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Road Capacity Reduction and Road Traffic Congestion

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Abstract

This paper presents a mathematical model to predict the average number of vehicles in a road segment when there is road capacity reduction. Empirical study of road traffic flow in Nigeria was undertaken using a digital camera (Samsung ES65) to collect video films on volume of vehicles and other scenes at different locations in Benin City. Result show that the average number of vehicles in the queue will increase over time when a road capacity is reduced. Also, the transient nature of the analyses can be used in building road traffic alert system.

1.0 Introduction

Reduction in road capacity is usually caused by incidents and this lead to a large part of the delays in road networks. A detailed understanding of the expected queue length can improve for instance the traffic prediction and thereby improve delay information or routing guidance. Therefore, this study determines the expected number of vehicles in queue when a road facility is reduced. The paper is outlined as follows: we give a brief review of related literature in section 2, observations from empirical study of road traffic flow are considered in section 3. Section 4 presents the effect of reduction of service capacity on road traffic congestion. Also in this section, a mathematical model is used to describe the effect of reduction in road capacity on road traffic congestion. Section 5 presents the application of the model. Finally, conclusion is presented in section 6.

2.0 Literature Review

Road traffic congestions are mostly caused by incidents. In the United States of America for example, Kwon et al [1] have shown that this amount is about 25 percent. A similar number is established for the Netherlands [2]. Knoop et al [3] assert that 'the amount of delay depends on the demand and the supply on the road. In fact, if the supply (the remaining capacity) reduces to a level lower than the demand, it causes a traffic jam.' Sinha et al [4] use several microscopic simulation tools to predict the reduction of capacity. Many capacity reductions are caused by a change of driving behaviour. Several definitions of deviant driving behaviours have been proposed in literature. For example, Tasca [5] summarises a driving behaviour to be aggressive or deviant "if it is deliberate, likely to increase the risk of collision. It may be motivated by impatience, annoyance, hostility and/or an attempt to save time." aggressive driving is seen as the operation of a motor vehicle in a manner which endangers or is likely to endanger people or property (for more definitions of deviant drivers' behaviour, see [6, 7]).

However, it is not uncommon to see some uniform personnel disobeying traffic lights restrictions and driving against traffic. Observation has shown that even a considerable number of the more literate road users in Nigeria are found to display irresponsible driving behaviour in most of the roads. A number of observed implications of deviant drivers' behaviour are: blocking of roads in order to gain entrance to another lane, too many people give instructions in one road junction leading to confusion. Some vehicle conductors including uniform personnel on their own journey take undue advantage of the chaotic condition and turn themselves to traffic wardens. They ensure that the lane in which their vehicles are moving in the queue is given priority to the detriment of the system. Some motorists do not obey the 'illegal' traffic wardens, thereby causing more confusion. Literature on deviant driving behaviour is mainly descriptive of drivers' behaviour. They avoid the use of mathematical models [8-10]. For each speed of vehicle, Igbinosun et al [11] used a mathematical model to explain the effect of road traffic frequency of vehicles and speed of vehicles on road traffic congestion. There is a frequency of vehicles that will induce road traffic gridlock and the authors argue that whatever will make the speed of the vehicles using a road to drop and approach the critical value is a potential cause of gridlock. These potential causes of road traffic congestion are called

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'gridlock vectors' [11]. These include road intersection, adverse weather conditions such as flood and failed sections on road segments.

Ismail et al [12] studied lane-changing behaviour based on queue length at an urban signalized intersection and developed models for drivers' lane-changing behaviour using logistic regression method. Ismail et al [12] also used the method of video filming and questionnaires to obtain pieces of information on why drivers change lanes. The authors concluded that drivers change lane when facing differences in queue lengths on a road segment.

It is believed that the road capacity in a work neighborhood is usually lower [13, 14] compared to an express road. The relationship between the work zones and incident situations are similar. Drivers' familiarity also is essential in the reduction of road capacity of the work environment [15, 16]. Even though, drivers can get used to a work neighborhood, they are never familiar to incident circumstances. Knoop et al [3] analyzed the maximum outflow out of a jam which is caused by an incident. The most important finding is that the capacity per lane reduces significantly due to a change in driving behaviour. The size of this reduction depends on the incident type. Omosigho and Igbinosun [17] also identified drivers' behaviour as contributing to road traffic congestion.

