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The Effect of Damping on the Dynamic Response of Beam Subjected To Distributed Load

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Abstract

Dynamics is a branch of Mathematical science that involves motion of bodies and the forces acting during the motion. This paper is concerned with the dynamic analysis of a structural damped system subjected to distributed moving load of constant magnitude and velocity. The governing equation of motion of the beam is transformed into a coupled ordinary differential equations. Method of Duhamel's integral is adopted in solving the resulting ordinary differential equations analytically. A numerical analysis of the effect of damping on the dynamic response of the beams, under moving loads with simply supported conditions is presented using tables and graphs. It was observed that when ζ>1 the deflection curves exhibit maximum amplitude at dimensionless time $\bar{t}$=4.00 for $\zeta = 1.5$. Also for ζ =1 the deflection curves exhibit maximum amplitude at $\bar{t}$= 6.0 and when ζ<1 the deflection curves exhibit maximum amplitude at dimensionless time $\bar{t}$= 8.0 for $\zeta =$ 0.0. Conclusively, it was observed that the time at which the curves exhibit the maximum amplitude increases as damping ratio ζ decreases.

Keywords: Damping, Distributed Moving Load, Simply supported Beams.

1.0 Introduction

Dynamics is a branch of Mathematical science that involves motion of bodies and the forces acting during the motion. A beam, or flexural member, is frequently encountered in structures and machines. It is a member subjected to loads applied transverse along the dimension, causing the member to bend. The dynamics effects of a load on beam and beam-like structural members play significant role in railway tracks and highway permanent designs. In modern engineering practices, beam-like structures resting on both variable and constant elastic foundation have wide applications and for this reason several authors have investigated the dynamic deflection of beam [1-7]. This paper is concerned with the dynamic analysis of a structural damped system resting on a foundation subjected to uniformly distributed moving load of constant magnitude and velocity. This type of problem is known as the moving load problem and has been of practical interest to many researchers. This is due to the fact that structure under the influence of moving vehicles have been subjected to vibration and dynamic stress which are far larger than before as a result of recent advances in speed of moving vehicles[3].

2.0 The Mathematical Model

We consider the case of a uniformly distributed load, advancing uniformly on a beam with velocity U, are shown as Figure 1. It is assumed that initially at time t = 0 the load is situated at the left hand support.

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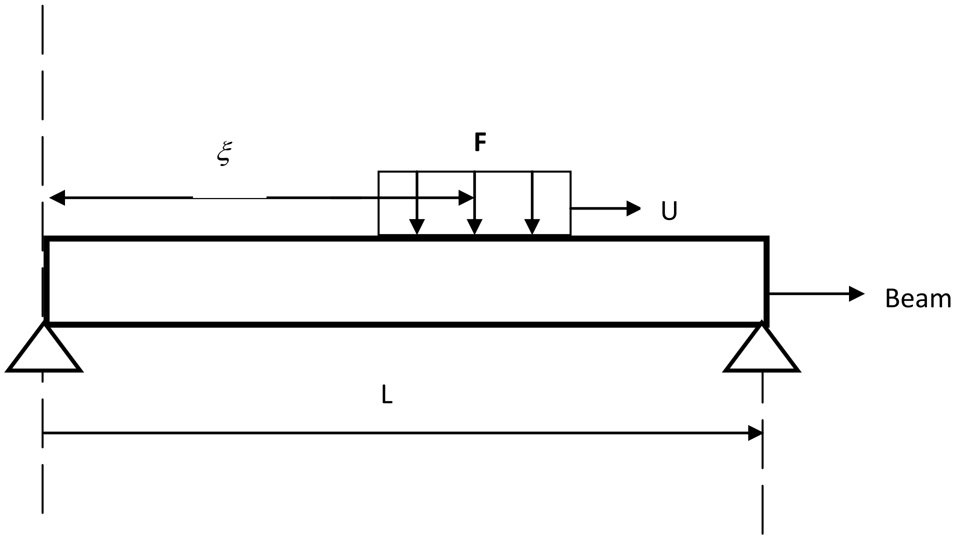

Figure 1: Diagrammatic illustration of the structural system

The following were assumed from the analysis;

1. The beam characteristics are described by Euler-Bernoulli equation.
2. The load moves at constant velocity and keeps in contact with the beam at all times.
3. The transverse displacement response V(t,x) is a product of position and time.

Based on the above descriptions and assumptions, the problem of interest is described to be a partial differential equation of the form:

$$\frac{\partial^2}{\partial x^2}\left[EI(x)\left(\frac{\partial^2 V(x,t)}{\partial x^2}+a_1\frac{\partial^3 V(x,t)}{\partial x^2\,\partial t}\right)\right]+m(x)\frac{\partial^2 V(x,t)}{\partial t^2}+C(x)\frac{\partial V(x,t)}{\partial t}$$

$$+K(x)V(x,t)-G(x)\frac{\partial^2 V(x,t)}{\partial x^2}=F(x,t) \qquad (1)$$

Where $F(x,t)$ is the applied moving Load. The symbols used in the equation (1) are defined as follows.

