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Variation of lumbar spine's centre of mass with changing posture of the lumbar spine

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Abstract

The lumbar spine is known to be the weight bearing section of the spine. The structures of the vertebrae are such as to allow the shape of the lumbar spine vary from one posture to another. The impact of the variation of the centre of mass and its implication on the stress experienced in different posture of the lumbar spine is the main thrust of this work. The result of this work shows that the centre of mass moves to the right of the original position when the lumbar spine bends backward with the vertical coordinate moved upward and the horizontal coordinate moved eastward while the reverse is the case when bending forward. Also, a quantity called virtual weight has been identified to be responsible for the seemingly increasing stress experienced during backward bending posture and the loss of weight or reduction in stress experienced during forward bending posture.

Keywords: Lumbar spine, Vertebra,Action parameter, posture, centre of mass, moment arm, weight and stress

1.0 Introduction

Eykhoff (1974) defined a mathematical model as 'a representation of the essential aspects of an existing system (or a system to be constructed) which presents knowledge of that system in usable form [1]. It is common to use idealized models in physics to simplify things. Massless ropes, point particles, ideal gases and the particle in a box are among the many simplified models used in physics. The laws of physics are represented with simple equations such as Newton's laws, Maxwell's equations and the Schrödinger equation. These laws serve as a basis for making mathematical models of real situations. Many real situations are very complex[2].In human anatomy, lumbar spine dynamics is an example of such complex phenomena. The lumbar vertebrae are the five vertebrae between the rib cage and the pelvis. They are the largest segments of the vertebral column and are characterized by the absence of the foramen transversarium within the transverse process, and by the absence of facets on the sides of the body. They are designated L1 to L5, starting at the top. The lumbar vertebrae help support the weight of the body, and permit movement [3]. This area is commonly called the "lower back". The lumbar vertebrae are the largest of the vertebrae because of their weight-bearing function supporting the torso and head [4,5]. The L5 vertebra (or the lumbar vertebra 5) is the last of the five lumbar vertebrae, positioned at the bottom of the lumbar curve of the back and just above the sacrum. Their articulating bodies are especially large and rather kidney-shaped with slightly concave faces above and below, plus a more deeply concave curve in the back. The L5additionally lacks costal facets. Located in the lumbar (from the Latin for 'loins') or pelvic region, the lumbar vertebrae provide substantial support to the rest of the spinal column rising above it. In particular, the fifth lumbar vertebra is distinct from the L1-4 vertebrae in being much larger on its front side than in the back. Its spinous process, on the other hand, is smaller than in the other lumbar vertebrae with a wide, four-sided shape that comes to a rough edge and a thick notch. The L5 vertebra's transverse process is particularly thick, and a wider space separates the inferior articular processes. However, like the other lumbar vertebrae, the L5 lumbar vertebra has strong pedicles, broad laminae, and long, thin transverse processes. The laminae are wider and they are tall, and the resulting vertebral arch encloses a triangular vertebral foramen somewhat smaller than that found in the cervical vertebrae but larger than the thoracic. Significant among its seven processes are three tubercles, among them the superior mammillary process and in the inferior position the accessory process [6].

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2.0 Methodology

In a paper Physical model of the lumbar spine [7], the equation of the lumbar spine for backward and forward movement is derived as

$$Y = \left(\frac{l}{\pi} \mp |\alpha|\right) \sin\frac{\pi\mu}{l}(x \pm |\beta|) \tag{2.01}$$

2.1 Centroid

The coordinates of the centre of mass can be calculated using element of area [8] as shown in the Figure 2.1. That is

$$dA = dxdy \tag{2.02}$$

To determine the coordinates of the centre of mass, further assumptions are made;

i. The lumbar spine is of uniform density,

ii.The area under the lumbar spine curve is homogenous.

The area of the Figure 2.1 is

$$A = \int_A dxdy = \int_0^X \int_0^Y dxdy = \int_0^X dx \int_0^Y dy = \int_0^X dx\{y\}_0^Y = \int_0^X Y dx \tag{2.03}$$

Using equation (2.01)

Therefore, equation (2.03) becomes

$$A = \int_0^X \left(\frac{l}{\pi} \mp |\alpha|\right) \sin\frac{\pi\mu}{l}(x \pm |\beta|)dx \tag{2.04}$$

$$A = \left\{-\frac{\left(\frac{l}{\pi} \mp |\alpha|\right)}{\left(\frac{\pi\mu}{l}\right)} \cos\frac{\pi\mu}{l}(x \pm |\beta|)\right\}_0^X \tag{2.05}$$