3.0 Empirical Observation of Road Traffic Flow

We undertake an empirical study of road traffic flow in Nigeria. We use a digital camera (Samsung ES65) to collect video films on volume of vehicles and other scenes at different locations in Benin City. Also, data on road traffic flow frequency was collected at different locations in Nigeria, one at kilometer thirty-four (34) on Kaduna-Lokoja Road and the other at kilometer three (3) on Benin-Asaba Road. The data were collected by manual count. From our empirical observation, the fundamental assumptions for the models in the literature on road traffic flow do not address our observed problem. For example, models in literature [3, 12, and 14] ignore many phenomena in our roads.

During our road traffic flow study, we observed different road traffic scenarios. We present a few of the observations relevant to our work. Firstly, we observed aggressive drivers' behavior; examples include, Tailgating, Driving against traffic (see Figure 1), Drivers blocking the road, Driving on road shoulder, Driving on unauthorized route. These behaviours lead to road capacity reduction and thereby causing road traffic gridlock.

Figure 1 shows an accident scene caused by a motorist driving against traffic. The end of the vehicle labeled A to the end of the vehicle labeled B in this instance is called 'the service point'. Note that the vehicle labeled B is on the wrong lane and has caused a head-on collision with vehicle labeled A which has the right of way. The accident has reduced the two-lane road to a single lane. We also observed variation in the number of vehicles using a road junction when the traffic light turns green. Time is wasted when a road junction is not used properly; cars are forced to stop in all directions. It is not uncommon to find broken down vehicles as a result of undue pressure on old vehicles. In addition, it was observed that when the free flow of traffic is tampered with (on many occasions, uniform personnel take undue advantage and pass a road by forcing all other vehicles to stop), it increases the pressure and traffic on the road. The exit velocity on the road will be affected because drivers will struggle to exit the service point arbitrarily. Figure 2 gives a picture of a secondary incident causing a long queue of vehicle in a road segment.

A two-lane road has been reduced to a single lane, because of a spilled paint on the road surface from a road accident. Note the long queue behind the bike labeled Y in the remaining lane left.

Normally, if unsignalized road intersections are free, only vehicles with the right of way use the road junction at a particular time. However, some road junctions are not well utilized because a traffic incident such as improper driving behaviour reduces the 'exit speed' of vehicles at road junctions. In addition, our observation revealed that some bus conductors take undue advantage of the chaos and turn themselves to traffic wardens and ensure that the lane in which their vehicles are in the queue is giving undue priority to the detriment of the system. Some motorists do not obey the 'illegal' traffic wardens, thereby causing more confusion. Also, the road capacity is reduced. The result of these behaviors is reduced speed of vehicles and congestion.

Arising from the observations above, it is seen that some drivers' behavior leads to road traffic congestion. However, our interest in this paper is to identify by how much the expected number of vehicles in a road segment will be if the road capacity is reduced.

4.0 Effect of Reduction of Service Capacity on Road Traffic Congestion.

Consider a road section AB with length x (measured in km), and an average speed of v (km/hr). From elementary physics, we analyze video shots of several locations taken in Benin City (see Table 1) under: (a) free flow speed, and (b) when there is reduction in the service capacity.

From Table 1, column 1 shows the location of study, column 2 shows the flow type, and column 3 shows the length of the road section under consideration. Column 4 shows the duration it takes a vehicle to travel through the length, while column 5 shows the sample variance of the time to cover the distance. It is observed that for a distance of 50meters (column

3, row 2), the free flow speed is 40km/hr on the average. It takes about 5 seconds to travel through the length AB. However, when there is an interruption, the time is increased to 112.2 seconds and the speed reduced to about 1.6km/hr. The cause of the reduction in capacity is the competing use of the junction by different road users. Cars access the main road (Ikpoba-Hill) from the Jemila end using the gap-acceptance technique (a situation where a vehicle at a road junction enters the road whenever there is a reasonable distance between it and any on-coming vehicle).

