Journal of the Nigerian Association of Mathematical Physics
Volume 26, (*March, 2014*), pp 593 – 596
© J. of NAMP

Fundamental changes in stability of a mutualistic interaction with two similar intrinsic growth rate parameter values and two dis-similar carrying capacities

[1]E.N. Ekaka-a, [2]N.E.S. Lale, [1]N.M. Nafo and [3]I.A. Agwu

[1]Department of Mathematics and Computer Science,
Rivers State University of Science and Technology, Nkpolu, Port Harcourt, Nigeria
[2]Department of Soil and Crop Science,
Faculty of Agriculture, University of Port Harcourt, Port Harcourt, Nigeria
[3]Department of Mathematics,
Abia State Polytechnic, Aba, Abia State

Abstract

The study of the stability with respect to a mutualistic interaction between two legumes with two dis-similar carrying capacities is a challenging crop science problem which can be successfully analysed using a mathematical simulation approach. In this paper, we propose to study the stability of a mutualistic interaction when the intra-competition parameter value for groundnut is varied while other parameter values remain fixed in the context when the doubling time for cowpea and groundnut is about 31 days. The results which we have obtained have not been seen elsewhere, they are presented here and discussed.

Keywords: co-existence steady-state solution, stability, interacting legumes

1.0 Introduction

Within the mathematical literatures, the idea of stability has been clearly defined and applied to tackle diverse mathematical and scientific problems [1, 2, 3, 4, 5, 6]. For the purpose of this study, we have considered when the two intrinsic growth rate parameter values are the same in the unit of grams. In a similar context, our present analysis is based on the data of Ekpo and Nkanang [7]. In this scenario, the carrying capacity of cowpea is 3.26 grams while the carrying capacity of groundnut is 1.69 grams. In this situation when the carrying capacity of cowpea is almost twice bigger than the carrying capacity of groundnut, crop scientists are keen to know how this change will affect the co-existence of these interacting legumes on one hand and the impact of the same change in the carrying capacity pattern on the stability of the co-existence steady-state solution in a mathematical sense.

In the work of Ekaka-a et al. [8], a mathematical model for two interacting yeast species was adapted based on the original formulation of Pielou [9] while our present mathematical model concerns the interaction between two legumes such as cowpea and groundnut. For the yeast species interaction the growth rate parameters have the values of 0.1 and 0.08 while the intra-species competition parameter values are 0.0014 and 0.001 which imply that the estimated carrying capacities are 71.4 and 80 in the unit of grams. For the present model description between two legumes, the estimated growth rate parameter value and the intra-species competition parameter value for cowpea are 0.0225 and 0.006902 implying that the carrying capacity value which can sustain the growth of cowpea is 3.26 grams. The estimated growth rate parameter value and the intra-species competition parameter value for groundnut are 0.0225 and 0.0133 implying that the carrying capacity value which can sustain the growth of groundnut is 1.69 grams. By comparing the two carrying capacities of yeast interaction model and that of the interaction between two legumes, we can see that it would take a bigger carrying capacity to sustain the growth of yeast species than that of our present model. While the mathematical structure in the work of Ekaka-a et al. [8] is a Lotka-Volterra competition system, our present mathematical structure is a Lotka-Volterra mutualistic system. In the work of Ekaka-a et al. [8], the effect of the inter-species competition parameters on the onset of stability and degeneracy of co-existence steady-state solutions between competing populations was examined in contrast to our present analysis in which we propose to determine the fundamental changes in stability of a mutualistic interaction with two similar intrinsic growth rate parameter values and two dis-similar carrying capacities when the intra-species competition coefficient of groundnut is varied

[1]Corresponding author: ***E.N. Ekaka-a,*** E-mail: alukere@yahoo.co.uk, Tel.: +2347066441590

and other model parameter values are fixed. The new contribution of our present analysis lies in the alternative application of a mutualistic interaction between two legumes to explore the changes in the pattern of the co-existence steady-state solution and its stability especially as cowpea and groundnut characteristically have a mutualistic type of interaction because they have similar abiotic-stress effect.

In the work of Ekaka-a et al. [10], the effect of the intra-species competition coefficients on the onset of stability, instability and degeneracy of co-existence steady-state solutions between competing yeast species was studied while we propose to determine the fundamental changes in stability of a mutualistic interaction with two similar intrinsic growth rate parameter values and two dis-similar carrying capacities.

