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Stability of a mutualistic interaction with a similar doubling time: a variation of the intra-competition coefficient of cowpea

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Abstract

Attempting to find the fundamental changes in the stability of a mutualistic interaction with a similar doubling time which is due to a variation of the intra-competition coefficient of cowpea when every other model parameter value is fixed is essentially an agricultural subject which can be tackled by a mathematical reasoning. The method of numerical simulation was implemented to solve this challenging ecological-agricultural problem. The results which we have obtained have not been seen elsewhere, they are reported here and discussed.

1.0 Introduction

The study of stability and its application is an active aspect of research within the field of mathematical ecology [1, 2, 3, 4, 5, 6]. Despite this observation, the numerical modelling of co-existence of two interacting populations and its stability behaviour with respect to a mutualistic interaction when the two populations have the same doubling time does seem to be a less popular approach. This present analysis is based on the data of Ekpo and Nkanang [7]. The concept of a doubling time can be calculated by dividing the logarithm of the whole number to base e by the intrinsic growth rate value which in this study has the value of 0.0225. When the doubling time is calculated, it turns out to be 30.80654136 which we estimated to have the value of 31 days approximately. In this context when the growth rate values permit the same doubling time in a mutualistic interaction between cowpea and groundnut, how a variation of the intra-species coefficient of cowpea affect the co-existence and stability behaviour which is primarily a crop science problem can be tackled using a mathematical reasoning. It is against this background that we propose to utilize our previous method of analysing a competition system [8, 9, 10, 11] to analyse an alternative mutualistic system with a similar doubling time of 31 days.

In the work of Ekaka-a et al. [8], a mathematical model for two interacting yeast species was adapted based on the original formulation of Pielou [9] while our present mathematical model concerns the interaction between two legumes such as cowpea and groundnut. For the yeast species interaction the growth rate parameters have the values of 0.1 and 0.08 while the intra-species competition parameter values are 0.0014 and 0.001 which imply that the estimated carrying capacities are 71.4 and 80 in the unit of grams. For the present model description between two legumes, the estimated growth rate parameter value and the intra-species competition parameter value for cowpea are 0.0225 and 0.006902 implying that the carrying capacity value which can sustain the growth of cowpea is 3.26 grams. The estimated growth rate parameter value and the intra-species competition parameter value for groundnut are 0.0225 and 0.0133 implying that the carrying capacity value which can sustain the growth of groundnut is 1.69 grams. By comparing the two carrying capacities of yeast interaction model and that of the interaction between two legumes, we can see that it would take a bigger carrying capacity to sustain the growth of yeast species than that of our present model. While the mathematical structure in the work of Ekaka-a et al. [8] is a Lotka-Volterra competition system, our present mathematical structure is a Lotka-Volterra mutualistic system. In the work of Ekaka-a et al. [8], the effect of the inter-species competition parameters on the onset of stability and degeneracy of co-existence steady-state solutions between competing populations was examined in contrast to our present analysis in which we propose to determine the fundamental changes in stability of a mutualistic interaction with two similar intrinsic growth rate

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parameter values and two dis-similar carrying capacities when the intra-species competition coefficient of cowpea is varied and other model parameter values are fixed. The new contribution of our present analysis lies in the alternative application of a mutualistic interaction between two legumes to explore the changes in the pattern of the co-existence steady-state solution and its stability especially as cowpea and groundnut characteristically have a mutualistic type of interaction because they have similar abiotic-stress effect.

In the work of Ekaka-a et al. [10], we examined the effect of the intra-species competition coefficients on the onset of stability, instability and degeneracy of co-existence steady-state solutions between competing yeast species while we propose to determine the type of stability of a mutualistic interaction with two similar intrinsic growth rate parameter values and two dis-similar carrying capacities.