Putting in the limits of integration we have

$$A = \frac{\left(\frac{l}{\pi} \mp |\alpha|\right)}{\left(\frac{\pi\mu}{l}\right)} \cos(\pm|\beta|) - \frac{\left(\frac{l}{\pi} \mp |\alpha|\right)}{\left(\frac{\pi\mu}{l}\right)} \cos\frac{\pi\mu}{l}(X \pm |\beta|) \tag{2.06}$$

Now, the first moment [7] of the area about y axis is

$$M_y = \int_A x dA = \int_0^X \int_0^Y x dxdy = \int_0^X x dx \int_0^Y dy = \int_0^X x dx\{y\}_0^Y = \int_0^X xY dx \tag{2.07}$$

substituting$Y$ in equation (2.07) we have

$$M_y = \int_0^X \left(\frac{l}{\pi} \pm |\alpha|\right) x \sin\frac{\pi\mu}{l}(x \pm |\beta|)dx \tag{2.08}$$

Integrating equation (2.08) by parts we get

$$M_y = \left(\frac{l}{\pi} \mp |\alpha|\right) \left\{\frac{-x}{\left(\frac{\pi\mu}{l}\right)} \cos\frac{\pi\mu}{l}(x \pm |\beta|) + \frac{1}{(\frac{\pi\mu}{l})^2} \sin\frac{\pi\mu}{l}(x \pm |\beta|)\right\}_0^X \tag{2.09}$$

Also, the first moment [8] of the area about x axis is

$$M_x = \int_A y dA = \int_0^X \int_0^Y y dxdy = \int_0^X dx \int_0^Y y dy = \int_0^X \left\{\frac{y^2}{2}\right\}_0^Y dx = \int_0^X \frac{Y^2}{2} dx \tag{2.10}$$

substituting$Y$ in equation (2.10) we have

$$M_x = \int_0^X \frac{1}{2}\left(\frac{l}{\pi} \mp |\alpha|\right)^2 sin^2 \frac{\pi\mu}{l}(x \pm |\beta|)dx \tag{2.11}$$

Now,

$$\int_0^X sin^2 \frac{\pi\mu}{l}(x \pm |\beta|)dx = \int_0^X \frac{1}{\left(\frac{\pi\mu}{l}\right)} \sin\frac{\pi\mu}{l}(x \pm |\beta|).\, d(-\cos\frac{\pi\mu}{l}(x \pm |\beta|))$$

$$= \left\{\frac{-\sin\frac{\pi\mu}{l}(x\pm|\beta|).\cos\frac{\pi\mu}{l}(x\pm|\beta|)}{\left(\frac{\pi\mu}{l}\right)}\right\}_0^X + \int_0^X \frac{cos^2\frac{\pi\mu}{l}(x\pm|\beta|)dx}{\left(\frac{\pi\mu}{l}\right)}$$

$$\therefore \int_0^X sin^2\frac{\pi\mu}{l}(x\pm|\beta|)dx + \int_0^X \frac{sin^2\frac{\pi\mu}{l}(x\pm|\beta|)dx}{\left(\frac{\pi\mu}{l}\right)} = \left\{\frac{-\sin\frac{\pi\mu}{l}(x\pm|\beta|).\cos\frac{\pi\mu}{l}(x\pm|\beta|)}{\left(\frac{\pi\mu}{l}\right)} + \frac{x}{\left(\frac{\pi\mu}{l}\right)}\right\}_0^X$$

Hence,

$$\int_0^X sin^2\frac{\pi\mu}{l}(x\pm|\beta|)dx = \left(\frac{\left(\frac{\pi\mu}{l}\right)}{\left(\frac{\pi\mu}{l}\right)+1}\right)\left\{\frac{-\sin\frac{\pi\mu}{l}(x\pm|\beta|).\cos\frac{\pi\mu}{l}(x\pm|\beta|)}{\left(\frac{\pi\mu}{l}\right)} + \frac{x}{\left(\frac{\pi\mu}{l}\right)}\right\}_0^X \tag{2.12}$$

Substituting equation (2.12) in equation (2.11) we have

$$M_x = \frac{1}{2}\left(\frac{l}{\pi}\mp|\alpha|\right)^2\left(\frac{\left(\frac{\pi\mu}{l}\right)}{\left(\frac{\pi\mu}{l}\right)+1}\right)\left\{\frac{-\sin\frac{\pi\mu}{l}(x\pm|\beta|).\cos\frac{\pi\mu}{l}(x\pm|\beta|)}{\left(\frac{\pi\mu}{l}\right)} + \frac{x}{\left(\frac{\pi\mu}{l}\right)}\right\}_0^X \tag{2.13}$$