In the work of Ekaka-a et al. [11], the stability analysis for a system of competing legumes with a dis-similar carrying capacity having two dis-similar intrinsic growth rate parameter values (a = 0.0225 and d = 0.05) and two dis-similar carrying capacities (a = 0.0225, b = 0.006902, d = 0.05, f = 0.0133) was studied while we propose to determine the fundamental changes in stability of a mutualistic interaction with two similar intrinsic growth rate parameter values and two dis-similar carrying capacities. In the current analysis of Ekaka-a et al. [11], the focus of the interaction between two legumes is a pure (-, -) response which corresponds to a scenario when cowpea competes to inhibit the growth of groundnut and vice-versa. In contrast, our new contribution proposes to look at the effect of a mutualistic interaction between cowpea and groundnut with an inherent abiotic-stress factor advantage to calculate several co-existence steady-state solutions and their qualitative patterns of stability.

While the interaction between two legumes with a similar carrying capacity [12] in which several co-existence steady-state solutions and their stability type were examined due to a variation of the intrinsic growth rate parameter has been current studied, our current simulation analysis is a clear point of departure from our most recent analysis. In the theory of ecology, the competition interaction between two legumes having a similar abiotic-stress factor with a similar carrying capacity can differ significantly from the mutualistic interaction between two legumes having a similar abiotic-stress factor with two-similar growth rates and two dis-similar carrying capacities. This open complex ecological problem is yet to be solved by another group of numerical mathematicians because this proposed problem would require a mathematical reasoning in order to provide a successful insight.

2.0 Mathematical Formulation

Following Ekaka-a [5], we consider the following system of model equations of continuous nonlinear first order ordinary differential equations

$$\frac{dC(t)}{dt} = C(t)[a - bC(t) + cG(t)] \tag{1}$$

$$\frac{dG(t)}{dt} = G(t)[d - fG(t) + eC(t)] \tag{2}$$

Here, the notations C(0) > 0 and G(0) > 0 define the starting biomasses of cowpea and groundnut at the start of the growing season otherwise called the initial conditions when t = 0. The duration of growth is in the unit of days hereby denoted by the independent variable t. For the purpose of this simulation study, the best-fit model parameters such as a and d that define the intrinsic growth rates for cowpea and groundnut were selected using the data of Ekpo and Nkanang [7]. The next best-fit parameters such as b and f define the intra-species competition parameters which measure the inhibiting factors on the growth of cowpea and groundnut due to self-interaction whereas the parameters c and e define the inter-species competition parameters which also measure other inhibiting factors on the growth of cowpea and groundnut due to interspecific interaction between cowpea and groundnut. In this study, we have considered the following parameter values: a = 0.0225, d = 0.0225, b = 0.006902, f = 0.0133, c = 0.0005, e = 0.01.

3.0 Method of Solution

Following the recent idea of Ekaka-a and Agwu [2013], the two carrying capacities for the interacting legumes were defined and coded using a Matlab programming language. Secondly, the co-existence steady-state solution which was derived analytically by solving the two simultaneous linear equations in terms of C and G which were obtained by equating the growth rate equations to zero was also coded. Thirdly, the four partial derivatives of the two interaction functions in terms of C and G with respect to C and G were derived and evaluated at the arbitrary co-existence steady-state solution or point (C, G). Fourthly, a Jacobian matrix of four elements were constructed and coded from which two eigenvalues were calculated computationally and tested to be consistent with their counterpart analytical calculations. From the theory of the sign method in the study of stability of a steady-state solution, the qualitative values of the eigenvalues were determined which form the basis for each type of stability of a co-existence steady-state solution. If upon the evaluation of the Jacobian matrix and we obtain either two positive eigenvalues or eigenvalues of opposite signs then the co-existence steady-state solution can be classified as being unstable. On the other hand, if two negative eigenvalues were obtained then the co-existence steady-state solution is said to be stable. It should also be noted that if any of the co-ordinates of the co-existence steady-state solution bears a negative sign, this observation has a counter-intuitive ecological meaning. In this scenario, such a steady-state solution can be classified as being degenerate. When a steady-state solution is degenerate, it should be considered as a quantitative indication in which one of the interacting legumes can go into the ecological risk of extinction escaping survival while the other legume can tend to survive.