The stability analysis for a system of competing legumes with a dis-similar carrying capacity having two dis-similar intrinsic growth rate parameter values (a = 0.0225 and d = 0.05) and two dis-similar carrying capacities (a = 0.0225, b = 0.006902, d = 0.05, f = 0.0133) was studied [11] but in this study it is our aim to determine the changes in the stability behaviour of a mutualistic interaction with two similar intrinsic growth rate parameter values and two dis-similar carrying capacities. In the current analysis of Ekaka-a et al. [11], the focus of the interaction between two legumes is a pure (-, -) response which corresponds to a scenario when cowpea competes to inhibit the growth of groundnut and vice-versa. In contrast, our new contribution proposes to look at the effect of a mutualistic interaction between cowpea and groundnut with a similar doubling time on the type of a co-existence steady-state solution and its stability due to a variation of the intra-competition coefficient of cowpea. The inclusion of abiotic-stress characteristic of two interacting legumes such as cowpea and groundnut for limited resource makes this present analysis quite unique and distinct from any of our current analyses.

While the interaction between two legumes with a similar carrying capacity [12] in which several co-existence steady-state solutions and their stability type were examined due to a variation of the intrinsic growth rate parameter has been current studied, our current simulation analysis is a clear point of departure from our most recent analysis. In the theory of ecology, the competition interaction between two legumes having a similar abiotic-stress factor with a similar carrying capacity can differ significantly from the mutualistic interaction between two legumes having a similar abiotic-stress factor with two-similar growth rates and two dis-similar carrying capacities. As previously mentioned earlier in one of our papers in this volume, it is clear that this open complex ecological problem is yet to be analysed by another group of numerical mathematicians because this proposed problem would require a mathematical reasoning in order to provide a successful insight.

2.0 Mathematical Formulation

Following Ekaka-a [5], we consider the following system of model equations of continuous nonlinear first order ordinary differential equations

$$\frac{dC(t)}{dt} = C(t)[a - bC(t) + cG(t)] \qquad (1)$$

$$\frac{dG(t)}{dt} = G(t)[d - fG(t) + eC(t)] \qquad (2)$$

Here, the notations C(0) > 0 and G(0) > 0 define the starting biomasses of cowpea and groundnut at the start of the growing season otherwise called the initial conditions when t = 0. In this study, we have considered the following parameter values: a = 0.0225, d = 0.0225, b = 0.006902, f = 0.0133, c = 0.0005, e = 0.01. This range of parameterization clearly shows that the two interacting legumes have the same doubling time in which the logarithm to base e is simply divided by the same intrinsic growth rate parameter value (a = 0.0225 or d =0.0225). That is on the basis of the theory of growth of plant species; it would take about 31 days to double their starting biomass. The impact of having the same doubling time on the co-existence of these interacting legumes and its stability behaviour is a challenging ecological problem which can be fully studied using a mathematical reasoning.

3.0 Method of Solution

Without a loss of generality, we have used the theory of stability to examine the sign changes of the eigenvalues which can either indicate a stable (or unstable) co-existence steady-state solution when the two interacting legumes are expected to survive together or a degenerate co-existence steady-state solution in which either of the two interacting legumes can go into the ecological risk of extinction.

4.0 Results and Discussion

The notations a and d represent the intrinsic growth rate parameter values for cowpea and groundnut, the notation ss stands for the co-existence steady-state solution while the notations λ_1 and λ_2 stand for the two eigenvalues whose signs define the type of stability for the co-existence steady-state solution. The parameter value denoted by b represents the intra-competition coefficient of cowpea. The empirical results which we have obtained in this present analysis are presented and discussed in the following Tables below.

Table 1: Calculating the qualitative stability of a co-existence steady-state solution due to a variation of the intra-competition coefficient ‘b’ of cowpea: summary of results 1

Example	b	ss	λ_1	λ_2	Type of Stability
1	0.006902	3.58:4.38	-0.0225	-0.0605	Stable
2	0.00035	-757:-567.5	-0.0225	7.83	Degenerate
3	0.00069	74.30:57.55	-0.0225	-0.7942	Stable
4	0.00104	35.41:28.31	-0.0225	-0.3907	Stable
5	0.0014	23.24:19.17	-0.0225	-0.2645	Stable
6	0.0017	17.30:14.70	-0.0225	-0.2028	Stable
7	0.00207	13.78:12.05	-0.0225	-0.1663	Stable
8	0.00242	11.45:10.30	-0.0225	-0.1421	Stable
9	0.00276	9.80:9.05	-0.0225	-0.1249	Stable
10	0.00311	8.55:8.12	-0.0225	-0.1121	Stable

