Journal of the Nigerian Association of Mathematical Physics
***Volume 26*, (*March, 2014*), pp 583 – 588**
© J. of NAMP

Determining the impact of the intrinsic growth rates on the qualitative behaviour of a co-existence steady-state solution and its stability in a competition interaction for biogas solids and yeast species : Part 1

Enu-Obari Nwamue Ekaka-a

Department of Mathematics and Computer Science,
Rivers State University of Science and Technology, Nkpolu, Port Harcourt, Nigeria

Abstract

The analytical dimension of determining the impact of the intrinsic growth rate on the co-existence steady-state solution and its stability is one of the standard methods in the application of numerical mathematics in the biogas production sector. An example of the competition between two yeast species was also considered and compared with that of interaction between biogas solids. The innovation of this simulation technique which is computationally more efficient than the tedious process of analytical calculation has been used to determine the impact of the intrinsic growth rates on the qualitative behaviour of stability and co-existence steady-state solution in a competition interaction between two biogas solids and compared with the interaction between two yeast species. The results which one has obtained have not been seen elsewhere, they are presented here and discussed.

Keywords: competition, co-existence steady-state solution, stability, interacting populations, biogas solids

1.0 Introduction

The idea of quantifying the impact of the intrinsic growth rates on the qualitative behaviour of stability and its effect on the co-existence steady-state solution between two and more interacting populations is a popular theoretical and analytical hypothesis which has received lots of contributions within the literatures of mathematical ecology and other related disciplines [1, 2, 3, 4, 5, 6] but of rare application in the simulation analysis of biogas production. Although, the variation of the two intrinsic growth rates-at-a-time when other model parameters are fixed on the pattern of stability is a popular experimental deductions, it would require a mathematical reasoning using the technique of a simulation analysis before a detailed analysis and results can be found for a broad variation of these two growth rates. In the context of this study, our present analysis is based on the biogas data of Iyagba et al. [7].

In our previous paper [8], the analysis to determine the type of co-existence steady-state solution and its stability was on the basis of a variation of the model parameter values c and e otherwise called the inter-species competition coefficients between two yeast species [9] while this present contribution based on the yeast species formulation has looked at the variation of the model parameter values a and d otherwise called the intrinsic growth rate parameter values. In our earlier contribution [8], a variation of the specified parameter values produced 25 instances of stable co-existence steady-state solutions, 4 instances of degenerate co-existence steady-state solutions and 2 instances of unstable co-existence steady-state solutions. In contrast, this present contribution has produced 40 instances of stable co-existence steady-state solutions and zero instances of degenerate and unstable co-existence steady-state solutions. What is novel about this present contribution is that a variation of the two intrinsic growth rates between two yeast species guarantees the total loss of degenerate co-existence steady-state solution which does not provide a sound biological/ecological meaning and lacks an intuitive ecological planning. In this present scenario, biodiversity gain is in place which is more of an advantage over the last contribution.

In our previous paper [10], we determined the type of co-existence steady-state solution and its stability on the basis of a variation of the model parameter values b and f otherwise called the intra-species competition coefficients between two yeast species while this present contribution based on the yeast species formulation has looked at the variation of the model parameter values a and d otherwise called the intrinsic growth rate parameter values. In our earlier contribution [10], a variation of the specified parameter values produced 1 instance of stable co-existence steady-state solution, 3 instances of

Corresponding author: E-mail: alukere@yahoo.co.uk, Tel.: +2347066441590

degenerate co-existence steady-state solutions and 16 instances of unstable co-existence steady-state solutions. In contrast, this present contribution has produced 40 instances of stable co-existence steady-state solutions and zero instances of degenerate and unstable co-existence steady-state solutions.

