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Compressional wave propagation and deformation in rocks

Agha S.O[1]

[1]Dept. of Industrial Physics, Ebonyi state University,
P.M.B. 053, Abakaliki, Nigeria.

Abstract

The behaviour of compressional body waves as they propagate within the earth's interior has been studied. The constant of elastic deformation investigated was the Young's modulus(E). The study was done using a 3-channel signal enhancement seismograph, compressional (P) wave sources, detectors and other accessories involved in seismic refraction surveying. The study area is Afikpo (latitude $5^0 52'$-$5^0 57'$N; longitude $7^0 52'$-$7^0 58'$E) in south-eastern Nigeria. The survey was carried out in three locations within the study area. The result shows that the average speed of propagation of the compressional waves in Afikpo for the first two layers from the surface were 290m/s and 700m/s accordingly giving mean values of Young's modulus as 0.33 $x10^9 N/m^2$ and 1.90 $x10^9 N/m^2$ for the first and second layers respectively. The result indicates that the deformation of rocks in the upper(first) layer is greater than that in the lower(second) layer.

Keywords: Compressional, seismic, propagation, Young's modulus.

1.0 Introduction

Compressional waves are seismic body waves. When they propagate through a material medium, their direction of motion is parallel to the direction of particles of the medium. Hence they are longitudinal waves. These waves are the first to arrive at the sensor and the first to be recorded in the seismograph during explosions or shootings. For this reason, they are called primary or P-waves, in contrast to shear waves which are latter/secondary arrivals and are described as S-waves. Being elastic waves, compressional waves have the ability to deform the materials through which they propagate[1].

The propagation of seismic disturbance through the earth is generally governed by physical properties such as density and the way in which the material of the earth's interior reacts to the disturbance[2]. The behaviour of materials of the earth in the presence of a seismic field is determined by their elastic properties[3]. Materials within the seismic source suffer permanent deformation but outside the source, the passage of a seismic disturbance takes place predominantly by elastic displacement of the medium(elastic deformation).

For a one-dimensional compressional (P) wave propagating in the x-direction (Fig.1),the area of the wavefront normal to this direction is $A_{x.}$ At an arbitrary position x, the passage of the wave produces a displacement u and a force, $F_{x.}$ At the position $x+dx$, the displacement is u+du and the force is $F_x + dF_x$ where dx is the infinitesimal length of a small element of the volume having mass ρdxA_x. The effective force acting on the volume element is given by:

$$(F_x + dF_x) - F_X = dF_x = \frac{\partial F}{dx} dx \tag{1}$$

Since the stress element σ_{xx} acting on the area A_x is the cause of the force, F_x, we can write the one-dimensional equation of motion as

$$(\rho dxA_x)\frac{\partial^2 u}{\partial t^2} = dxA_x \frac{\partial \sigma_{xx}}{\partial x} \tag{2}$$

But σ_{xx} is defined mathematically as:

$$\sigma_{xx} = E\varepsilon_{xx} = E\frac{du}{dx} \tag{3}$$

Corresponding author: E-mail: stanleyaghao@gmail.com Tel.: +2348037540994

where ε_{xx} is the strain and E is a constant of proportionality between stress and strain known as Young's modulus of the material.

Substituting equation (3) into equation (2) gives the one-dimentional deformation:

$$\frac{d^2u}{dt^2} = V^2 \frac{d^2u}{dx^2} \tag{4}$$

where V is the velocity of the wave, given by

$$V = \sqrt{\frac{E}{\rho}} \tag{5}$$

E is the elastic constant of the material called it's Young's modulus, and ρ is the density [2].

This work is an attempt to estimate the Young's modulus of the subsurface materials. The knowledge of the Young's modulus of materials gives information about the deformational characteristics of such materials when compressional waves travel through them.

The study area selected is Afikpo (Lat.*$5^0$52'-$5^0$57'N and Long. $7^0$52'-$7^0$58'E)* and the geophysical method chosen is the seismic refraction method.

A number of geophysical explorations have been carried out in Afikpo and environs. Some of them are mentioned here.

Okwueze[4] carried out a seismic refraction survey around $6^0$19' N and $8^0$39' E in Ikom, within the Ezeaku shale formation which passes laterally into Afikpo in South - Eastern Nigeria. His objective was to observe the passage of shear waves through media. He found out that the S-waves utilized in the study delineated a 2m thick top layer with a velocity of 330m/s overlying a 1000m/s second layer giving the modulus of rigidity of the first two media as 0.2 and 2.4 x 10^{10}N/m^2 respectively. Agha et al[5] in their determination of the strength of foundation materials in Afikpo using the seismic refraction method, found out that the Poisson's ratio values of the first two layers from the earth's surface were 0.16 and 0.23 respectively.

