability parameter decreases the velocity as shown in Figure 3. It is observed that fluid velocity decreses with increases in the heated wall slip in Figure 4. Figure 5 presents the increase in the heat source parameter significantly increase the fluid velocity.

Figure 6 illustrates the increase in heat source parameter significantly increase thermal bouyancy effects which raise the fluid temperature. It is observed that increase in the varying temperature at the cold wall slip produces raise in the thermal boundary which increases the temperature profile in Figure 7. Figure 8 depicts the varying temperature at the heated wall which show little or no significant effect on the temperature profile.

Table 1& 2 show the effect of heat source parameter on skin friction and Nusselt number. The result show that an increase in heat source parameter increases the shear stress and the heat transfer rate within the channel. Table 3&4 show the effect of vary temperature on the Nusselt number. It is observed that increases the vary temperature at the cold wall increseas the heat transfer rate but, increases in the vary temperature at the heated wall increase the heat transfer slightly.

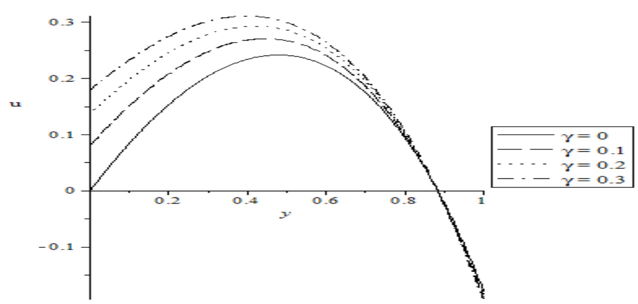

Figure 1: Velocity profile for different values of Naiver parameter at the cold wall

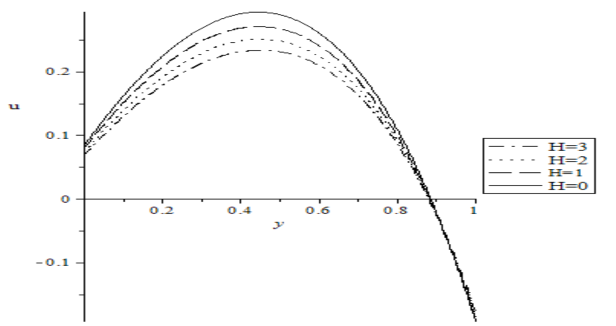

Figure 2: Velocity profile different values of Hartmann number

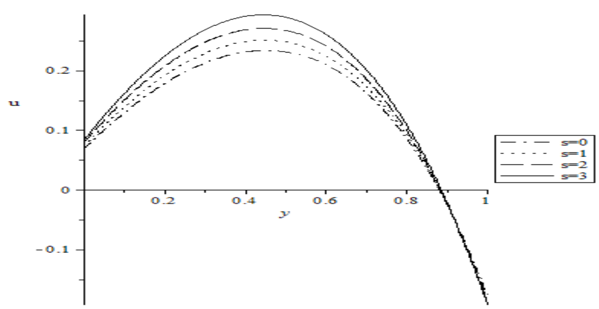

Figure 3: Velocity profile for different values of porous permeability parameter

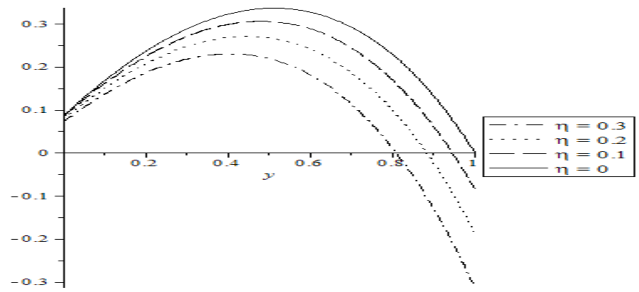

Figure 4: Velocity profile for different values of Navier slip at the heated wall