What do we learn from Table 1? The first row of Table 1 shows that when the intra-competition coefficient of cowpea is not varied, it produces a co-existence steady-state solution (3.58, 4.38) which is stable having two negative eigenvalues. Therefore, these two eigenvalues contribute to the decaying behaviour of the solution trajectories over a long time. But when the intra-competition coefficient of cowpea has the precise value of 0.00035, a degenerate co-existence steady-state solution has occurred indicating the extinction of both cowpea and groundnut. This fundamental change from a stable steady-state solution to a degenerate steady-state solution has some implication for the ecosystem functioning and stability which is beyond the scope of this present study. Thereafter, it is very clear on the basis of this systematic simulation analysis that the stability of the co-existence steady-state solution (*csss*) is sustained ranging from b = 0.00069 to b = 0.01104 when every other model parameter is fixed. The series of these results are presented in Table 2 and Table 3.

Table 2: Calculating the qualitative stability of a co-existence steady-state solution due to a variation of the intra-competition coefficient ‘b’ of cowpea: summary of results 2

Example	b	*csss*	λ_1	λ_2	Type of Stability
11	0.00345	7.60:7.40	-0.0225	-0.1021	Stable
12	0.00380	6.83:6.82	-0.0225	-0.0942	Stable
13	0.00414	6.20:6.35	-0.0225	-0.0877	Stable
14	0.00450	5.68:5.96	-0.0225	-0.0823	Stable
15	0.00483	5.24:5.63	-0.0225	-0.0777	Stable
16	0.00518	4.86:5.35	-0.0225	-0.0738	Stable
17	0.00552	4.54:5.10	-0.0225	-0.0704	Stable
18	0.00587	4.25:4.90	-0.0225	-0.0675	Stable
19	0.00621	4.00:4.70	-0.0225	-0.0649	Stable
20	0.00656	3.78:4.53	-0.0225	-0.0625	Stable

Table 3: Calculating the qualitative stability of a co-existence steady-state solution due to a variation of the intra-competition coefficient ‘b’ of cowpea: summary of results 3

Example	b	ss	λ_1	λ_2	Type of Stability
21	0.00697	3.54:4.35	-0.0225	-0.0601	Stable
22	0.00725	3.40:4.24	-0.0225	-0.0586	Stable
23	0.00759	3.24:4.12	-0.0225	-0.0569	Stable
24	0.00794	3.10:4.01	-0.0225	-0.0554	Stable
25	0.00828	2.95:3.91	-0.0225	-0.0540	Stable
26	0.00863	2.83:3.82	-0.0225	-0.0527	Stable
27	0.00897	2.72:3.73	-0.0225	-0.0515	Stable
28	0.00932	2.61:3.66	-0.0225	-0.0504	Stable
29	0.00966	2.51:3.58	-0.0225	-0.0494	Stable
30	0.0100	2.42:3.51	-0.0225	-0.0485	Stable
31	0.0104	2.34:3.45	-0.0225	-0.0476	Stable
32	0.0107	3.26:3.40	-0.0225	-0.0468	Stable
33	0.01104	2.20:3.34	-0.0225	-0.0461	Stable

5.0 Conclusion and Remarks

The hypothesis of this present study supports the fact that fundamental changes in stability of a mutualistic interaction with a similar doubling time due to a variation of the intra-competition coefficient of cowpea has occurred when other model parameter values which drive the deterministic dynamics of the interaction between cowpea and groundnut for a limited resource are fixed. Therefore, we have successfully applied a mathematical reasoning to solve a challenging crop science problem which was not previously considered in any other published scientific report. We would expect these empirical results which dominantly support the ecological theory of co-existence of two interacting legumes surviving together with their overall implication for a sustained biodiversity gain and sustainable development in the Niger Delta Region of Nigerian agricultural sector.

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