4.0 Results and Discussion

The results which we have obtained and have not been seen elsewhere are presented and discussed here in the Tables below: the notation a stands for the intrinsic growth rate parameter value for cowpea, the notation ss stands for the co-existence steady-state solution while the notations λ_1 and λ_2 stand for the two eigenvalues whose signs define the type of stability for the co-existence steady-state solution. Typical examples which clearly illustrate the application of our present methodology are presented in the Tables below and discussed.

Table 1: Calculating the qualitative stability of a co-existence steady-state solution due to a variation of the intra-competition coefficient 'f' of groundnut: summary of results 1

Example	f	ss	λ_1	λ_2	Type of stability
1	0.0133	3.58: 4.38	-0.0225	-0.0605	stable
2	0.0007	-63.91:-927.16	-0.0225	1.0801	degenerate
3	0.0013	9.85:90.99	-0.0225	-0.1665	stable
4	0.0020	6.40:43.37	-0.0225	-0.1082	stable
5	0.0027	5.32:28.47	-0.0225	-0.0900	stable
6	0.0033	4.80:21.20	-0.0225	-0.0810	stable
7	0.0040	4.48:16.87	-0.0225	-0.0758	stable
8	0.0047	4.28:14.02	-0.0225	-0.0723	stable
9	0.0053	4.13:11.99	-0.0225	-0.0698	stable
10	0.0060	4.02:10.47	-0.0225	-0.0679	stable

In thirty eight (33) typical examples as illustrated in Table 1, Table 2 and Table 3, we observe that in only one case, degeneracy of a co-existence steady-state solution has occurred while in 32 cases the phenomenon of degeneracy is lost having all stable co-existence steady-state solutions. The implication of the degeneracy indicates that the two interacting legumes are less vulnerable to the ecological risk of extinction whereas in 32 instances both cowpea and groundnut will only co-exist together and survive together. This contribution can have interesting ecosystem functioning and stability application and its rich application in the mathematical study of stability based on the popular sign method. We have used a mathematical approach to identify one region of degeneracy and several instances of stability which can rarely be deduced in a single inter-disciplinary experimental study.

It is worth noting that when the intra-competition coefficient parameter value is 0.0133, the expected estimated biomasses for cowpea and groundnut which guarantee the co-existence of these interacting legumes are 3.58 grams and 4.38 grams respectively. When the value of the parameter value f is 0.0007, there is a shift from a stable co-existence steady-state solution to several other typical examples of co-existence steady-state solutions which permit the survival of two growing legumes such as cowpea and groundnut. In this present study when a variation of the intra-specific competition coefficient of groundnut is considered, a dominant stable co-existence steady-state solution has been observed.

Table 2: Calculating the qualitative stability of a co-existence steady-state solution due to a variation of the intra-competition coefficient 'f' of groundnut: summary of results 2

Example	f	ss	λ_1	λ_2	Type of stability
11	0.0067	3.93:9.30	-0.0225	-0.0665	stable
12	0.0073	3.87:8.36	-0.0225	-0.0653	stable
13	0.0080	3.81:7.60	-0.0225	-0.0644	stable
14	0.0086	3.76:6.96	-0.0225	-0.0636	stable
15	0.0093	3.73:6.42	-0.0225	-0.0630	stable
16	0.0100	3.69:5.96	-0.0225	-0.0624	stable
17	0.0106	3.66:5.56	-0.0225	-0.0619	stable
18	0.0113	3.64:5.21	-0.0225	-0.0615	stable
19	0.0120	3.62:4.90	-0.0225	-0.0611	stable
20	0.0126	3.60:4.63	-0.0225	-0.0608	stable

Table 3: Calculating the qualitative stability of a co-existence steady-state solution (ss) due to a variation of the intra-competition coefficient 'f' of groundnut : summary of results 3