The coordinates of centre of mass is definedby [8] as

$$X_c = \frac{M_y}{A} \qquad Y_c = \frac{M_x}{A} \tag{2.14}$$

Substituting equations (2.06), (2.09) and (2.13) into equation (2.14)we have

$$X_c = \frac{\left(\frac{l}{\pi}\mp|\alpha|\right)\left\{\frac{-x}{\left(\frac{\pi\mu}{l}\right)}\cos\frac{\pi\mu}{l}(x\pm|\beta|)+\frac{1}{\left(\frac{\pi\mu}{l}\right)^2}\sin\frac{\pi\mu}{l}(x\pm|\beta|)\right\}_0^X}{\frac{\left(\frac{l}{\pi}\mp|\alpha|\right)}{\left(\frac{\pi\mu}{l}\right)}\left(\cos(\pm|\beta|)-\cos\frac{\pi\mu}{l}(X\pm|\beta|)\right)}$$

$$Y_c = \frac{\frac{1}{2}\left(\frac{l}{\pi}\mp|\alpha|\right)^2\left(\frac{\left(\frac{\pi\mu}{l}\right)}{\left(\frac{\pi\mu}{l}\right)+1}\right)\left\{\frac{-\sin\frac{\pi\mu}{l}(x\pm|\beta|).\cos\frac{\pi\mu}{l}(x\pm|\beta|)}{\left(\frac{\pi\mu}{l}\right)}+\frac{x}{\left(\frac{\pi\mu}{l}\right)}\right\}_0^X}{\frac{\left(\frac{l}{\pi}\mp|\alpha|\right)}{\left(\frac{\pi\mu}{l}\right)}\left(\cos(\pm|\beta|)-\cos\frac{\pi\mu}{l}(X\pm|\beta|)\right)} \tag{2.15}$$

Equation(2.15) simplifies to

$$X_c = \frac{\left\{\frac{\sin\frac{\pi\mu}{l}(x\pm|\beta|)}{\left(\frac{\pi\mu}{l}\right)}-x\cos\frac{\pi\mu}{l}(x\pm|\beta|)\right\}_0^X}{\cos(\pm|\beta|)-\cos\frac{\pi\mu}{l}(X\pm|\beta|)}$$

$$Y_c = \frac{\frac{1}{2}\left(\frac{l}{\pi}\mp|\alpha|\right)\left(\frac{\left(\frac{\pi\mu}{l}\right)^2}{\left(\frac{\pi\mu}{l}\right)+1}\right)\left\{\frac{-\sin\frac{\pi\mu}{l}(x\pm|\beta|).\cos\frac{\pi\mu}{l}(x\pm|\beta|)}{\left(\frac{\pi\mu}{l}\right)}+\frac{x}{\left(\frac{\pi\mu}{l}\right)}\right\}_0^X}{\cos(\pm|\beta|)-\cos\frac{\pi\mu}{l}(X\pm|\beta|)} \tag{2.16}$$

By substituting the limits of integration we have

$$X_c = \frac{\frac{\sin\frac{\pi\mu}{l}(X\pm|\beta|)}{\left(\frac{\pi\mu}{l}\right)}-X\cos\frac{\pi\mu}{l}(X\pm|\beta|)}{\cos(\pm|\beta|)-\cos\frac{\pi\mu}{l}(X\pm|\beta|)}$$

$$Y_c = \frac{\frac{1}{2}\left(\frac{l}{\pi}\mp|\alpha|\right)\left(\frac{\left(\frac{\pi\mu}{l}\right)}{\left(\frac{\pi\mu}{l}\right)+1}\right)\left(X-\sin\frac{\pi\mu}{l}(X\pm|\beta|).\cos\frac{\pi\mu}{l}(X\pm|\beta|)\right)}{\cos(\pm|\beta|)-\cos\frac{\pi\mu}{l}(X\pm|\beta|)} \tag{2.17}$$

Putting back the equation in the right vertical position of the lumbar spine, we have

$$Y_c = \frac{\frac{\sin\frac{\pi\mu}{l}(Y\pm|\beta|)}{\left(\frac{\pi\mu}{l}\right)} - Y\cos\frac{\pi\mu}{l}(Y\pm|\beta|)}{\cos(\pm|\beta|) - \cos\frac{\pi\mu}{l}(X\pm|\beta|)}$$

$$X_c = \frac{\frac{1}{2}\left(\frac{l}{\pi}\mp|\alpha|\right)\left(\frac{\left(\frac{\pi\mu}{l}\right)}{\left(\frac{\pi\mu}{l}\right)+1}\right)\left(Y - \sin\frac{\pi\mu}{l}(Y\pm|\beta|).\cos\frac{\pi\mu}{l}(Y\pm|\beta|)\right)}{\cos(\pm|\beta|) - \cos\frac{\pi\mu}{l}(Y\pm|\beta|)} \quad (2.18)$$

Equation (3.7) [9]$l = \mu Y$ (2.19)

2.2 Application

Now, for arbitrary values of l, Y as 20cm and 10cm respectively, and for various actions with action parameters, the table for the coordinates of centre of mass have been calculated, using equation (2.18) and equation (2.19), as shown in Table 2.1

Where, $d_{MA} = \frac{l}{\pi} \pm |\alpha| - X_c$ and is defined as the moment arm of a point at the extremum of the curvature of the lumbar spine.

