

Deformation Due to Inclined Load in Thermoelastic Half-Space with Voids

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Abstract

The Two- dimensional deformation of homogeneous, isotropic, thermoelastic half-space with voids as a result of inclined line load is investigated by applying the Laplace and Fourier transforms. The inclined load is assumed to be a linear combination of a normal load and a tangential load. The displacements, stresses, temperature distribution and change in volume fraction field are obtained in the physical domain and some isolated cases are stated with a particular case as results.

Keywords; Thermoelasticity, concentrated force, Laplace and Fourier transforms

1.0 Introduction

The Theory of linear elastic materials with voids is one of the most important generalizations of the classical theory of elasticity. This theory has practical utility for investigating various types of geological and biological materials for which the classical elastic theory is inadequate. This theory is concerned with elastic materials having a distribution of small pores (voids) in which the void volume is included among the kinematics variable and in the limiting case of volume tending to zero, the theory reduces to the classical theory of elasticity.

A nonlinear theory of elastic materials with voids was developed by Nunziato and Cowin [1]. Later, Cowin and Nunziato [2] developed a theory of linear elastic materials with voids for the mathematical study of the mechanical behavior of porous solids. They considered several applications of the linear theory by investigating the response of the materials to homogeneous deformations, pure bending of beams and small amplitudes of acoustic waves. Puri and Cowin [3] studied the behavior of plane waves in linear elastic materials with voids. Domain of influence theorem in the theory of elastic materials with voids was discussed by Dhaliwal and Wang [4]. Scarpetta [5] studied the well-posedness theorems for linear elastic materials with voids. Birsan [6] established the existence and uniqueness of weak solutions in the linear theory of elastic shells with voids

Rusu [7] studied the existence and uniqueness of solutions in thermoelastic materials with voids. Saccomandi [8] presented some remarks about the thermoelastic theory of materials with voids. Ciarletta and Scalia [9] discussed the nonlinear theory of nonsimple thermoelastic materials with voids. Dhaliwal and Wang [10] developed the heat-flux dependent theory of thermoelasticity with voids. Marin [11, 12] studied the uniqueness and domain of influence results for thermoelastic bodies with voids. Marin [13] presented the contributions on uniqueness in thermoelastodynamics for bodies with voids. Marin and Salca [14] obtained the relation of the KnopT-de Hoop type in thermoelasticity of dipolar bodies with voids. Chirita and Scalia [15] studied the spatial and temporal behaviour in linear thermoelasticity of materials with voids.

When the source surface is very long in one direction in comparison with the others, the use of two-dimensional approximation is justified and consequently, calculations are simplified to a great extent and one gets analytical solutions in closed form. A very long strip-sources and a very long line-source are examples of such two-dimensional sources. Love [16] obtained expressions for the displacements due to a line source in an isotropic elastic medium. Maruyama [17] obtained the displacement and stress fields corresponding to long strike-slip faults in a homogeneous isotropic half-space. Okada [18,19] presented a compact analytic expressions for the surface deformation and internal deformation due to inclined shear and tensile faults in a homogeneous isotropic half-space. Several authors [20, 21, 22, 23] discussed the problems of inclined load in the theory of elastic solids. No attempt has been made so far to study the response to inclined load in a thermoelastic body with voids.

We study the general plane strain problem of thermoelastic half-space with voids due to different sources. The integral transform techniques has been used to solve it. We have obtained the expression for displacements, stresses, temperature distribution and change in volume fractions field in a thermoelastic half-space with voids due to an inclined line

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load in the form of Laplace and Fourier transforms, which could be converted to the original solution numerically but was not done as it is not a subject of this paper. The deformation due to other sources such as strip loads, continuous line loads etc., can also be similarly obtained.

2.0 Basic Related Mathematical Equations

Following Lord-Shulman [24], Green-Lindsay [25] and Cowin and Nunziato [2], the field equations and constitutive relations in thermoelastic solid with voids without body forces, heat sources and extrinsic equilibrated body force can be written as:

$$\mu \nabla^2 u + (\lambda + \mu) \nabla(\nabla \cdot u) + b \nabla \phi - \beta \nabla(T + \delta_{2kT_1} \dot{T}) = \rho \ddot{u}, \tag{1}$$

$$\alpha \nabla^2 \phi - b(\nabla \cdot u) - \xi_1 \dot{\phi} - \omega_0 \ddot{\phi} + mT = \rho \chi \ddot{\phi}, \tag{2}$$

$$K \nabla^2 T - \beta T_0 (\nabla \cdot \dot{u} + \tau_0 \delta_{1k} \nabla \cdot \ddot{u}) - mT_0 \dot{\phi} = \rho c_e (\dot{T} + \tau_0 \ddot{T}), \tag{3}$$

$$t_{ij} = \lambda u_{G,G} \delta_{ij} + \mu (u_{ij} + u_{j,i}) + b \phi \delta_{ij} - \beta (T + \delta_{2k} \tau_1 \dot{T}) \delta_{ij}, \quad (i, j = x, y, z). \tag{4}$$