Example	f	ss	λ_1	λ_2	Type of stability
21	0.0134	3.57:4.34	-0.0225	-0.0604	stable
22	0.0140	3.56:4.16	-0.0225	-0.0602	stable
23	0.0146	3.55:3.96	-0.0225	-0.0600	stable
24	0.0153	3.53:3.78	-0.0225	-0.0597	stable
25	0.0160	3.52:3.62	-0.0225	-0.0595	stable
26	0.0166	3.51:3.47	-0.0225	-0.0593	stable
27	0.0173	3.50:3.33	-0.0225	-0.0592	stable
28	0.0180	3.49:3.20	-0.0225	-0.0590	stable
29	0.0186	3.48:3.08	-0.0225	-0.0589	stable
30	0.0193	3.48:2.97	-0.0225	-0.0587	stable
31	0.0199	3.47:2.87	-0.0225	-0.0586	stable
32	0.0206	3.46:2.77	-0.0225	-0.0585	stable
33	0.0213	3.45:2.68	-0.0225	-0.0584	stable

5.0 Conclusion

In this study when the two intrinsic growth rate parameter values are dis-similar, we have used a mathematical approach called numerical simulation to identify one instance of bifurcation in the qualitative behaviour of stability. We have found a fundamental change in the behaviour of stability when the value of the intra-competition coefficient f lies between 0.0133 and 0.0007. In the first observation which corresponds to the closed interval of the intra-competition competition [0.0133, 0.0007], the value of 0.0133 responds to the stability of a co-existence steady-state solution while the value of 0.0007 responds to the degeneracy of a co-existence steady-state solution in which the two legumes are driven into extinction. The mathematical concept of bisection method can be utilized to check for further fundamental changes in the behaviour of stability when the intra-competition parameter value is varied as other model parameters are fixed. Our present contribution can provide further insight into ecosystem planning, stability and crop production in the event of climate change.

It is also clear that our present contribution also has a sound indication for a dominant biodiversity gain with its implication for food production and sustainable development in the Niger Delta Region of Nigeria.

References

[1] A. Halanay (1966), Differential Equations, Stability, Oscillations, Time Lags, Academic Press, New York.
[2] N.J. Ford, P.M. Lumb, E. Ekaka-a (2010), Mathematical modelling of plant species interactions in a harsh climate, Journal of Computational and Applied Mathematics, Volume 234, pp. 2732-2744.
[3] M. Kot (2001), Elements of Mathematical Ecology, Cambridge University Press.
[4] J.D. Murray (1993), Mathematical Biology, 2nd Edition Springer Berlin.
[5] E.N. Ekaka-a (2009), Computational and Mathematical Modelling of Plant Species Interactions in a Harsh Climate, PhD Thesis, Department of Mathematics, The University of Liverpool and The University of Chester, United Kingdom.
[6] Y. Yan, E.N. Ekaka-a (2011), Stabilizing a mathematical model of population system, Journal of the Franklin Institute 348, pp. 2744-2758.
[7] M.A. Ekpo and A.J. Nkanang (2010), Nitrogen fixing capacity of legumes and their Rhizosphereal microflora in diesel oil polluted soil in the tropics, Journal of Petroleum and Gas Engineering 1(4), pp. 76-83.
[8] E.N. Ekaka-a and A.I. Agwu (2013), Effect of the inter-species competition parameters on the onset of stability and degeneracy of co-existence steady-state solutions between competing populations, Journal of the Nigerian Association of Mathematical Physics, Volume 25, No. 2, pp. 309-312.
[9] E.C. Pielou, Mathematical Ecology, 2nd ed; Wiley, New York, 1977.
[10] A.I. Agwu, E.N. Ekaka-a and N.M. Nafo (2013), Effect of the intra-species competition parameters on the onset of stability, instability and degeneracy of co-existence steady-state solutions between competing populations, Journal of the Nigerian Association of Mathematical Physics, Volume 25, No. 2, pp. 313-316.
[11] E.N. Ekaka-a, A. Musa, E.C. Nwachukwu and E. Esekhaigbe (2013), Stability analysis for a system of interacting populations with a dis-similar carrying capacity, Journal of the Nigerian Association of Mathematical Physics, Volume 25, No. 2, pp. 317-320.
[12] E.N. Ekaka-a, N.M. Nafo, A. Weli and E.H. Amadi (2013), Stability analysis for a system of interacting populations with a similar carrying capacity, Journal of the Nigerian Association of Mathematical Physics, Volume 25, No. 2, pp. 321-324.