Definition 2.1. Due to the variation of moment arm with changing curvature, we are defining two important quantities namely: virtual weight W_v and virtual stress S_v.

VIRTUAL WEIGHT $\boldsymbol{W_v}$ is defined as the weight additional to the true weight W_t required at neutral posture of the lumbar spine to produce the same moment at a point as the original weight W_t at any other posture of the lumbar spine.

Mathematically,

$$(W_v + W_t)d_N = dW_t \quad (2.20)$$

and,

$$W_v = \left(\frac{d}{d_N} - 1\right)W_t \quad (2.21)$$

Where d is the moment arm at other posture of the lumbar spine and d_N is the moment arm at neutral posture.

VIRTUAL STRESS $\boldsymbol{S_v}$ is the ratio of virtual weight to the area under the stress.

Mathematically,

$$S_v = \frac{W_v}{A} = \left(\frac{d}{d_N} - 1\right)\frac{W_t}{A} \quad (2.22)$$

but

$$A = A_0 cos\theta$$

Where A_0, is the original surface area of the vertebra under consideration and θ is the tilting angle. Hence equation (2.22) becomes

$$S_v = \left(\frac{d}{d_N} - 1\right)\frac{W_t}{A_0 cos\theta} \quad (2.23)$$

3.0 Results and Discussion

Equation (2.18) gives the equation of the coordinates of centre of mass. Figure 2.2shows that the centre of mass shifts upward as the lumbar spine bends backward while in forward bending, Figure 2.6 seems to show upward displacement of centre of mass, this may not be true as the chord formed by the lumbar spine increases with forward bending (see Figure 2.1). However, to understand the true behaviour of the centre of mass with forward bending, Figure 2.7 gives the true picture which implies a downward displacement opposite to what is seen in backward bending and this explains why the action of raising up of hands reduces stability [10] as raising of hands can bring about little backward bending of the lumbar spine [9] (see Figure 2.9). It can therefore be concluded that backward bending of the spine reduces it stability and can have negative consequences on the weight bearing capability of the lumbar spine.

According toMafuyai (2013), Figure 3.13 [9], figure 2.2.2 and 2.2.3 can both be seen as variations of moment arms of some vertebrae with backward bending and since the slope of the graphs are positive, this means that the moment arm increases with backward bending and hence increases the tendency for the vertebrae to be unstable (turning effect) [11]. But Figure 2.5 and 2.8 gives the reverse process during forward bending. From the foregoing, the action of backward bending generally reduces stability of the lumbar spine and this goes to explain why the lumbar spine easily sustain injuries in backward bending than in forward bending.

Equations (2.21) and (2.23) are the virtual weight and virtual stress equations respectively, and from figure 3.13 [9], the ratio of the moment arm at backward bending to that at neutral posture is greater than unity and this means the virtual weight is positive. And this explain the seemingly increase in weight people experience when they bend backward and the reason for getting tired easily as well while the ratio of the moment arm at forward bending is less than unity, and this will make the value of virtual weight to be negative which means reduction in the overall weight borne by the lumbar spine. Even though the virtual weight may not be much compare to the true weight of the body, but the stress that is created in the inter-vertebral disc can be great due to the fact that a little tilt between the vertebrae can greatly reduce the surface area of contact of the vertebrae. Under such condition, any twist of the lumbar spine can lead to the wearing and possibly the inflammation of the inter-vertebral disc owing to the increased friction that result from the virtual stress. This is one of the causes of lower back pain that anatomists don't seem to know the cause [12,13,14] and other models could not explain.

4.0 Conclusion

Backward bending creates an additional weight on the lumbar spine which greatly stresses it. 'Action' thatincrease stress on lumbar spine such as obesity, pregnancy, facing the sky, sitting upright, use of high heel shoes etc are suspected to cause a backward bend of the lumbar spine. This explain the reason why students may easily become bored with lecture after a short while when prevailed upon to seat upright during the lesson.

Forward bending however reduces stress from the lumbar spine and this explains the natural tendency for human being to bend forward at old age.