$EI(x)$ is the flexural rigidity of the structure

E is the modulus of elasticity

I is the cross moment of inertia

C is the damping coefficient

a_1 is the stiffness proportionality factor define for Rayleigh damping

$V(x,t)$ is the displacement of the structure at point x and time t, measure from the equilibrium position when unloaded.

t is the time

χ is the axial co-ordinate

m is the constant mass per unit length of the structure.

ξ is the position of the moving load from left of the structure.

The associated boundary and initial conditions are respectively given as

$$V(0,t)=V(L,t)=V''(L,t)=0 \qquad (2a)$$

and

$$V(x,t)=\frac{\partial V(x,0)}{\partial t}=0 \qquad (2b)$$

3.0 The Analytic Solution Of The Governing Equation Of Motion

The governing equation for the problem is given as :

$$\frac{\partial^2}{\partial x^2}\left[EI(x)\left(\frac{\partial^2 V(x,t)}{\partial x^2}+a_1\frac{\partial^3 V(x,t)}{\partial x^2\,\partial t}\right)\right]+M(x)\frac{\partial^2 V(x,t)}{\partial t^2}+C(x)\frac{\partial V(x,t)}{\partial t}+K(x)(x,t)-G(x)\frac{\partial^2 V(x,t)}{\partial x^2}$$

$$=Mg\left[H\left(x-\left(\xi+\frac{\varepsilon}{2}\right)\right)-H\left(x-(\xi-\frac{\varepsilon}{2})\right)\right]-M\left[\frac{\partial^2 V(x,t)}{\partial t^2}+u^2\frac{\partial^2 V(x,t)}{\partial x^2}+2u\frac{\partial^2 V(x,t)}{\partial x\,\partial t}\right]$$

$$\left[H\left(x-(\xi+\frac{\varepsilon}{2})\right)-H\left(x-(\xi-\frac{\varepsilon}{2})\right)\right] \qquad (3)$$

We make use of constant values of the distributed parameters such as $EI(x),\ m(x),\ C(x),\ K(x)\ and\ G(x)$ in equation (3) to obtain:

$$\left[EI\frac{\partial^4 V(x,t)}{\partial x^4}+a_1 EI\frac{\partial^5 V(x,t)}{\partial x^4\,\partial t}\right]+Mt\frac{\partial^2 V(x,t)}{\partial t^2}+C\frac{\partial V(x,t)}{\partial t}+KV(x,t)-G\frac{\partial^2 V(x,t)}{\partial x^2} \qquad (4)$$

$$=Mg\left[H\left(x-(\xi+\frac{\varepsilon}{2})\right)-H\left(x-(\xi-\frac{\varepsilon}{2})\right)\right]-M\left[\frac{\partial^2 V(x,t)}{\partial t^2}+u^2\frac{\partial^2 V(x,t)}{\partial x^2}+2u\,\partial^2\frac{V(x,t)}{\partial x\,\partial t}\right]$$

Then we assume a solution of the form

$$V(x,t)=\sum_n X_n(x)T_n(t) \qquad (5)$$

So that equation (4) becomes

$$\sum_n X_n^{IV}(x)T_n(t)+a_1\sum X_n^{IV}(x)\dot{T}_n(t)+\frac{Mt}{EI}\sum_n X_n(t)\ddot{T}_n(t)+\frac{K}{EI}\sum_n X_n(x)T_n(t)-\frac{G}{EI}\sum_n X_n^{II}(x)T_n(t)$$

$$=\frac{1}{EI}\left\{Mg-M\left[\sum_n X_n(x)\ddot{T}_n(t)+u^2\sum_n X_n^{II}(x)T_n(t)+2u\sum_n X_n^{I}(x)\dot{T}_n(t)\right]\right\}$$

$$\left\{H\left(x-(\xi+\frac{\varepsilon}{2})\right)-H\left(x-(\xi-\frac{\varepsilon}{2})\right)\right\} \qquad (6)$$

Simplification of equation (6) yields

$$\ddot{T}_n(t)+2\omega_n\zeta\dot{T}_n(t)+\omega_n^2T_n(t)=P(t) \qquad (7)$$

Where $P(t)$ *is*

$$\left\{-\frac{Mg}{m_t}\int_O^L X_K(x)dx-\frac{M}{m_t}\sum_n\left[\begin{array}{l}\ddot{T}(t)\int_O^L X_n(x)X_K(x)dx+u^2T_n(t)\int_O^L X_n^{11}(x)X_K(x)dx\\ 2u\,\dot{T}_n(t)\int_O^L X_n^{1}(x)X_K(x)dx\end{array}\right]\right\}$$

$$\left\{H\left(x-(\xi+\frac{\varepsilon}{2})\right)-H\left(x-(\xi-\frac{\varepsilon}{2})\right)\right\}dx$$

Solving the set of coupled Linear Second Order differential equations (7) the values of $T_n(t)^{'}s$ can be determined. When substituting these values into the assumed solution i.e equation (5), the desired solution for the vibration of the beam, under different boundary conditions and with any number of modal shapes can be determined.