In Eqs. (1) – (4) we have used the notations: λ, μ as Lamme constants, α (stress times squared length), b, ξ_1 (Stress), ω_0 (stress times time), χ (equilibrated inertia) which are material constants due to the presence of voids, m (stress temperature coefficient) material constant due to the presence of voids and temperature, T – temperature change, $\beta = (3\lambda + 2\mu)\alpha_t, \alpha_t$ – linear thermal expansion coefficient u – displacement vector, t_{ij} - stress tensor, ρ, c_e - density and specific heat at constant strain, respectively, K -thermal conductivity, temperature; a superposed dot denotes differentiation with respect to time, variables t, τ_0, τ_1 are thermal relaxation times. For the L-S theory, $\tau_1 = 0, \delta_{1k} = 1$ and for the G-L theory $\tau_1 > 0, \delta_{1k} = 0$ (i.e., $k=1$ for the L-S theory and $k=2$ for the G-L theory). The thermal relaxations τ_0 and τ_1 satisfy the inequality $\tau_1 \geq \tau_0 > 0$ for the G-L theory only,

$$\nabla = \hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z}, \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \tag{5}$$

are the gradient and Laplacian operators respectively.

3.0 Formulation and solution of the problem

We consider a homogeneous, isotropic, thermally conducting elastic half-space with voids in the undeformed state at uniform temperature T_0 . The rectangular Cartesian coordinate system (x,y,z) with z-axis pointing vertically into the medium is introduced.

Suppose that an inclined line load F_0 per unit length is acting on the y-axis and its inclination to z-direction is θ . To simplify the algebra, only problems with zero initial conditions are considered.

For a two-dimensional problem, we assume $u = (u, 0, w)$ and Eqs. (1) – (4).

We introduce the dimensionless quantities:

$$\begin{aligned} x' &= \frac{\omega_1^*}{c_2} x, & z' &= \frac{\omega_1^*}{c_2} z, & t' &= \omega_1^* t, & u &= \frac{\omega_1^*}{c_2} u. \\ u' &= \frac{\omega_1^*}{c_2} w, & T' &= \frac{T}{T_0}, & \phi' &= \frac{\omega_1^{*2} \chi}{c_2^2} \phi, \end{aligned} \tag{6}$$

$$\begin{aligned} \epsilon_1 &= \frac{\beta c_2^2}{K \omega_1^*}, & \tau_0' &= \omega_1^* \tau_0, & \tau_1' &= \omega_1^* \tau_1, & a' &= \frac{\omega_1^*}{c_2} a, \\ t'_{zz} &= \frac{t_{zz}}{\beta T_0}, & t'_{zx} &= \frac{t_{zx}}{\beta T_0}, & h' &= \frac{h c_2}{\omega_1^*}, \end{aligned} \tag{7}$$

Where

$$c_2 = \left(\frac{\mu}{\rho}\right)^{1/2} \quad \text{and} \quad \omega_1^* = \frac{\rho c_e c_2^2}{K} \tag{8}$$

The expression relating the displacement components $u(x,z,t)$ and $w(x,z,t)$ to the scalar potential functions $\psi_1(x,z,t)$ and $\psi_2(x,z,t)$ in dimensionless form are given by

$$u = \frac{\partial \psi_1}{\partial x} - \frac{\partial \psi_2}{\partial z}, \quad w = \frac{\partial \psi_1}{\partial z} + \frac{\partial \psi_2}{\partial x} \tag{9}$$

Applying the Laplace and Fourier transforms

$$\hat{f}(x,z,p) = \int_0^\infty f(x,z,t)e^{-pt} \quad \text{and} \quad \hat{f}(\xi,z,p) = \int_{-\infty}^\infty \hat{f}(x,z,p)e^{i\xi x} dx \tag{10}$$