In our previous paper [11], we determined the type of co-existence steady-state solution and its stability on the basis of a variation of the model parameter value a otherwise called the intrinsic growth rate of cowpea between two legumes while this present contribution based on the yeast species formulation has looked at the variation of the model parameter values a and d otherwise called the intrinsic growth rate parameter values. In our earlier contribution [11], a variation of the specified parameter value produced 41 instances of stable co-existence steady-state solution, 8 instances of degenerate co-existence steady-state solutions and zero instance of unstable co-existence steady-state solution. In contrast, this present contribution has produced 40 instances of stable co-existence steady-state solutions and zero instances of degenerate and unstable co-existence steady-state solutions. Since both yeast species and legumes belong to the plant species and a stable co-existence steady-state solution is more likely to favour effective ecological planning, the contributions of our earlier paper and the present paper can provide similar insight. Efforts should be made on the part of the agricultural policies to mitigate the incidence of degeneracy that may involve further data collection and model validation in order to avoid the bias of parameter selection which might affect the outcome of competition and its pattern of stability.

In our previous paper [12], we determined the type of co-existence steady-state solution and its stability on the basis of a variation of the model parameter value a otherwise called the intrinsic growth rate of cowpea between two legumes with a similar carrying capacity while this present contribution based on the yeast species formulation has looked at the variation of the model parameter values a and d otherwise called the intrinsic growth rate parameter values. In our earlier contribution [12], a variation of the specified parameter value produced 27 instances of stable co-existence steady-state solution, 13 instances of degenerate co-existence steady-state solutions and zero instance of unstable co-existence steady-state solution. In contrast, this present contribution has produced 40 instances of stable co-existence steady-state solutions and zero instances of degenerate and unstable co-existence steady-state solutions.

2.0 Mathematical Formulation

Following Ekaka-a [5], we consider the following system of model equations of continuous nonlinear first order ordinary differential equations

$$\frac{dC(t)}{dt} = C(t)[a - bC(t) - cG(t)] \qquad (1)$$

$$\frac{dG(t)}{dt} = G(t)[d - fG(t) - eC(t)] \qquad (2)$$

Here, the notations C(0) > 0 and G(0) > 0 define the starting weights of two biogas solids at the start of the biogas production season otherwise called the initial conditions when t = 0. The duration of growth is in the unit of days hereby denoted by the independent variable t. For the purpose of this simulation study, the best-fit model parameters such as ‘a’ and ‘d’ that define the intrinsic growth rates for two biogas solids were selected using the data of Iyagba et al. [7]. The next best-fit parameters such as b and f define the intra-species competition parameters which measure the inhibiting factors on the growth of biogas solids due to self-interaction whereas the parameters c and e define the inter-species competition parameters which also measure other inhibiting factors on the growth of two biogas solids due to interspecific interaction. In this study, we have considered the following parameter values: a = 0.1067, d = 0.1064, b = 0.0099, f = 0.0079, c = 0.01, e = 0.0002.

3.0 Method of Solution

Following the recent method of Ekaka-a et al. [8], we considered a variation of the intrinsic growth rates of a competition interaction between two biogas solids when other model parameters are fixed and utilize these variations to quantify the qualitative behaviour of stability and its effect on the co-existence steady-state solution.

4.0 Results and Discussion

The results which we have obtained and have not been seen elsewhere are presented and discussed here in the Tables below: the notation ‘a’ stands for the intrinsic growth rate parameter value for the first biogas solid, the notation ‘d’ stands for the intrinsic growth rate parameter value for the second biogas solid, the notation ‘css’ stands for the co-existence steady-state solution while the notations λ_1 and λ_2 stand for the two eigenvalues whose signs define the type of stability for the co-existence steady-state solution. Typical examples which clearly illustrate the application of our present methodology are presented in the Tables below and discussed. The notation ‘TOS’ stands for the type of stability. The notations ‘css1’ and ‘css2’ stand for the co-ordinates of the co-existence steady-state solution.