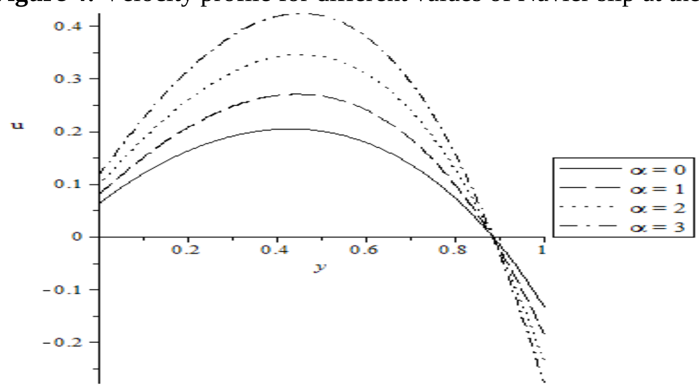

Figure 5: Velocity profile for different values of heat source parameter

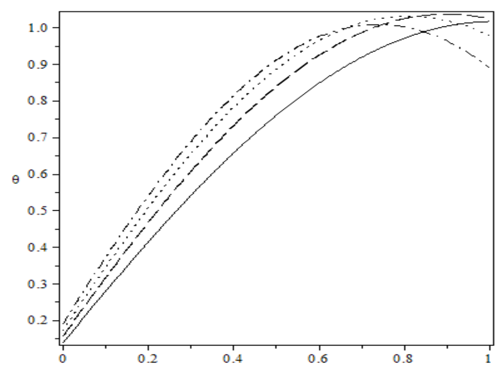

Figure 6: Temperature profile for different values of Heat source

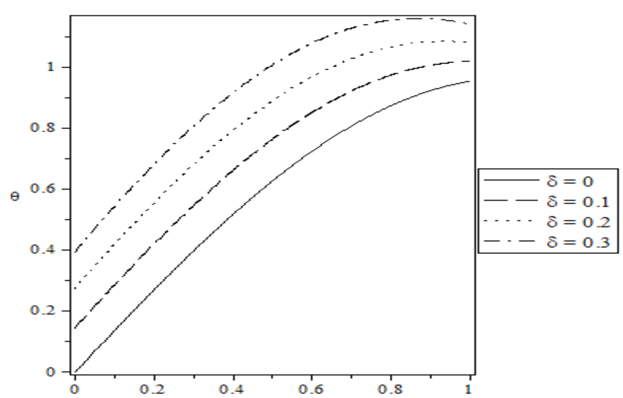

Figure 7: Temperature profile for different values of δ

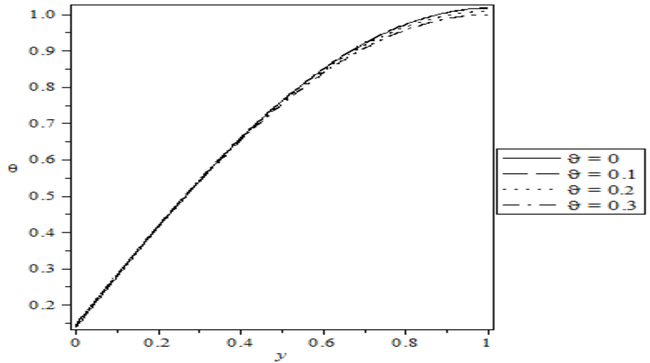

Figure 8: Temperature profile for different values of ϑ

Table 1: Effects of heat sorce parameter on the skin-friction

y	α	τ	α	τ	α	τ
0	0	-0.63975	1	-0.78808	2	-0.95695
0.25	0	-0.28245	1	-0.40442	2	-0.53869
0.5	0	0.12448	1	0.12937	2	0.14885
0.25	0	0.63912	1	0.86209	2	1.10821
1	0	1.33504	1	1.851615	2	2.348744

Table 2: Effects of heat sorce parameter on the Nusselt number

y	α	Nu	α	Nu	α	Nu
0	0	-0.85147	1	-1.42519	3	-1.89859
0.25	0	-0.79589	1	-1.26831	3	-1.46338
0.5	0	-0.69428	1	-0.95454	3	-0.66988
0.25	0	-0.54286	1	-0.52268	3	-0.28761
1	0	-0.36104	1	-0.02617	3	-1.74709

Table 3: Effects of δ on the skin-friction

y	δ	Nu	δ	Nu	δ	Nu
0	0	-0.02527	0.2	-1.48467	0.3	-1.54414
0.25	0	-1.36572	0.2	-1.26085	0.3	-1.25824
0.5	0	-1.28125	0.2	-0.88107	0.3	-0.81670
0.25	0	-1.03828	0.2	-0.39522	0.3	-0.27412
1	0	-0.6668	0.2	0.45007	0.3	0.30235

Table 4: Effects of ϑ on the skin-friction

y	ϑ	Nu	ϑ	Nu	ϑ	Nu
0	0	-1.42810	0.1	-1.42519	0.2	-1.42291
0.25	0	-1.27119	0.1	-1.26832	0.2	-1.26543
0.5	0	-0.95704	0.1	-0.95454	0.2	-0.95037
0.25	0	-0.52449	0.1	-0.52268	0.2	-0.52087
1	0	-0.02970	0.1	-0.02617	0.2	-0.02527

5.0 Conclusion

This investigated the effects of heat transfer on MHD oscillatory slip flow with temperature dependent heat source in a channel filled with porous medium. The closed form solution were obtained for the velocity and temperature field. There is good agreement betwen the present work and Adesanya and Makinde [4] when $\alpha = 0, \delta = 0, \vartheta = 0$. Skin friction increases as the heat source parameter increases. The varying temperature at the cold slip wall increase the Nusselt number but, decerase the Nusselt number at the heated slip wall.

Reference

[1] *O. D. Makinde, P. Y. Mhone*, Heat transfer to MHD oscillatory flow in achannel filled withporous medium, Rom. Journ. Phys., **50** (2005) 931–938

[2] *A. Mehmood, A. Ali*, The effect of slip condition on unsteady MHD Oscillatory flow of aviscous fluid In a planer channel, Rom. Journ. Phys., **52** (2007) 85–91

[3] *A. S. Eegunjobi, O.D. Makinde*, Combined Effect of Buoyancy Force and Navier Slip onEntropy Generation in a Vertical Porous Channel, Entropy, **14** (2012), 1028-1044

[4] *S. O. Adesanya, O. D. Makinde*, MHD oscillatory slip flow and Heat transfer in a channel filled with porous media, Rom. Journ.Phys.,**76**(2014) 197-204

[5] *A. C. L. Cogley, W. G. Vinvent, E. S. Giles,* American Institute of Aeronautics and Astronautics (1968) 551