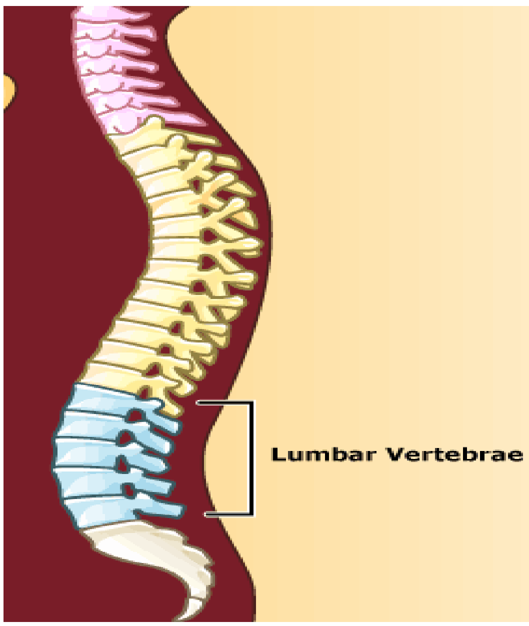

Figure 1.1: Structure of the Spine showing the Lumbar Vertebrae [5]

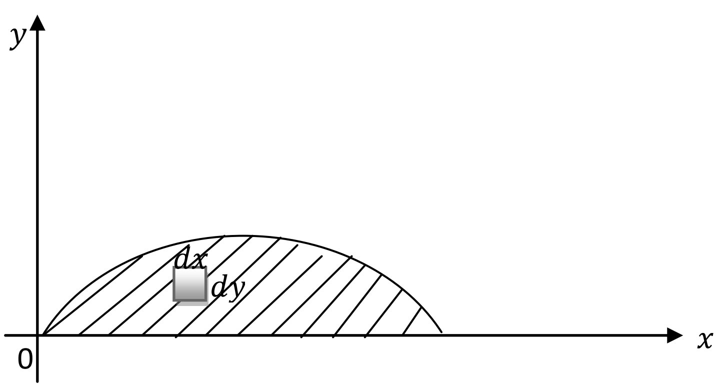

Figure 2.1 A SCHEMATIC OF LUMBAR

Table 2.1: COORDINATES OF CENTER OF MASS FOR VARIOUS ACTION PARAMETERSAT AN INTERVAL OF $0.5°$FOR BACKWARD BENDING

$\|\beta\|/°$	$\|\beta\|/rad$	μ	$Y(cm)$	$X_c(cm)$	$Y_c(cm)$	$d_{MA}(cm)$
0.00	0.0000	2.0000	10.0000	3.80460	4.99935	2.56077
0.50	0.0044	2.0009	9.9956	3.80723	4.99933	2.56254
1.00	0.0087	2.0017	9.9913	3.80993	4.99955	2.56414
1.50	0.0131	2.0026	9.9869	3.81281	4.99963	2.56566
2.00	0.0175	2.0035	9.9825	3.81572	4.99972	2.56715
2.50	0.0218	2.0044	9.9782	3.81862	4.99980	2.56855
3.00	0.0262	2.0053	9.9738	3.82161	5.00003	2.56996
3.50	0.0305	2.0061	9.9695	3.82451	5.00045	2.57136
4.00	0.0349	2.0070	9.9651	3.82759	5.00076	2.57268
4.50	0.0393	2.0079	9.9607	3.83070	5.00106	2.57397
5.00	0.0436	2.0088	9.9564	3.83381	5.00140	2.57516
5.50	0.0480	2.0096	9.9520	3.83709	5.00159	2.57628
6.00	0.0524	2.0105	9.9476	3.84016	5.00256	2.57761
6.50	0.0567	2.0114	9.9433	3.84339	5.00300	2.57868
7.00	0.0611	2.0122	9.9389	3.84663	5.00382	2.57984
7.50	0.0655	2.0132	9.9345	3.85007	5.00418	2.58080
8.00	0.0698	2.0141	9.9302	3.85342	5.00478	2.58175
8.50	0.0742	2.0150	9.9258	3.85687	5.00545	2.58270
9.00	0.0786	2.0158	9.9214	3.86028	5.00643	2.58369
9.50	0.0829	2.0167	9.9171	3.86375	5.00719	2.58452
10.00	0.0873	2.0176	9.9127	3.86732	5.00802	2.58535