Table 1: Calculating the qualitative stability of a co-existence steady-state solution due to a variation of the intrinsic growth rates **a** and **d**: summary of results 1

Example	a	d	css1	css2	λ_1	λ_2	TOS
1	0.1067	0.1064	-2.901	13.54	0.028	13.54	2
2	0.0053	0.0053	-0.1450	0.677	0.0014	-0.0053	2
3	0.0107	0.0106	-0.2901	1.354	0.0028	-0.011	2
4	0.0160	0.0160	-0.4351	2.031	0.0042	-0.016	2
5	0.0213	0.0213	-0.5802	2.708	0.0056	-0.0213	2
6	0.0267	0.0266	-0.7252	3.385	0.007	-0.027	2
7	0.0320	0.0319	-0.8702	4.063	0.0084	-0.032	2
8	0.0373	0.0372	-1.0153	4.739	0.0098	-0.037	2
9	0.0427	0.0426	-1.1603	5.417	0.0113	-0.043	2
10	0.0480	0.0479	-1.3054	6.094	0.0127	-0.048	2
11	0.0534	0.0532	-1.4504	6.771	0.0141	-0.053	2
12	0.0587	0.0585	-1.5954	7.448	0.0155	-0.0585	2
13	0.0640	0.0638	-1.7405	8.125	0.0169	-0.0638	2
14	0.0694	0.0692	-1.8855	8.802	0.0183	-0.0692	2
15	0.0747	0.0745	-2.0306	9.479	0.0197	-0.0745	2
16	0.0800	0.0798	-2.1756	10.156	0.0211	-0.0798	2
17	0.0854	0.0851	-2.3206	10.833	0.0225	-0.0851	2
18	0.0907	0.0904	-2.4657	11.511	0.0239	-0.0904	2
19	0.0960	0.0958	-2.6107	12.188	0.0253	-0.0958	2
20	0.1014	0.1011	-2.7558	12.865	0.0267	-0.1011	2

The number 2 of the last column represents the degeneracy of the co-existence steady-state solution having a negative sign for the C_e co-ordinate of the point (C_e, G_e) or co-existence steady-state solution. A negative biomass for cowpea does not exist and has no biological intuitive meaning. These results were obtained for the range of percentage variations between 5% and 95%. The notation TOS represents the type of stability. Next, we would consider the scenario when the two intrinsic growth rates were varied from 101% to 120%.

Table 2: Calculating the qualitative stability of a co-existence steady-state solution due to a variation of the intrinsic growth rates **a** and **d**: summary of results 2

Example	a	d	css1	css2	λ_1	λ_2	TOS
21	0.1078	0.1075	-2.9298	13.677	0.0284	-0.1075	2
22	0.1088	0.1085	-2.9588	13.813	0.0287	-0.1085	2
23	0.1099	0.1096	-2.9878	13.948	0.0290	-0.1096	2
24	0.1110	0.1107	-3.0168	14.084	0.0293	-0.1107	2
25	0.1120	0.1117	-3.0458	14.219	0.0295	-0.1117	2
26	0.1131	0.1128	-3.0748	14.354	0.0298	-0.1128	2
27	0.1142	0.1138	-3.1039	14.490	0.0301	-0.1138	2
28	0.1152	0.1149	-3.1329	14.625	0.0304	-0.1149	2
29	0.1163	0.1160	-3.1619	14.761	0.0307	-0.1160	2
30	0.1174	0.1170	-3.1909	14.896	0.0310	-0.1170	2
31	0.1184	0.1181	-3.2199	15.031	0.0312	-0.1181	2
32	0.1195	0.1192	-3.2489	15.167	0.0315	-0.1192	2
33	0.1206	0.1202	-3.2779	15.302	0.0318	-0.1202	2
34	0.1216	0.1213	-3.3069	15.438	0.0321	-0.1213	2
35	0.1227	0.1224	-3.3359	15.573	0.0324	-0.1224	2
36	0.1238	0.1234	-3.3649	15.709	0.0326	-0.1234	2
37	0.1248	0.1245	-3.3939	15.844	0.0329	-0.1245	2
38	0.1259	0.1256	-3.4229	15.979	0.0332	-0.1256	2
39	0.1270	0.1266	-3.4520	16.115	0.0335	-0.1266	2
40	0.1280	0.1277	-3.4810	16.250	0.0338	-0.1277	2

In these series of variations for the two intrinsic growth rates, the phenomenon of degeneracy of their co-existence steady-state solutions was sustained. What if the intrinsic growth rate parameter value ‘a’ is varied in the combination of another intrinsic growth rate parameter value ‘d’ in the context of two interacting yeast species populations [9], how would these variations affect the co-existence steady-state solution and its type of stability? Our next calculations are presented and displayed in Table 3.