At Ugowo-Lagos road by Adolo junction, for a distance of 20meters, it takes an average of 122.7 seconds to complete the distance when there is an incident. The cause of the road capacity reduction is a bad portion of the road. The free flow speed for cities and urban arterials is usually 48km/hr [18, 19], but at the junction described, the average free flow speed is about 19km/hr (this is due to the fact that the area is highly populated and the entry-exit dynamics of the junction affects the flow rate). At Ikpoba Hill, by Jemila junction, the free flow speed is 40km/hr on the average. Further down the road at the University of Benin Teaching Hospital (UBTH) junction, the average speed is between 30km/hr to 35km/hr. It is reasonable therefore, to argue that a free flow speed is location dependent. However, from the analysis of the film taken, it was observed that the average time taken to cover a distance of 20m under a free flow speed was 2.2 seconds. When there is an incident (in this case, a road traffic accident and a depression on the road surface) which causes a reduction in road capacity, it took an average time of 182.9 seconds to cover the same distance.

We use a mathematical model to explain the effect of reducing the service capacity of a road traffic system on the expected number of vehicles in the facility at time *t*. We consider a road segment having free flow traffic until interrupted by an incidence causing lane reduction, the service rate will reduce and the number of vehicles served will also reduce. Some causes of reduction in service rate are: bad portions of road, road incidence such as spilled loads, road construction, accidents, etc. We shall present the mathematical model for reduction in road capacity in the next section; first we shall begin with the assumptions made.

4.1 Model Assumptions

We consider that the arrival rate of vehicles in the system is independent of the number of vehicles at the road at time *t*, and also, the duration of service is mutually independent. The number of vehicle at the start of service is less than the service capacity. We assume that the service rate is exponential and dependent on a service control factor, *r*.

4.2 Model Notation and Formulation

We shall use the following parameters in the formulation of the model.

Let λ = number of vehicles arriving for service per unit time.

μ_m = mean service rate given *m* vehicles in the queue.

r = service control factor

$P_m(t)$ = the probability that *m* vehicles are in the queue at time *t*.

$E\{N(t)\}$ = expected number of vehicles in the queue at time *t*.

We also assume that there must be infinitely many servers. However, in reality we do not have infinite number of lanes in the roads. We have introduced the assumption of infinitely many servers because it is the only one that gives transient results of the expected number of vehicles in the queue. Our interest in this paper is in the reduction of the road capacity.

Consider a case where a vehicle joins a busy multi-lane road and stays until service is available. If more than one road lane is available when a new vehicle arrives (which in essence implies that the service point is empty) then the incoming vehicle may enter any of the free lanes. Vehicles arrive according to a Poisson process with constant arrival rate *λ*.

The assumption of exponential service times implies that if at any time all the *m* vehicles in the system are in service, then the rate at which service completion occurs is $m\mu$; if all the lanes are busy, only the vehicle in service is eligible to leave.

Therefore, $$\begin{cases} \lambda_m = \lambda & (m = 0,1,2,...) \\ \mu_m = m\mu & (m = 1,2,...) \end{cases} \qquad (1)$$

The probability that *m* vehicles will be in the queue at time *t* is a function of λ and $r\mu_m$.

We suppose that the probability that one of the *m* vehicles in the queue will be served within time *t* and $t+h$ is $rm\mu h + o(h)$. The probability of a new arrival of vehicle is $\lambda h + o(h)$. Since the arrival rate of vehicles in the system is independent of the number of vehicles at the road at time *t*, and also, the duration of service is mutually independent, then the scenario just described is a birth and death process [20, 21] with the condition that $\lambda_m = \lambda, \quad \mu_m = mr\mu$.The basic differential equations take the form;

$$P_m(t+h) = P_m(t)\{1-\lambda_m h - r\mu_m h\} + \lambda_{m-1} h P_{m-1}(t) + r\mu_{m+1} h P_{m+1}(t) + o(h) \tag{2}$$