In order to investigate and achieve this aims, the case of simply supported beam is considered. Beams vibrating either freely or under an applied force assume various vibrating configuration depending on the associated boundary conditions.

The most common and the simplest vibrating configuration, which is also adopted in this work is the simply supported one with the corresponding boundary conditions described as

$$V\ (o,t) = V^{II}\ (o,t) = o$$

$$V\ (L,t) = V^{II}\ (L,t) = o$$

and the initial condition as $V(x,t) = \dfrac{\partial V(x,0)}{\partial t} = 0$

The normalized deflection curves, X_n for a simply supported beam is

$$\left.\begin{array}{l} X_n(x) = \sqrt{\dfrac{2}{L}}\ Sin\dfrac{n\prod x}{L} \\ n = 1,2,3 \end{array}\right\} \qquad (8)$$

Substitution of eq.(8) into (7) and further Simplification yields

$$\ddot{T}_n(t) + 2\omega_n \zeta \dot{T}_n(t) + \omega_n^2 T_n(t) = \left\{\begin{array}{l} -\dfrac{Mg}{Mt}\int_0^L \sqrt{\dfrac{2}{L}}\int_O^L Sin\dfrac{K\prod x}{L}dx - \dfrac{M}{Mt}\sum_n \\ \left[\ddot{T}_n(t)\int_O^L \sqrt{\dfrac{2}{L}} Sin\dfrac{n\prod x}{L} Sin\dfrac{K\prod x}{L}dx\right] + U^2\, T_n(t)\int_O^L -\left(\dfrac{n\prod}{L}\right)^2 \\ \sqrt{\dfrac{2}{L}} Sin\dfrac{n\prod x}{L}.\sqrt{\dfrac{2}{L}} Sin\dfrac{K\prod x}{L}dx + 2u\dot{T}_n(t)\int_O^L \dfrac{n\prod}{L}\sqrt{\dfrac{2}{L}} \\ Cos\dfrac{n\prod x}{L}\sqrt{\dfrac{2}{L}} Sin\dfrac{K\prod x}{L}dx\ \Big] \end{array}\right\} \qquad (9)$$

$$\left\{H\left(x-(\xi+\frac{\varepsilon}{2})\right) - H\left(x-(\xi-\frac{\varepsilon}{2})\right)\right\}$$

$$= \left\{\begin{array}{l} -\dfrac{Mg}{Mt}\sqrt{\dfrac{2}{L}}\int_O^L Sin\dfrac{K\prod x}{L}dx - \dfrac{2M}{Lmt}\sum_n \left[\ddot{T}_n(t)\int_O^L \sqrt{\dfrac{2}{L}} Sin\dfrac{n\prod x}{L}.Sin\dfrac{K\prod x}{L}dx\right] \\ -\left(+\dfrac{n\prod}{L}\right)^2 U^2\, T_n(t)\int_O^L -^2\, Sin\dfrac{n\prod x}{L}.Sin\dfrac{K\prod x}{L}dx + 2u\left(\dfrac{n\prod}{L}\right)\dot{T}_n(t)\int_O^L Cos\dfrac{n\prod x}{L}Sin\dfrac{K\prod x}{L}dx\ \Big] \end{array}\right\}$$

$$\left\{H\left(x-(\xi+\frac{\varepsilon}{2})\right) - H\left(x-(\xi-\frac{\varepsilon}{2})\right)\right\} \qquad (9b)$$

4.0 Application of Duhamel's Integral

The solution of equation (9) subject to the initial conditions stated above is given by Duhamel's integral as follows

$$T_n(t) = \frac{1}{\omega_n}\int_0^t P(\tau)\exp\left[\frac{-1}{2}\mu_N^2(\bar{t}-\tau)\sin\omega_n(\bar{t}-l)d\tau\right] \qquad (10)$$