To equations (1) – (3), after using Eqs. (6), (7) (suppressing the primes) and eliminating $\tilde{\psi}_1, \tilde{\phi}, \tilde{T}$ and $\tilde{\psi}_2$ from the resulting expressions, we obtain

$$\left(\frac{d^6}{dz^6} + Q\frac{d^4}{dz^4} + N\frac{d^2}{dz^2} + I\right)(\tilde{\psi}_1, \tilde{\phi}, \tilde{T}) = 0, \tag{11}$$

and

$$\left(\frac{d^2}{dz^2} - \lambda_4^2\right)\tilde{\psi}_2 = 0, \tag{12}$$

The roots of Eqs. (11) and (12) are $\pm \lambda_\ell (\ell = 1,2,3,4)$. Assuming the regularity condition at $z = \infty$, the solution of Eqs. (11) and (12) may be written as

$$\tilde{\psi}_1 = A_1 \bar{e}^{\lambda_1 z} + A_2 \bar{e}^{\lambda_2 z} + A_3 \bar{e}^{\lambda_3 z} \tag{13}$$

$$\tilde{\phi} = d_1 A_1 \bar{e}^{\lambda_1 z} + d_2 A_2 \bar{e}^{\lambda_2 z} + d_3 A_3 \bar{e}^{\lambda_3 z} \tag{14}$$

$$\tilde{T} = e_1 A_1 \bar{e}^{\lambda_1 z} + e_2 A_2 \bar{e}^{\lambda_2 z} + e_3 A_3 \bar{e}^{\lambda_3 z}, \tag{15}$$

$$\tilde{\psi}_2 = A_4 \bar{e}^{\lambda_4 z} \tag{16}$$

with $A_\ell (\ell = 1,2,3,4)$ being arbitrary constants and e_ℓ and d_ℓ are given in Appendix C.

4.0 Application

Consider a normal line load of intensity F_1 , per unit length, acting in the positive z-direction on the plane boundary $z = 0$ along the y-axis and a tangential line load F_2 , per unit length, acting at the origin in the positive x-direction then the boundary conditions are

$$t_{zz}(x,z,t) = F_1 \psi(x) \delta(t), \quad t_{zx}(x,z,t) = -F_2 \zeta(x) \delta(t),$$

$$\frac{\partial \phi}{\partial z} = 0, \quad \frac{\partial T}{\partial z} + hT = 0 \quad z = 0, \tag{17}$$

where $\delta(t)$ is the Dirac delta function, $\psi(x)$ and $\zeta(x)$ denote the vertical and horizontal load functions, respectively, distributed along the x-axis, h is the heat transfer coefficient, F_1 and F_2 are force intensities.

Using Eqs. (5) and (6) along with $F_1' = \frac{F_1}{\beta T_0}$, $F_2' = \frac{F_2}{\beta T_0}$ in the boundary conditions (17) (suppressing the primes for convenience) and applying the Laplace and Fourier transforms defined by Eq. (10), we obtain the transformed boundary conditions as

$$\begin{aligned} \tilde{t}_{zz}(\xi, z, p) &= -F_1 \tilde{\psi}(\xi), & \tilde{t}_{zx}(\xi, z, p) &= -F_2 \tilde{\zeta}(\xi), \\ \frac{d\tilde{\phi}}{dz} &= 0, & \frac{d\tilde{T}}{dz} + h\tilde{T} &= 0 \end{aligned} \tag{18}$$

where $\tilde{\psi}(\xi)$ and $\tilde{\zeta}(\xi)$ are the Fourier transforms of $\psi(x)$ and $\zeta(x)$ respectively.

Making use of Eqs. (4) – (9) (suppressing the primes for convenience) and applying the Laplace and Fourier transform defined by (10) in the transformed boundary conditions (18) and substituting the values of $\tilde{\psi}_1, \tilde{\psi}_2, \tilde{T}, \tilde{\phi}$ from equations (13) - (16), we obtain the expressions for displacement components, stresses, temperature distribution and change in the volume fraction field respectively as follows:

$$\begin{aligned} \tilde{u} = & -\frac{i\xi}{\Delta} \left\{ F_1 \tilde{\psi}(\xi) (\Delta_1 e^{-\lambda_1 z} - \Delta_3 e^{-\lambda_3 z} + \Delta_5 e^{-\lambda_3 z} - \Delta_7 e^{-\lambda_4 z}) \right. \\ & \left. + F_2 \tilde{\zeta}(\xi) (\Delta_2 e^{-\lambda_1 z} - \Delta_4 e^{-\lambda_3 z} + \Delta_6 e^{-\lambda_3 z} - \Delta_8 e^{-\lambda_4 z}) \right\}, \end{aligned} \tag{C.1}$$

$$\begin{aligned} \tilde{\omega} = & -\frac{1}{\Delta} \left\{ F_1 \tilde{\psi}(\xi) (\lambda_1 \Delta_1 e^{-\lambda_1 z} - \lambda_2 \Delta_3 e^{-\lambda_3 z} + \lambda_3 \Delta_5 e^{-\lambda_3 z} - i\xi \Delta_7 e^{-\lambda_4 z}) \right. \\ & \left. + F_2 \tilde{\zeta}(\xi) (\lambda_1 \Delta_2 e^{-\lambda_1 z} - \lambda_2 \Delta_4 e^{-\lambda_3 z} + \lambda_3 \Delta_6 e^{-\lambda_3 z} - i\xi \Delta_8 e^{-\lambda_4 z}) \right\}, \end{aligned}$$

$$\begin{aligned} \tilde{t}_{zz} = & \frac{1}{\Delta} \left\{ F_1 \tilde{\psi}(\xi) (s_1 \Delta_1 e^{-\lambda_1 z} - s_2 \Delta_3 e^{-\lambda_3 z} + s_3 \Delta_5 e^{-\lambda_3 z} - s_4 \Delta_7 e^{-\lambda_4 z}) \right. \\ & \left. + F_2 \tilde{\zeta}(\xi) (s_1 \Delta_2 e^{-\lambda_1 z} - s_2 \Delta_4 e^{-\lambda_3 z} + s_3 \Delta_6 e^{-\lambda_3 z} - s_4 \Delta_8 e^{-\lambda_4 z}) \right\}, \end{aligned} \tag{C.2}$$

$$\begin{aligned} \tilde{t}_{zx} = & \frac{1}{\Delta} \left\{ F_1 \tilde{\psi}(\xi) (\lambda_1 \Delta_1 e^{-\lambda_1 z} - \lambda_2 \Delta_3 e^{-\lambda_3 z} + \lambda_3 \Delta_5 e^{-\lambda_3 z} + n_1 \Delta_7 e^{-\lambda_4 z}) \right. \\ & \left. + F_2 \tilde{\zeta}(\xi) (\lambda_1 \Delta_2 e^{-\lambda_1 z} - \lambda_2 \Delta_4 e^{-\lambda_3 z} + \lambda_3 \Delta_6 e^{-\lambda_3 z} + n_1 i \xi \Delta_8 e^{-\lambda_4 z}) \right\}, \end{aligned}$$

$$\begin{aligned} \tilde{T} = & \frac{1}{\Delta} \left\{ F_1 \tilde{\psi}(\xi) (e_1 \Delta_1 e^{-\lambda_1 z} - e_2 \Delta_3 e^{-\lambda_3 z} + e_3 \Delta_5 e^{-\lambda_3 z}) \right. \\ & \left. + F_2 \tilde{\zeta}(\xi) (e_1 \Delta_2 e^{-\lambda_1 z} - e_2 \Delta_4 e^{-\lambda_3 z} + e_3 \Delta_6 e^{-\lambda_3 z}) \right\}, \end{aligned} \tag{C.3}$$

$$\begin{aligned} \tilde{\phi} = & \frac{1}{\Delta} \left\{ F_1 \tilde{\psi}(\xi) (d_1 \Delta_1 e^{-\lambda_1 z} - d_2 \Delta_3 e^{-\lambda_3 z} + d_3 \Delta_5 e^{-\lambda_3 z}) \right. \\ & \left. + F_2 \tilde{\zeta}(\xi) (d_1 \Delta_2 e^{-\lambda_1 z} - d_2 \Delta_4 e^{-\lambda_3 z} + d_3 \Delta_6 e^{-\lambda_3 z}) \right\}, \end{aligned}$$