Table 3: Calculating the qualitative stability of a co-existence steady-state solution due to a variation of the intrinsic growth rates **a** and **d** between two interacting yeast species: summary of results 3

Example	a	d	css1	css2	λ_1	λ_2	TOS
41	0.1	0.08	12.50	68.75	-0.0033	-0.083	1
42	0.005	0.004	0.625	3.44	-0.0002	-0.0041	1
43	0.010	0.008	1.250	6.88	-0.0003	-0.0083	1
44	0.015	0.012	1.875	10.31	-0.0005	-0.0124	1
45	0.020	0.016	2.50	13.75	-0.0007	-0.0166	1
46	0.025	0.020	3.13	17.20	-0.0008	-0.0207	1
47	0.030	0.024	3.75	20.63	-0.001	-0.0249	1
48	0.035	0.028	4.38	24.06	-0.0012	-0.0290	1
49	0.040	0.032	5.00	27.50	-0.0013	-0.0332	1
50	0.045	0.036	5.63	30.94	-0.0015	-0.0373	1
51	0.050	0.040	6.25	34.38	-0.00017	-0.0415	1
52	0.055	0.044	6.88	37.81	-0.0018	-0.0456	1
53	0.060	0.048	7.50	41.25	-0.0020	-0.0498	1
54	0.065	0.052	8.13	44.70	-0.0022	-0.0539	1
55	0.070	0.056	8.75	48.13	-0.0023	-0.0581	1
56	0.075	0.060	9.38	51.56	-0.0025	-0.0622	1
57	0.080	0.064	10.00	55.00	-0.0027	-0.0663	1
58	0.085	0.068	10.63	58.44	-0.0028	-0.0705	1
59	0.090	0.072	11.25	61.88	-0.0030	-0.0746	1
60	0.095	0.076	11.88	65.31	-0.0032	-0.0788	1

In this scenario, the whole number 1 represents the stability of the co-existence steady-state solution. We have observed that a variation of the intrinsic growth rates for two interacting yeast species unanimously supports the stability of the co-existence steady-state solution in which the phenomenon of degeneracy is totally lost. The same conclusion can be observed for other variations of these two intrinsic growth rates as reported in Table 4.

Table 4: Calculating the qualitative stability of a co-existence steady-state solution due to a variation of the intrinsic growth rates **a** and **d** between two interacting yeast species: summary of results 4

Example	a	d	css1	css2	λ_1	λ_2	TOS
61	0.1010	0.0808	12.625	69.44	-0.0033	-0.084	1
62	0.1020	0.0816	12.75	70.13	-0.0034	-0.085	1
63	0.1030	0.0824	12.88	70.81	-0.0034	-0.085	1
64	0.1040	0.0832	13.00	71.50	-0.0035	-0.086	1
65	0.1050	0.0840	13.13	72.20	-0.0035	-0.087	1
66	0.1060	0.0848	13.25	72.88	-0.0035	-0.088	1
67	0.1070	0.0856	13.38	73.56	-0.0035	-0.089	1
68	0.1080	0.0864	13.50	74.25	-0.0036	-0.090	1
69	0.1090	0.0872	13.63	74.94	-0.0036	-0.090	1
70	0.1100	0.0880	13.75	75.63	-0.0036	-0.091	1
71	0.1110	0.0888	13.88	76.31	-0.0037	-0.092	1
72	0.1120	0.0896	14.00	77.00	-0.0037	-0.093	1
73	0.1130	0.0904	14.13	77.70	-0.0037	-0.094	1
74	0.1140	0.0912	14.25	78.38	-0.0038	-0.095	1
75	0.1150	0.0920	14.38	79.06	-0.0038	-0.095	1
76	0.1160	0.0928	14.50	79.75	-0.0038	-0.096	1
77	0.1170	0.0936	14.63	80.44	-0.0039	-0.097	1
78	0.1180	0.0944	14.75	81.13	-0.0039	-0.098	1
79	0.1190	0.0952	14.88	81.81	-0.0039	-0.099	1
80	0.1200	0.0960	15.00	82.50	-0.0040	-0.100	1

