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Sunshine-Based Estimation of Global Solar Radiation in Maiduguri – Nigeria

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Abstract

Sunshine-based regression models have been evaluated for the estimation of the monthly averaged daily global solar radiation in Maiduguri. Five models in total, all Angstrom-Prescott type models, were investigated. The models are linear, quadratic, cubic, exponential and logarithmic with respect to the sunshine duration ratio used as independent variable. The Angstrom coefficients of all five models were determined and is very good agreement of the models' estimation with the observed data. The cubic model out perform all the other models and is closely followed by the linear and the exponential models.

1.0 Introduction

Solar radiation is central to most of the significant activities happening on the earth surface that effect life and/or its quality – physical, chemical, biological. Engineers, architects, agriculturists, hydrologists etc. all require accurate information of the availability solar radiation for the proper applications of their devices/services at their location of interest. For the energy engineer, for example, this information is needed for the design of solar collectors for solar heaters and/or other photovoltaic equipment that depend on solar energy. This requirement imposes a dire need for the availability of daily recordings of solar radiation. For regions where there are no actual measured values available, reasonable estimates of the average daily global solar radiation are needed. These estimates often use empirical correlation based on measured relevant data at those locations. These correlation estimates the global solar radiation from the more readily available meteorological, climatological and geographic parameters.

There has been increased interest in the modeling of solar radiation recently. A vast number of empirical models have been proposed for predicting daily or monthly global solar radiation at global and/or local scale. The principle meteorological and physical parameters such as extraterrestrial radiation, sunshine hours, actual duration of sunshine, air temperatures, relative humidity, precipitation, latitude, elevation etc. have all been used in these models. The review by Besharat et al [1] gives a very good account of recent efforts in this exercise. Besides the temperature-based models[2—5] and cloud-based models[6—9] a good number of models for estimating daily and/or monthly averaged global solar radiation use the sunshine ration – a model first proposed by Angstrom and later modified by Prescott. This model is often considered more accurate than the temperature-basedand cloud-based models[10 and 11]

This study attempts to compare five variations of the Angstrom-Prescott model in estimation of the monthly averaged global solar radiation in Maiduguri.

2.0 Data and Methodology

2.1 Data

The monthly averaged daily global solar radiation and the actual sunshine hours used in this study were obtained from the Nigerian Meteorological Agency (NIMET) measurement station in Maiduguri. The station has 12.8333^{o}N and 13.1500^{o}E as latitude and longitude respectively. A ten year (2002 – 2011) record of the monthly averaged data was utilized. The first eight-year data was used in developing the regression and their analyses while the models were tested using the later two-year records.

2.2 Models

Of the several regression models available in the literature five main models were examined and validated in this work for the estimation of monthly averaged daily global solar radiation. Table 1 enumerates the models

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Table 1: The regression models employed in the work

	Model
Linear	$H/H_0 = a + b\left(\frac{n}{N}\right)$
Quadratic	$H/H_0 = a + b(n/N) + c(n/N)^2$
Cubic	$H/H_0 = a + b(n/N) + c(n/N)^2 + d(n/N)^3$
Exponential	$H/H_0 = ae^{b(n/N)}$
Logarithmic	$H/H_0 = a + log\big(b(n/N)\big)$

These are the linear model, the actual Angstrom-Prescott model and is the most commonly adopted model[12], the quadratic and cubic (second and third order polynomial) regression models first derived by [13] and [14] respectively, the logarithmic and exponential models proposed by [15] and [16] respectively.

The correlation coefficients of the models were calculated from the regression analyses between the clearness index (H/H0) and the sunshine duration ration (n/N). The clearness index is the ratio of the monthly averaged daily global solar radiation (H) and the extraterrestrial radiation H_0. The sunshine duration ratio is the ratio between the observed daily sunshine hours (n) and the day light hours (N). The monthly averaged daily extraterrestrial radiation is given by

$$H_0 = \frac{24 \times 60}{\pi} G_{sc} d_r [\omega_s \sin\varphi \sin\delta + \cos\varphi \cos\delta \sin\omega_s] \qquad (1)$$

in which G_{sc} is the solar constant given by

$$G_{sc} = 0.0820\ MJm^{-2}min^{-1} \qquad (2)$$

andd$_r$ is the is the relative earth-sun distance given by

$$d_r = 1 + 0.033\cos\left(\frac{2\pi J}{365}\right) \qquad (3)$$

Here as usual, J stand for the day number; a number between 1 and 365 with I indicating the fist of January and 365 (366) representing the 31st of December. The sunset hour angle ω_s ,solar declination δ and solar sunshine hours N are given by

$$\omega_s = \cos^{-1}(-\tan\varphi \tan\delta) \qquad (4)$$

$$\delta = 0.409\sin\left(\frac{2\pi J}{365} - 1.39\right) \qquad (5)$$

$$N = \frac{2\pi}{15}\omega_s \qquad (6)$$

Finally, ω represent the latitude of the station (location of measurement)