where

$$\begin{aligned} \Delta &= \Delta_1^* + h\Delta_2^*, \\ \Delta_1^* &= \lambda_2 \lambda_3 (s_4 \lambda_1 - n_1 s_1) (d_3 e_2 - d_2 e_3) + \lambda_1 \lambda_3 (s_4 \lambda_2 + n_1 s_2) (d_1 e_3 + e_4 d_3) \\ &\quad + \lambda_1 \lambda_2 (s_4 \lambda_3 + n_1 s_3) (d_1 e_2 - e_1 d_2), \\ \Delta_2^* &= (s_4 \lambda_1 - n_1 s_1) (e_3 \lambda_2 d_2 - e_2 d_3 \lambda_3) + (e_3 \lambda_1 d_1 - e_1 \lambda_3 d_3) (s_4 \lambda_2 + n_1 s_4) \\ &\quad - (s_4 \lambda_3 + n_1 s_3) (e_1 \lambda_2 d_2 - e_2 \lambda_1 d_1), \\ \Delta_{1,2} &= (r_4, s_4) \Delta_{10} & \Delta_{3,4} &= (-n_1 s_4) \Delta_{20}, & \Delta_{5,6} &= (-n_1, s_4) \Delta_{30}, \\ \Delta_7 &= [\lambda_1 \lambda_2 \lambda_3 \{ e_3 (d_1 - d_2) + e_2 (d_3 - d_1) - e_1 (d_3 - d_2) \} \\ &\quad + h \{ \lambda_1 (e_3 d_2 \lambda_2 - e_2 d_3 \lambda_3) - \lambda_2 (e_3 \lambda_1 - e_1 \lambda_3 d_3) \\ &\quad + \lambda_3 (\lambda_1 d_1 e_2 - e_1 \lambda_2 d_2) \}]. \end{aligned}$$

$$\Delta_8 = \left[\left\{ s_2 \lambda_1 \lambda_3 e_3 (d_1 e_3 - e_1 d_2) + s_1 \lambda_2 \lambda_3 (e_2 d_3 - e_3 d_2) - s_3 \lambda_2 \lambda_1 (e_1 d_2 - e_2 d_1) \right\} \right. \\ \left. + h \left\{ s_1 (e_3 d_2 \lambda_2 - e_2 d_3 \lambda_3) - s_2 (e_3 \lambda_1 d_1 - e_1 \lambda_3 d_3) \right. \right. \\ \left. \left. + s_3 (\lambda_1 d_1 e_2 - e_1 \lambda_2 d_2) \right\} \right]$$

$$\Delta_{10} = (e_2 d_3 - d_2 e_3) \lambda_2 \lambda_3 + h(e_3 \lambda_2 d_2 - \lambda_3 d_3 e_2),$$

$$\Delta_{20} = (e_1 d_3 - d_1 e_3) \lambda_1 \lambda_3 + h(e_3 \lambda_1 d_1 - e_1 \lambda_3 d_3),$$

$$\Delta_{30} = (e_1 d_2 - d_1 e_2) \lambda_1 \lambda_2 + h(e_2 \lambda_1 d_1 - \lambda_2 d_2 e_1),$$

$$s_\ell = -i\xi s'_{10} - \lambda_\ell^2 s'_{20} + d_\ell s'_{30} + \epsilon_\ell s'_{40} \quad (\ell = 1, 2, 3),$$

$$s_4 = -(s'_{10} + i\xi s'_{20}) \lambda_4, \quad s'_{10} = \frac{-i\xi \lambda}{\beta T_0},$$

$$s_{20} = \frac{-(\lambda + 2\mu)}{\beta T_0}, \quad s'_{30} = \frac{bc_2^2}{\beta T_0 \omega_1^{*2} \chi},$$

$$s_{40} = (1 + p\tau_1 \delta_{2k}), \quad n_1 = \frac{\lambda_4^2 + \xi^2}{2i\xi}.$$

4.1 Some Important Results

4.1.1 Inclined line load

For an inclined line load F_0 per unit length, we have

$$F_1 = F_0 \cos \theta, \quad F_2 = F_0 \sin \theta. \tag{19}$$

CASE 1. Concentrated force

In this case

$$\zeta(x) = \delta(x), \quad \psi(x) \delta(x) \\ \text{with} \\ \tilde{\zeta}(x) = 1, \quad \tilde{\psi}(\xi) = \xi \tag{20}$$