Predictive models in the works of experts in chemical engineering working on some aspect of biogas production [13] have indicated an important unexplained issue in which daily biogas yield for different substrate loadings show some characteristic changes in the changing patterns of the carrying capacity in a biogas production in Nigeria. In their work, the potential of vegetable (putriscible) component of municipal solid wastes was examined in terms of biogas production. In the theory and application of these predictive models in biogas production, the expected model parameter values 'a' and 'd' otherwise called the intrinsic growth rate parameter values can change over a retention time in the unit of days. One of the key contributions in the work of Ojolo et al [13] is here stated as follows: The total biogas yield from 10 kg putriscible waste is12.325 dm3 over a retention time of 40 days; 13.338 dm3 from 12 kg waste; 12.254 dm3 from18 kg waste all at 8% Total Solids (TS). In the total biogas yield for different substrate loadings scenario, it can be seen that the two carrying capacities were 12.325 dm3 and 13.338 dm3 while in another situation the two carrying capacities were 13 dm3 and 12.254 dm3. However, in the daily biogas yield for different substrate loadings scenario, biogas production has indicated an instance in which the carrying capacity of biogas production or saturation level of biogas production tends to extinction. What these interesting ideas simply mean is that for instances when either of the carrying capacities is changing and the other carrying capacity is fixed one-at-a-time, would degeneracy be either sustained or lost to full stability? As far as one knows, there is no numerical mathematics contribution which translates the ideas of Ojolo et al. [13] and Iyagba et al. [7] to study the impact of changing intrinsic growth rates on the co-existence of the steady-state solution and its stability behaviour for some sort of anticipated biogas production policy in Nigeria. It is against this background that an attempt to study the implication of these ideas using the technique of simulation analysis over repeated simulations was conducted in the context of biogas growth is proposed in this present study. Our results are presented in Table 5.

Table 5: Calculating the qualitative stability of a co-existence steady-state solution due to a variation of the intrinsic growth rate **a** and fixing **d** between two interacting biogas solids: summary of results 5

Example	a	d	css1	css2	λ_1	λ_2	TOS
81	0.0053	0.1064	-13.4084	13.808	0.1312	-0.1075	2
82	0.0107	0.1064	-12.86	13.79	0.1258	-0.1075	2
83	0.0160	0.1064	-12.302	13.78	0.1203	-0.1074	2
84	0.0213	0.1064	-11.75	13.77	0.1149	-0.1073	2
85	0.0267	0.1064	-11.20	13.75	0.1094	-0.1072	2
86	0.0320	0.1064	-10.64	13.74	0.1040	-0.1072	2
87	0.0373	0.1064	-10.09	13.72	0.0986	-0.1071	2
88	0.0427	0.1064	-9.54	13.71	0.0931	-0.1070	2
89	0.0480	0.1064	-8.98	13.70	0.0877	-0.1069	2
90	0.0534	0.1064	-8.43	13.68	0.0823	-0.1069	2
91	0.0587	0.1064	-7.88	13.67	0.0768	-0.1068	2
92	0.0640	0.1064	-7.33	13.65	0.0714	-0.1067	2
93	0.0694	0.1064	-6.77	13.64	0.0660	-0.1067	2
94	0.0747	0.1064	-6.22	13.63	0.0606	-0.1066	2
95	0.0800	0.1064	-5.67	13.61	0.0551	-0.1066	2
96	0.0854	0.1064	-5.11	13.60	0.0497	-0.1065	2
97	0.0907	0.1064	-4.56	13.58	0.0443	-0.1065	2
98	0.0960	0.1064	-4.01	13.57	0.0389	-0.1065	2
99	0.1014	0.1064	-3.45	13.56	0.0335	-0.1064	2

When two biogas solids are competing for limited resources in which the model parameter ‘a’ is changing and the parameter ‘d’ is fixed, degeneracy is sustained. That is in a situation when the carrying capacity of the first biogas solid is changing while the carrying capacity of the second biogas solid is not changing, degeneracy would rarely disappear. A similar degeneracy occurrence can be deduced when the parameter value ‘a’ is varied over 100% while the parameter value d is fixed. As such, this list of results will not be presented in this contribution.