2.3 Model Performance Estimation

The Angstrom coefficients of the five regression models were determined by curve estimation techniques and the goodness of the fitting were measured by correlation coefficient R, the coefficient of determination R^2 and root mean square error б.

Two year data of the record of the sunshine hour and measured monthly averaged daily global solar radiation were also used to test the models. The performances of the models were measured using the normal parameters – mean bias error, root mean square error mean percentage error given by the following relations.

$$MBE = \sum_1^k (H_{est} - H_{est})/k \qquad (7)$$

$$RMSE = \frac{1}{k}\sqrt{\sum_1^k (H_{est} - H_{meas})^2} \qquad (8)$$

$$MPE = \frac{1}{k}\sum_1^k \left(\frac{H_{meas} - H_{est}}{H_{meas}}\right) \times 100 \qquad (9)$$

In all three relations, k stand for the number of observations while H_{est} and H_{meas} are the calculated and measured monthly averaged daily global solar radiation respectively.

3 Results and Discussion

The results of the analyses carried out are present below in two sections. The first describe and appraise the determined coefficients of the regression models while in the second section an evaluation of the models' performances is presented

3.1 Regression

Table 2 gives the typical values of the Angstrom coefficients (computed) required for the models studied. Scatter plots,as exemplified in figure 1, of the clearness index and sunshine duration ratio were used in the analyses. The fitting equations determined were compared with the actual data and the limits of confidence/prediction evaluated at 95% as shown in the typical figure.

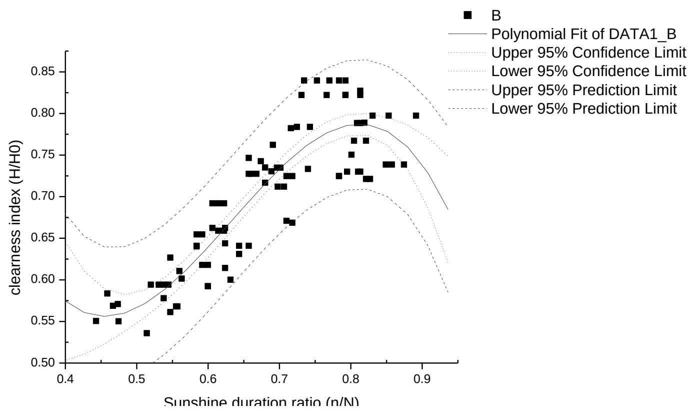


Figure 1: Variation of the measured H/Ho and n/N for the period 2002 – 2009. The cubic fit to the data is shown together with curves indicating the confidence and prediction limits of the fit.

The coefficients of determination (R^2) of the models studied, using the H/H0 and n/N data sets, of the eight-year record varies between 0.7363 (linear) to 0.8069 (cubic). The cubic regression model presented the highest value of the coefficient while the linear model showed the lowest indicated, Table 2.

Table 2: The Angstrom coefficients of the regression models

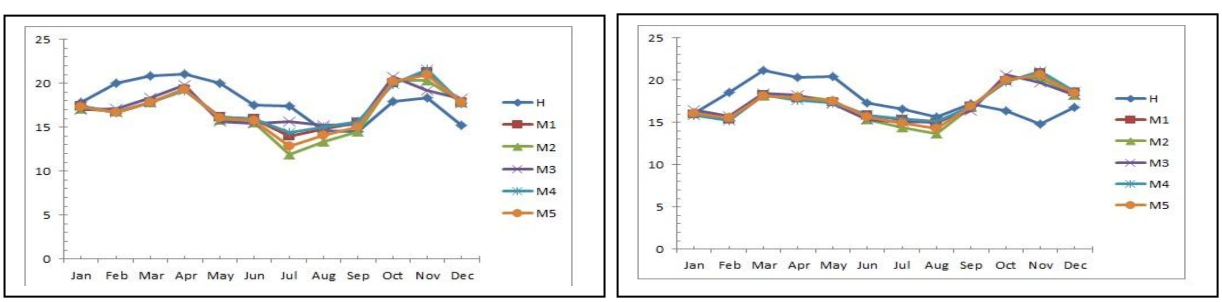
Model	a	b	c	d	R^2
Linear	0.2682	0.6313	-	-	0.7363
Quadratic	-0.2285	2.1368	-1.1097	-	0.7672
Cubic	2.6371	-11.247	19.28	-10.142	0.8069
Exponential	0.3685	0.9268	-	-	0.7497
Logarithmic	0.8671	0.423	-	-	0.7535