$$p_m(t+h) = p_m(t)\{1-\lambda h - mr\mu h\} + \lambda h p_{m-1}(t) + (m+1) r\mu h p_{m+1}(t) + o(h) \tag{3}$$

$$p'_m(t) = -(\lambda + mr\mu) p_m(t) + \lambda p_{m-1}(t) + (m+1) r\mu p_{m+1}(t), \quad m \geq 1 \tag{4}$$

$$p'_0(t) = -\lambda p_0(t) + r\mu p_1(t), \qquad 0 < r \leq 1 \tag{5}$$

For the initial state E_i then $E\{N(0)\} = i$ (the average number of vehicles in the service point equals the value of vehicle at the start of epoch $t = 0$), and the average number of vehicles in the queue at time t is:

$$E\{N(t)\} = \frac{\lambda}{r\mu}(1 - e^{-r\mu t}) + ie^{-r\mu t} \tag{6}$$

However, for the case $i = 0$ (the number of vehicles initially at the service point), the expected number of vehicles in the queue is

$$E\{N(t)\} = \frac{\lambda}{r\mu}(1 - e^{-r\mu t}), \quad t \geq 0 \tag{7}$$

5.0 Application

We use the road traffic flow data collected at kilometer three (3) on Benin-Asaba Road in Benin City. Table 2 shows the data collected at kilometer three (3) on Benin-Asaba Road. The manual count was done at a place called 'Jemila junction' at Ikpoba hill. We assume that all vehicles leaving the observation point were coming from the Benin City axis of the road and going to Asaba.

For a relatively low speed, the calculated average arrival rate is $\lambda = 0.3572$ and the service rate is $\mu = 0.364539$ (for an average of 1286 vehicles per hour for a Monday in Benin-Asaba road and $c = 3, s = 2$ i.e. three cars passing a double lane road). The average number of vehicles that will be in the queue is 10 as shown in Figure 3.
Note the build-up to the apparent steady state at time *t=90seconds* for *r=0.1* The system experiences reduction in service and an increase in traffic for about 100 seconds and remains so over time. For various values of *r*, there are varying degrees of build up on the road network. However, when *r=1*, there is no reduction at the service point, hence no congestion.

We consider a road length of about 100m in Benin-Auchi road by Big Joe motors (a road segment in Benin City). Table 3 shows the number of vehicles arriving at the service point and the number of vehicles departing the service point. We used personal observation to collect our traffic flow data for a period of one hour. The count was carried out for a 5 minutes interval period each.

The average arrival rate of vehicles from Auchi end of the road to Benin was $\lambda = 0.3136$ (that is an average of 1129 vehicles entering the segment of the road in 1 hour). The service rate is 0.1131 (an average of 407 vehicles leaving in 1 hour).

Note that the reduction in capacity is as a result of different bad spots very close to Big Joe Motors which reduced the service capacity (Figure 4).

Figure 5 shows the average number of vehicles in the queue when the service capacity is affected by *r=0.1, 0.02 and 0.05* respectively. $\lambda=0.5298$, $\mu=0.1845$. The graph shows higher level of congestion when the road capacity is reduced by higher percentage.

Failed portion of a road segment compel drivers to reduce their speed. Therefore, we can say that failed portion of a road segment will induce reduction in the capacity of the affected road segment. The extent of the reduction in the capacity of the road segment will depend on the extent of damage to the failed portion of the road segment or traffic incident involved. A road segment may be totally reduced to a state where it is not passable (total blockade) caused by either road traffic accident or total road failure. One example of total failure of a road segment is a failed or collapsed bridge.