5.0 Conclusion

Our present simulation analysis of the interacting populations of yeast species supports a dominant occurrence of the stability of each co-existence steady-state solution. In contrast, the simulation analysis of the interacting populations of biogas solids unanimously supports the occurrence of the phenomenon of degeneracy of each co-existence steady-state solution.

Since the environment can be heterogeneous and stochastic due to the effect of other extrinsic factors which are capable to alter the growth of biogas solids, we would be interested to investigate the impact of these two factors on the co-existence steady-state solution and its stability behaviour in a future contribution.

We would expect the insight from the stability regions of a co-existence steady-state solution to provide useful information which can be utilized in the effective policy planning of the ecosystem for the purpose of a sustainable development and the control of biodiversity loss.

The present method of simulation analysis in the differentiation of the type of stability for each co-existence steady-state solution between two competing populations can be extended to tackle other types of interactions such as mutualism, commensalism and predation which were not considered in this present study.

References

[1] A. Halanay (1966), Differential Equations, Stability, Oscillations, Time Lags, Academic Press, New York.
[2] N.J. Ford, P.M. Lumb, E. Ekaka-a (2010), Mathematical modelling of plant species interactions in a harsh climate, Journal of Computational and Applied Mathematics, Volume 234, pp. 2732-2744.
[3] M. Kot (2001), Elements of Mathematical Ecology, Cambridge University Press.
[4] J.D. Murray (1993), Mathematical Biology, 2nd Edition Springer Berlin.
[5] E.N. Ekaka-a (2009), Computational and Mathematical Modelling of Plant Species Interactions in a Harsh Climate, PhD Thesis, Department of Mathematics, The University of Liverpool and The University of Chester, United Kingdom.
[6] Y. Yan, E.N. Ekaka-a (2011), Stabilizing a mathematical model of population system, Journal of the Franklin Institute 348, pp. 2744-2758.
[7] E.T. Iyagba, I.A. Mangibo and Y.S. Mohammad (2009), The study of cow dung as co-substrate with rice husk in biogas production, Scientific Research and Essay Vol. 4 (9), pp.861-866.
[8] E.N. Ekaka-a and A.I. Agwu (2013), Effect of the inter-species competition parameters on the onset of stability and degeneracy of co-existence steady-state solutions between competing populations, Journal of the Nigerian Association of Mathematical Physics, Volume 25, No. 2, pp. 309-312.
[9] E.C. Pielou, Mathematical Ecology, 2nd ed; Wiley, New York, 1977.
[10] A.I. Agwu, E.N. Ekaka-a and N.M. Nafo (2013), Effect of the intra-species competition parameters on the onset of stability, instability and degeneracy of co-existence steady-state solutions between competing populations, Journal of the Nigerian Association of Mathematical Physics, Volume 25, No. 2, pp. 313-316.
[11] E.N. Ekaka-a, A. Musa, E.C. Nwachukwu and E. Esekhaigbe (2013), Stability analysis for a system of interacting populations with a dis-similar carrying capacity, Journal of the Nigerian Association of Mathematical Physics, Volume 25, No. 2, pp. 317-320.
[12] E.N. Ekaka-a, N.M. Nafo, A. Weli and E.H. Amadi (2013), Stability analysis for a system of interacting populations with a similar carrying capacity, Journal of the Nigerian Association of Mathematical Physics, Volume 25, No. 2, pp. 321-324.
[13] S.J. Ojolo, A.I. Bamgboye, B.S. Ogunsina, S.A. Oke (2008), Analytical approach for predicting biogas generation in a municipal solid waste anaerobic digester, Iran J. Environ. Health Sci. Eng; Vol. 5, No.3, pp. 179-186.