CASE 2. Uniformly distributed force

The solution due to uniformly distributed force applied to the half-space surface is obtained by setting

$$\{\zeta(x), \psi(x)\} = \begin{cases} 1 & \text{if } |x| \leq a, \\ 0 & \text{if } |x| > a, \end{cases} \tag{21}$$

in Eq. (18). The Laplace and Fourier transforms with respect to the pair (x, ξ) for the case of a uniform strip load of unit amplitude and width $2a$ applied at the origin of the coordinate system $(x = z = 0)$ in dimensionless form, after suppressing the primes, becomes

$$\{\tilde{\zeta}(\xi), \tilde{\psi}(x)\} = \left[2 \sin \left(\frac{\xi c_2 a}{\omega_1^*} \right) / \xi \right], \quad \xi \neq 0. \tag{22}$$

CASE 3. Linearly distributed force

The solution due to linearly distributed force applied to the half-space surface is obtained by setting.

$$\{\zeta(x), \psi(x)\} = \begin{cases} 1 - \frac{|x|}{a} & \text{if } |x| \leq a, \\ 0 & \text{if } |x| > a, \end{cases} \quad (23)$$

in Eq. (17), where $2a$ is the width of the strip load. Using Eqs. (6) – (7) (suppressing the primes) and applying the transforms defined by Eq. (10), we get

$$\left\{ \tilde{\zeta}(\xi), \tilde{\psi}(x) \right\} = \left[\frac{2 \left\{ 1 - \cos \left(\frac{\xi c_2 a}{\omega_1^*} \right) \right\}}{\frac{\xi^2 c_2 a}{\omega_1^*}} \right]. \quad (24)$$

Using Eq. (20) in Eqs. (C.1) and with the aid of Eqs. (21) – (22), we obtain the expressions for displacements, stresses, temperature distribution and change in the volume fraction field for different sources applied on the surface of the thermoelastic half-space with voids.

4.1.2 Particular case

If we neglect the voids effect, [27], i.e. $(\alpha = b = \xi = m\chi = \omega_0 = 0)$ in the Eqs. (C.1) along with Eqs. (20), we can also obtain the expressions for displacement components, stresses and temperature distribution in the thermoelastic half-spaces.

SUB-CASE 1: If $h \rightarrow 0$, Eqs. (C.1) yield the expressions of displacements, stresses, temperature distribution and change in the volume fraction field for the insulated boundary.

SUB-CASE 2: If $h \rightarrow \infty$, Eqs. (C.1) yield the expressions of displacements, stresses, temperature distribution and change in the volume fraction field for the isothermal boundary.

SPECIAL CASE 1: By putting $k=1$ and $\tau_1 = 0$ in Eqs. (C. 1), we recover the displacements, stresses, temperature distribution and change in volume fraction field for L.S theory.

SPECIAL CASE 2: For the G-L theory, we obtain the corresponding expressions for displacements, stresses, temperature distribution and change in the volume fraction field by substituting $k=2$ in Eqs. (C.1).

SPECIAL CASE 3: The expressions for displacements, stresses, temperature distribution and change in the volume fraction field for the theory of coupled thermoelasticity (CT) are obtained by putting $k = 3, \tau_0 = \tau_1 = 0$ in Eqs. (C.1).

5.0 Inversion of the transforms

To obtain the solution of the problem in the physical domain, we must invert the transforms in equations (C.1) for the two theories, i.e., L-S and G-L. These expressions are functions of z , the parameters of Laplace and Fourier transforms p and ξ , respectively, and hence they are of the form $\tilde{f}(\xi, z, p)$. To get the function $f(x, z, t)$ in the physical domain, first we invert the Fourier transform using the formula:

$$\tilde{f}(x, z, p) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\xi x} \tilde{f}(\xi, z, p) d\xi = \frac{1}{\pi} \int_0^{\infty} (\cos(\xi x) f_e - i \sin(\xi x) f_0) d\xi, \quad (25)$$

where f_e and f_0 are, respectively, even and odd parts of the function $\hat{f}(\xi, z, p)$.

Thus, expression (25) gives us the Laplace transform $\hat{f}(x, z, p)$ can be inverted to $f(x, z, t)$.

The last step is to calculate the integral in Eq. (5.1). The method for evaluating this integral is described by Press *et al* [26] and Kumar and Ranu [27], and it involves the application of Romberg's integration with adaptive step size. This uses also the results of successive approximations of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

The numerical results of this analysis is not the subject of this paper and will be shown in the next paper.

6.0 Conclusion

The expressions for displacements, stresses and temperature distribution in the case of inclined line load can be obtained for concentrated, uniformly, and linearly distributed force by application of equations (20) – (23) in (C.1).

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