6.0 Conclusion

Several factors termed 'gridlock vectors' are responsible for road capacity reduction which leads to road traffic congestion. In this paper, we highlighted some of these gridlock vectors to include drivers' deviant behaviour, road incidents, spilled loads, road construction, etc. We have demonstrated, mathematically, that whenever there is reduction in service by a factor, $r,\ 0 < r \leq 1$, the number of vehicles in the queue will increase over time. Also, result shows that the transient nature of the analyses will help in building road traffic alert system. If alternative routes are available, re-routing can be implemented. Dynamic route information panels [22] will provide the drivers with information about alternative routes and their expected travel times from the knowledge of the expected number of vehicles in a queue when the road is reduced due to the presence of road traffic incidents.

We recommend that incidents should be avoided if practicable, or whenever they occur, the clearance rate should be immediate. If factors that cause reduction in service are promptly cleared, the congestion rate will be reduced. Reducing the road capacity leads to increase in the number of vehicles in the queue.

Figure 1: Accident scene causing reduction in service capacity.

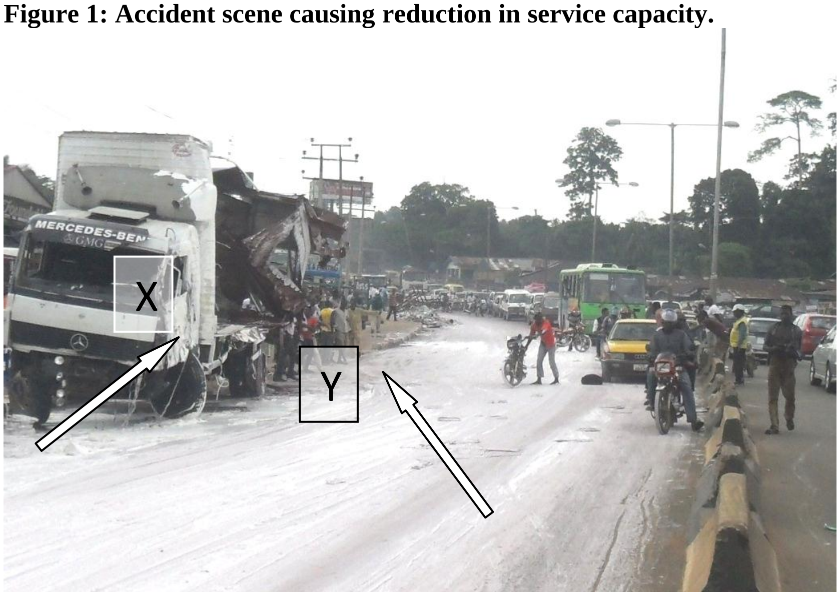

Fig 2: A long queue of vehicles behind an incident when a road capacity is reduced.

Table 1: Free flow speed and flow reduction by location.

Location	*Flow type*	*Distance(km)*	*Average time taken to cover distance(sec)*	*Variance of time taken to cover distance (sec)*
Ikpoba-Hill (Jemila junction)	Free flow	0.05	4.60	4.27
	Reduction	0.05	112.20	28.40
Ugbowo (Adolo junction)	Free flow	0.02	3.80	3.73
	Reduction	0.02	122.70	3.34
Ugbowo (UBTH junction)	Free flow	0.02	2.20	0.84
	Reduction	0.02	182.90	128.33

Table 2: Traffic distribution by time of day at kilometer three (3) on Benin-Asaba Road

	Average Hourly Traffic Flow At Km 3 On Benin-Asaba Road.						
Time	**Mon**	**Tues**	**Wed**	**Thur**	**Fri**	**Sat**	**Sun**
6.00-7.00	1286	1038	1061	995	1045	962	661
7.00-8.00	1373	1303	1429	1335	1452	1165	1072
8.00-9.00	1334	1385	1367	1256	1362	1239	983
9.00-10.00	1107	1112	1042	1163	1242	1153	923
10.00-11.00	990	1021	998	1000	1000	1059	833
11.00-12.00	942	950	1013	1042	949	961	748
12.00-13.00	701	915	1027	998	979	1033	859
13.00-14.00	817	1033	1094	1077	992	1050	1011
14.00-15.00	997	1233	1299	1139	1260	1240	1241
15.00-16.00	1243	1238	1273	1289	1224	1294	1238
16.00-17.00	1245	1278	1409	1435	1445	1378	1230
17.00-18.00	1377	1466	1490	1526	1485	1472	1370