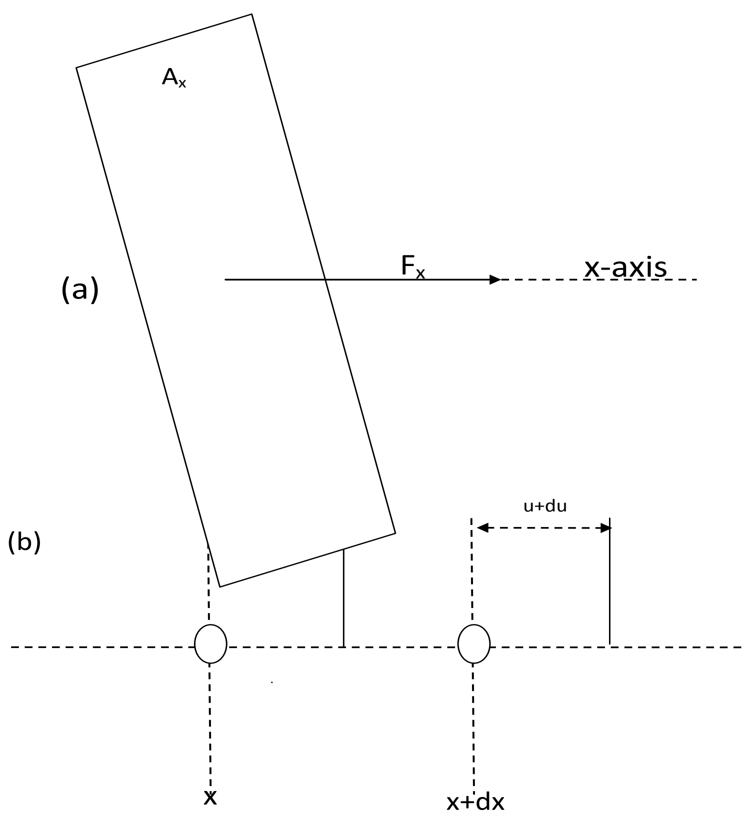

Fig.1(a) One- dimensional P-wave propagating in the x-direction.
(b) Particle displacement along the x-axis

2.0 Instrumentation and Procedure

Instrumentation: The main instrument used in this study is a signal enhancement seismograph with model number MOD. S79. Its accessories consisted of a 10Hz electromagnetic type P-wave geophone and a 9kg sledge-hammer/metal plate which served as seismic source. Other components in the instrumentation includes a geophone cable, twines, pegs, surveyor's tape and a piezoelectric starter.

Procedure: Seismic refraction surveys were carried out in three locations within the study area. The wave source was mechanical and comprised a sledgehammer and a metal plate. Only P-waves were generated. As the waves propagated through the earth from the point of impact, P-wave electromagnetic geophones sensitive to ground motions received the signals and transmitted them via cable to the seismograph. The seismograph displays the waveform of these signals on its screen from where the arrival times T(ms) of the waves are determined. The geophone cable connecting the geophones to the seismograph has about 12 terminals but since the seismograph used was a 3-channel type, only the ground response from 3 geophones are fed into it at each impact of the sledgehammer and metal plate, after which the geophones were shifted along the profile line and subsequent impacts made. The inter-geophone spacing was 4.5m. Both forward and reverse profiles were carried out. The arrival times of the compressional waves were plotted against the offset distances, X(m) of the sensors from the wave source. Average velocities were determined from both forward and reverse profiles.

3.0 Results And Discussion

Results: Figs. 2-4 are the T-X plots of the compressional waves (forward and reverse) for the three locations chosen in the study area.