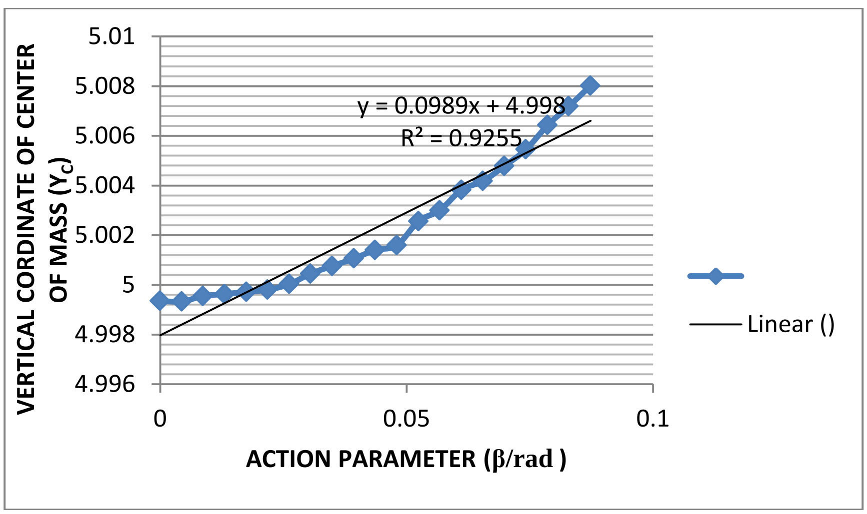

Figure2.2: GRAPH OF THE VERTICAL COORDINATE OF THE CENTER OF MASSAGAINST ACTION PARAMETER FOR BACKWARD BENDIN

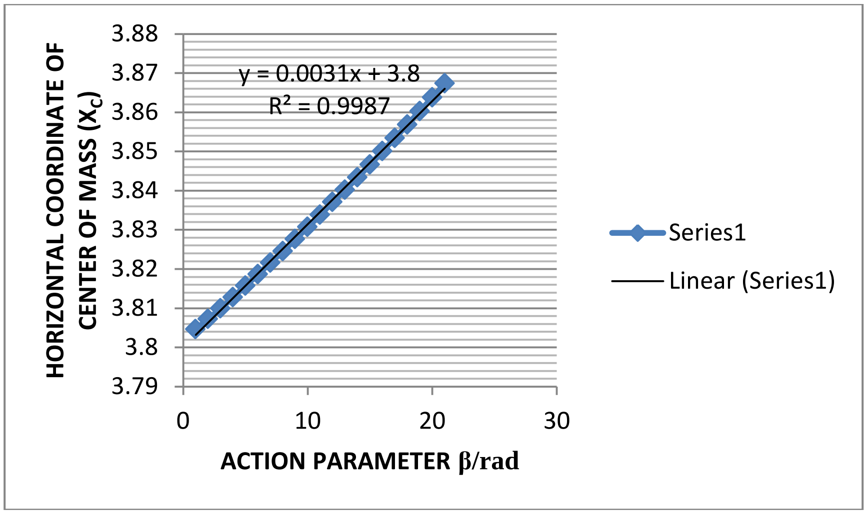

Figure 2.3: GRAPH OF THE HORIZONTAL COORDINATES OF THE CENTER OF MASSAGAINST ACTION PARAMETER FOR BACKWARD BENDING

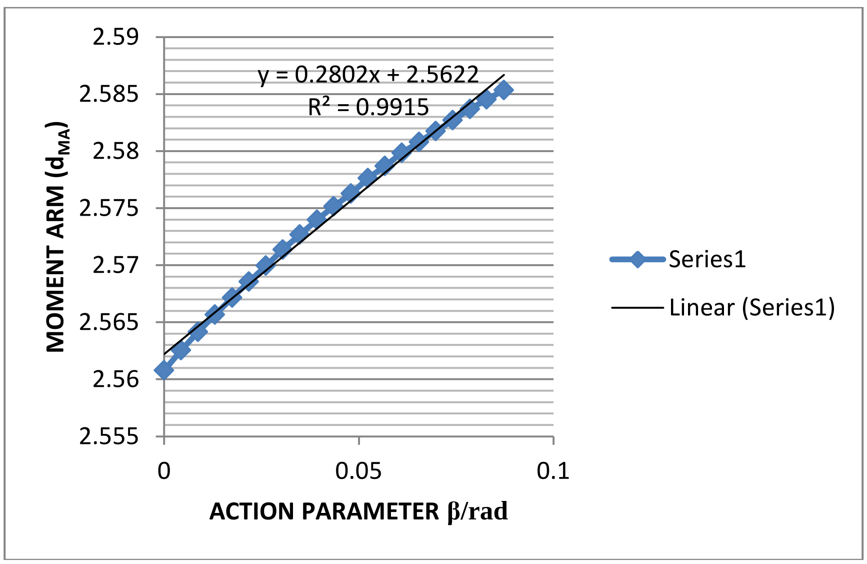

Figure 2.4: GRAPH OF THE MOMENT ARMAGAINST ACTION PARAMETER FOR BACKWARD BENDING

Table 2.2: COORDINATES OF CENTER OF MASS FOR VARIOUS ACTION PARAMETERS AT AN INTERVAL OF 1. 0° DURING FORWARD BENDING

$\|\beta\|/°$	$\|\beta\|/rad$	μ	$Y(cm)$	$X_c(cm)$	$Y_c(cm)$	$d_{MA}(cm)$	Y/Y_c
0.00	0.0000	2.0000	10.0000	3.8046	4.9994	2.5608	2.0002
1.00	0.0087	1.9983	10.0087	3.7991	4.9992	2.5576	2.0021
2.00	0.0175	1.9965	10.0175	3.7938	4.9997	2.5541	2.0036
3.00	0.0262	1.9948	10.0262	3.7888	5.0000	2.5503	2.0052
4.00	0.0349	1.9930	10.0349	3.7838	5.0008	2.5467	2.0067
5.00	0.0436	1.9913	10.0436	3.7791	5.0015	2.5427	2.0081
6.00	0.0524	1.9896	10.0524	3.7746	5.0024	2.5384	2.0095
7.00	0.0611	1.9879	10.0611	3.7702	5.0034	2.5341	2.0109
8.00	0.0698	1.9861	10.0698	3.7658	5.0049	2.5298	2.0120
9.00	0.0786	1.9844	10.0786	3.7617	5.0063	2.5251	2.0132
10.00	0.0873	1.9827	10.0873	3.7578	5.0080	2.5208	2.0142