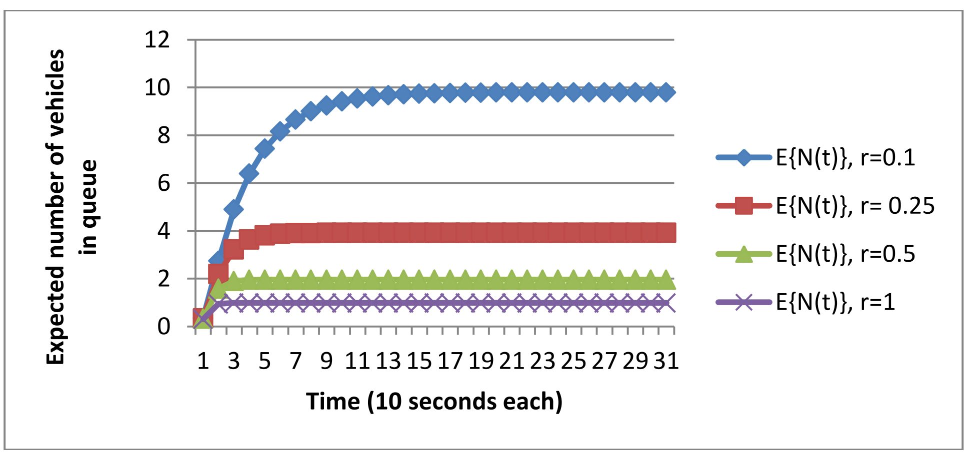

Figure 3: Average number of vehicles in the queue when the service channel is affected by *r=0.1, 0.25, 0.5 and 1.0 respectively. λ=0.3572, µ=0.364539.*

Table 3: Number of vehicles arriving at Benin-Auchi road by Big Joe Motors

Observation time (minutes)	Number of vehicles arriving	Number of vehicles departing
9.00am-9.05am	90	23
9.06am-9.10am	98	20
9.11am-9.15am	90	14
9.16am-9.20am	92	21
9.21am-9.25am	93	76
9.26am-9.30am	94	45
9.31am-9.35am	96	32
9.36am-9.40am	96	33
9.41am-9.45am	98	47
9.46am-9.50am	92	31
9.51am-9.55am	92	36
9.56am-10.00am	98	29

Figure 4: Failed portion in a road segment. Observe the effect of failed portion of road segment: Note that the vehicle marked Q has free space in front. Observe that the vehicle marked X is driving against traffic.

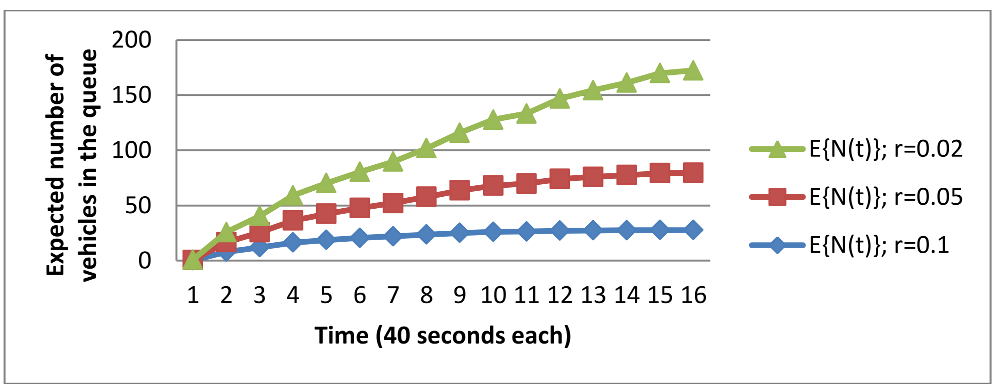

Figure 5: Average number of vehicles in the queue when the service channel is affected by *r=0.1, 0.02 and 0.05* respectively. *λ=0.3136, μ=0.1131.*

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