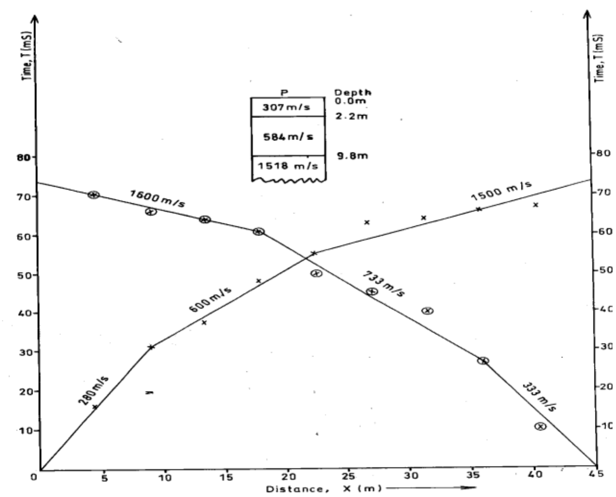

Fig. 2: P-wave (T-X) plots and the equivalent geoseismic layers for location 1

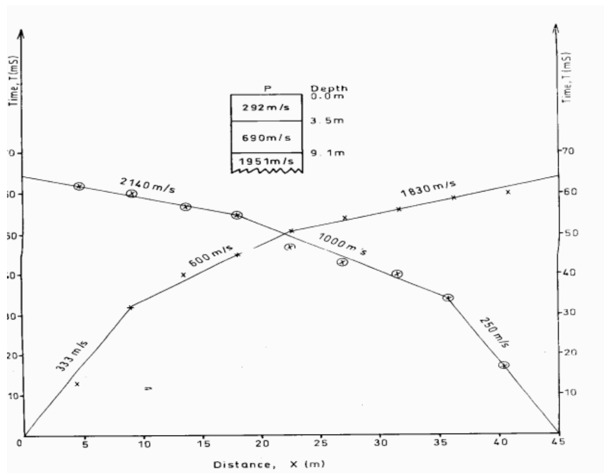

Fig. 3: P-wave (T-X) plots and the equivalent geoseismic layers for location 2

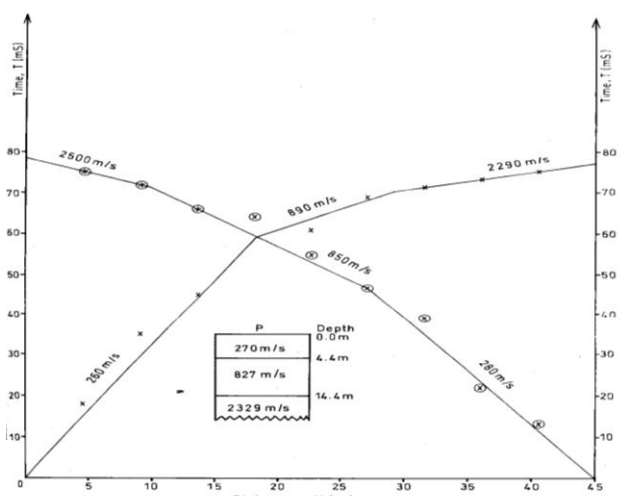

Fig. 4: P-wave (T-X) plots and the equivalent geoseismic layers for location 3

4.0 Discussion:

Each of the T-X plots revealed a three-layer case. Average velocities of layers and the depth to refracting beds in each location are as shown (inset) in the figures. The depth/thickness of the third layers could not be determined. From Fig.1, it is observed that the 307m/s top layer (probably shale) is 2.2m thick and overlies a 7.6m thick second layer (probably dry sand with gravel) with an average velocity of 584m/s. The third layer has an average velocity of 1518m/s and this is interpreted to be wet coarse gravel. These gave values of the Young's modulus, E as 0.3 and $1.4x10^9 N/m^2$ for the first and second layers respectively.

From Fig. 2, the average compressional wave velocities of the first three layers from the earth's surface are 292m/s, 690m/s and 1951m/s which are interpreted to be probably shale, sand with gravel and a consolidated layer respectively. The depths to the first and second refractors are 3.5m and 9.1m respectively. The Young's moduli of the layers were hence 0.5 and1.6 x $10^9 N/m^2$ for the first and second layers respectively.

From Fig. 3, the average P-wave velocities for the first three layers are 270m/s, 827m/s and 2329m/s which are interpreted as probably shale, clay and a saturated layer respectively. These velocities gave values of Young's modulus of the layers as 0.2 x $10^9 N/m^2$ and 2.7 x $10^9 N/m^2$ for the first and second layers respectively.

5.0 Conclusion:

In the light of the foregoing discussion, we conclude that:

i. Compressional waves propagated through materials in the body of the earth in the study area.
ii. In the course of the wave propagation, the subsurface materials experienced elastic deformation.
iii. The average speed of propagation of the waves in rock layers increased with depth; 290m/s for topmost(first layer) and 700m/s for second layer.
iv. The values of the Young's modulus (E) varied from one material type to another. In contrast to the wave velocities, the E-values generally decrease with depth of rock layers. Materials in the upper layers had lower values of Young's modulus than those in lower layers.
v. The mean values of Young's modulus of materials in the study area are 0.33 x 10^9 N/m^2 and 1.90 x 10^9 N/m^2 for the first and second layers respectively.
vi. Hence, given the same amount of stress, the deformation (strain) caused by the compressional waves on materials in the upper medium is greater than the deformation caused by the same waves in the lower medium.

Through this survey, an attempt has been made to study the deformational characteristics of compressional waves in Afikpo using estimates of the Young's modulus of the underlying materials. The limitation of the survey, however, was in the fact that only P-wave source was available and utilized.

For more accurate result, combined use of both P- and S- wave sources/sensors are therefore recommended to future researchers, so that the Poisson's ratio values and Bulk modulus of the materials could be determined in addition to the values of the Young's modulus.

6.0 References

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