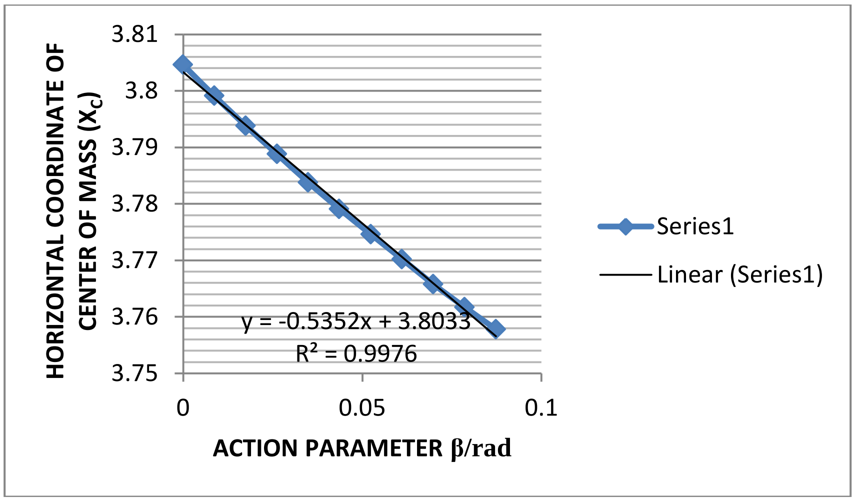

Figure 2.5: GRAPH OF THE HORIZONTAL COORDINATES OF THE CENTER OF MASSAGAINST ACTION PARAMETER FOR FORWARD BENDING

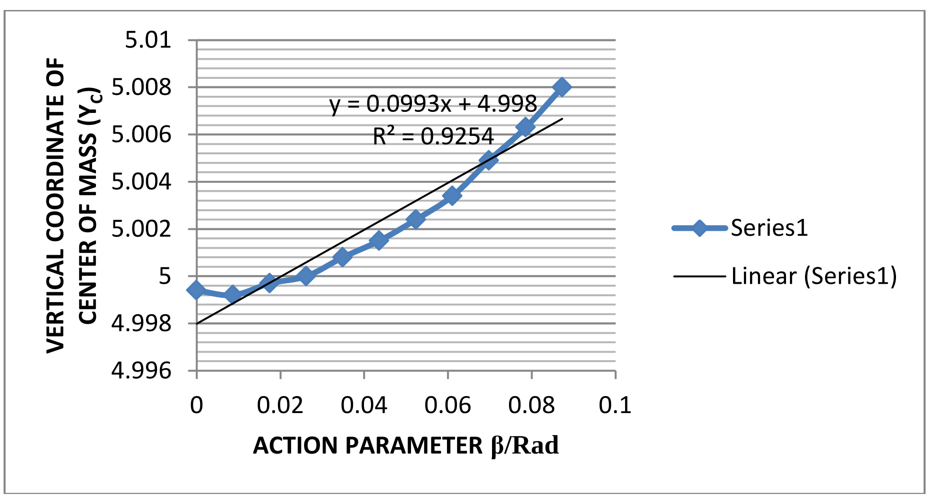

Figure 2.6:GRAPH OF THE VERTICAL COORDINATE OF THE CENTER OF MASSAGAINSTACTION PARAMETER FOR FORWARD BENDING

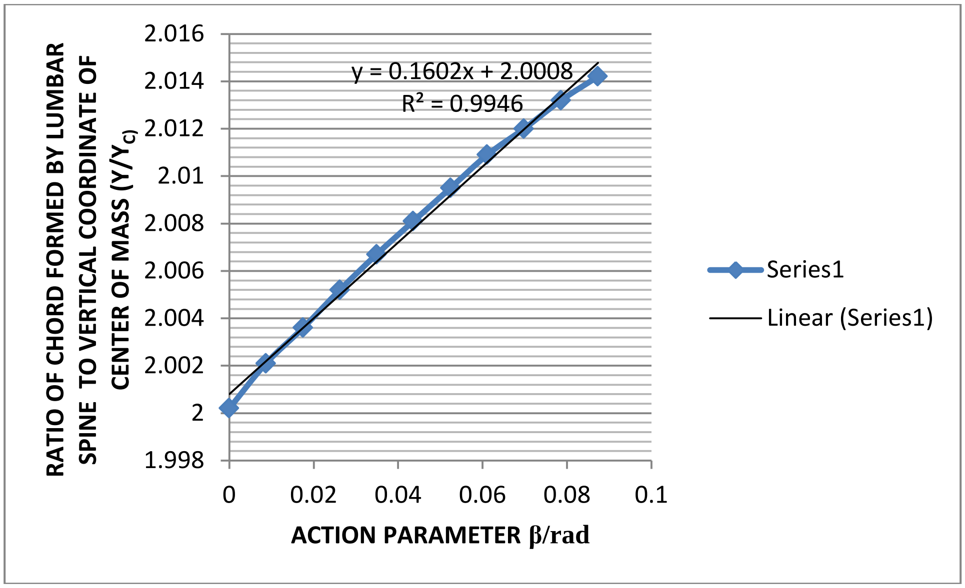