The values of coefficient of determination (R^2) are within the range of values obtained in similar studies involving different models.In works employing Hargreaves and two forms Bristow and Campbell models, for example, Ball et al [17] reported values between 0.53 to 0.97 while Alvareze et al [18] reported values ranging between 0.86 to 0.90 at different sites in the USA and Chile respectively.

3.2 Performance Evaluation

The recorded and estimated values, from the five models studied, of the monthly averaged daily global solar radiation falling in Maiduguri for the two years, 2010 and 2011, are presented in figure 2. The models seem to provide reliable performances in general. The quality of the estimates are especially better between the months of June to September In table 3, the mean biases associated with the models are presented.

The quadratic model showed the largest bias errors, 6.07 and 2.14% mean percent error for the year 2010 and 2011 studied respectively. These (largest errors) are within the acceptable limit of 10% [19 —21]. The largest errors, being lower than the maximum expected error, indicate the goodness of all the models for the prediction of the global solar radiation at the study area. The cubic model, in particular, shows the least errors -- of.0.662355 (0.146MJm^{-2}/day) and 0.750589 (3.205 MJ m^{-2}/day) RMSE (energy) for 2010 and 2011 respectively. The linear and the exponential models also present low values of the root mean square error. The mean bias errors are low with the quadratic model presenting the largest value of 1.204



(a) (b)

Figure 2: comparison between observed and estimated values of the monthly averaged daily solar radiation (MJm-2/day) by the regression models for (a) 2010 and (b) 2011

All the regression equations give good results. The variations observed in the errors do not significantly affect the predictability of any of the models. However, by the consistent low values of MBE, MPE and RMSE shown by the cubic model it seem to best fit the data (and the study area). The exponential and the linear models follow closely behind the cubic model. The performances of the quadratic and the logarithmic models, though good, do lag behind the others.

Table 3: Performances of the regression models as measured by the statistical measures selected

	2010			**2011**		
Model	MBE	MPE	RMSE	MBE	MPE	RMSE
Linear	0.710074563	3.040178	0.715396	0.357907662	0.753657	0.796184
Quadratic	1.203865957	6.071836	0.80002	0.567851466	2.137891	0.786593
Cubic	0.62470998	2.528597	0.662355	0.375995311	0.967839	0.750589
Exponential	0.635187621	2.574247	0.714	0.354755228	0.69016	0.811053
Logarithmic	0.953874912	4.54957	0.757409	0.450783644	1.371043	0.792481

All the models investigated show clear underestimation of the solar radiation in the months of January to May and overestimation in the months of October to December. This may be associated with the climatic condition of the study area. In the months of October through February, the area is often under the cover of harmatan dust. The dust cover due harmatan and due to the sand storm associated with each early rainy season would limit the amount of solar radiation available in the indicated months. A model based on the use of extraterrestrial radiation would definitely overestimate the received radiation. The difficulty or subjective nature that defines what a clear day reference may be, that which leads to the use of the extraterrestrial radiation may be responsible for the under/overestimation. This difficulty has been reported for sites, for example in semi-arid regions of Sudan. Elagib et al [22] reported similar discrepancies.

4.0 Conclusions

Five Angstrom type regression model have been tested for the predication of global solar radiation in Maiduguri. All the models were found to be applicable for the estimation proposed. They showed low errors and predicts values that are close to observed/ recorded ones.The performance of the model that involved a cubic relationship between clearness index and sunshine duration ratio showed the best fit to the observed data. Its coefficient of determination, mean bias error, mean percent error and root mean square error were the best recorded amongst the five models studied.

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