Figure 2.7: GRAPH OF RATIO OF CHORD FORMED BY THE LUMBAR SPINE TO VERTICAL COORDINATE OF THE CENTER OF MASS ACTION PARAMETER FOR FORWARD BENDING

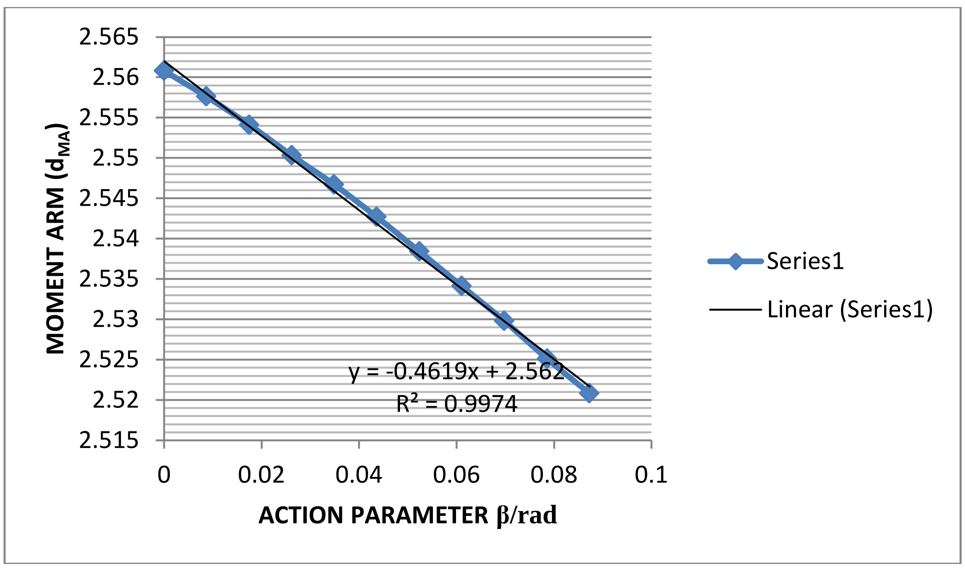

Figure 2.8: GRAPH OF THE MOMENT ARMAGAINST ACTION PARAMETER FOR FORWARD BENDING

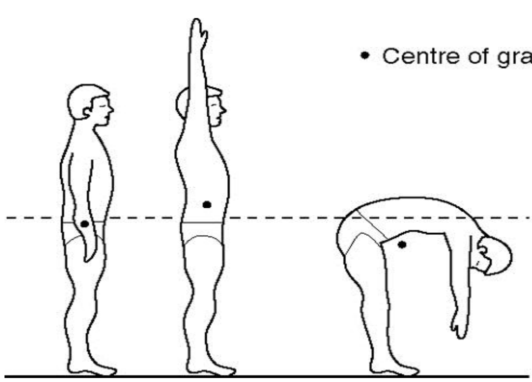

Figure 2.9: PICTURE SHOWING POSITIONS OF CENTRE OF GRAVITY (MASS) IN VARIOUS SHAPES OF THE LUMBAR SPINE[10]

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