Where tau (τ) is the time at which the effect of the Load is been considered on the structure, W_n is the natural angular frequency of undamped system, W_d is the free-vibration frequency of a damped system, ξ is the damping ratio and

$$P(\tau) = \frac{Q\cdot R}{\Pi n}\left[\cos J - \cos D\right] - QRF\sum_{k=1}^{\infty}\frac{1}{8k(n-k)}\lambda_1$$

$$\left[\sin L_3 - \sin L_4 + \sin L_5 - \sin L_8 - \sin L_9 + \sin L_{14} - SINL_{10} - \sin L_{12} + \sin L_{13} + \sin L_2\right] + QRF\sum_{k=1}^{\infty}\frac{1}{8k(n-k)}\lambda_2$$

$$\left[\sin L_3 - \sin L_4 + \sin L_5 - \sin L_8 - \sin L_9 + \sin L_{14} - SINL_{10} - \sin L_{12} + \sin L_{13} + \sin L_2\right]$$

$$-QRF \sum_{k=1}^{\infty} \frac{1}{4k(n-k)} \lambda_3 \left[\sin M_1 + \sin M_2 + \sin M_3 + \sin M_4 - \sin m_5 - \sin M_6 - \sin M_7 - \sin M_8\right]$$

$$-QRF \sum_{k=1}^{\infty} \frac{1}{8k(n-k)} \lambda_4$$

$$\left[\sin L_3 + \sin L_4 + \sin L_2 - 2\sin L_1 + \sin L_8 - \sin L_5 - \sin L_{10} - \sin L_{14} + \sin L_9 - \sin L_{13} + \sin L_{12}\right]$$

$$+QRF \sum_{k=1}^{\infty} \frac{1}{8k(n-k)} \lambda_5$$

$$\left[\sin L_3 + \sin L_4 + \sin L_2 - 2\sin L_1 + \sin L_8 - \sin L_5 - \sin L_{10} - \sin L_{14} \sin L_9 - \sin L_{13} + \sin L_{12}\right]$$

$$-QRF \sum_{k=1}^{\infty} \frac{1}{4k(k-n)} \lambda_6 \left[\cos M_1 + \cos M_2 + \cos M_3 + \cos M_4 - \cos M_5 - \cos M_6 - \cos M_7 - \cos M_8\right]$$

$$-QRF \sum_{k=1}^{\infty} \frac{1}{8k(k-n)} \lambda_1 [$$

$$2\sin N_1 - \sin N_2 + \sin N_3 + \sin N_3 + \sin N_4 + \sin N_5 - 2\sin N_6 + 2\sin N_7 + \sin N_8 - \sin N_9 - \sin N_{10}$$

$$-\sin N_{11} - \sin N_{12} - \sin N_{13}]$$

$$-QRF \sum_{k=1}^{\infty} \frac{1}{8k(n-k)} \lambda_2 \left[\sin N_3 - \sin N_4 + \sin N_2 + \sin N_9 - \sin N_8 + \sin N_{13} + \sin N_{11} - \sin N_{10} - \sin N_{12}\right]$$

$$-QRF \sum_{k=1}^{\infty} \frac{1}{4k(k-n)} \lambda_3 [\sin q_1 + \sin\sin q_2 + \sin q_3 + \sin q_4 - \sin q_5 - \sin q_6 - \sin q_7 - \sin q_8]$$

$$+QRF \sum_{k=1}^{\infty} \frac{1}{8k(n-k)} \lambda_4 [\cos N_4 + \cos N_2 - 2\cos N_1 - \cos N_3 + \cos N_8 + \cos N_{14} - \cos N_5 - 2\cos N_7$$

$$-\cos N_{11} - \cos N_1 + \cos N_{10} - \cos N13 + \cos N_{12} + \cos N6]$$

$$+QRF \sum_{k=1}^{\infty} \frac{1}{8k(n-k)} \lambda_5 [$$

$$\cos N_4 + \cos N_2 - 2\cos N_1 - \cos N_3 + \cos N_8 + \cos N_{14} - \cos N_5 - 2\cos N7 - \cos N11 + \cos N_9$$

$$-\cos N_{11} + \cos N_9 + \cos N_{10} - \cos N_{13} + \cos N_{12} + \cos N_6]$$

$$+QRF \sum_{k=1}^{\infty} \frac{1}{4k(n+k)} \lambda_6 \left[\cos q_1 + \cos q_2 + \cos q_3 + \cos q_4 - \cos q_5 - \cos q_6 - \cos q_7 - \cos q_8\right]$$

$$+QRF \sum_{k=1}^{\infty} \frac{1}{8k(n-k)} \lambda_7$$

$$\left[2\sin L_{16} - \sin L_4 + \sin L_3 + \sin L_2 + \sin L_5 - \sin L_8 \sin L_{16} - \sin L_9 + \sin L_{10} - \sin L_{12} + \sin L_{13}\right]$$

$$+QRF \sum_{k=1}^{\infty} \frac{1}{8k(n-k)} \lambda_8$$

$$\left[2\sin L_1 - \sin L_2 + \sin L_3 + \sin L_4 + \sin L_5 - 2\sin L_6 + 2\sin L_7 + \sin L_8 - \sin L_9 - \sin L_{10} - \sin_{13} - \sin L_{14}\right]$$

$$-QRF \sum_{k=1}^{\infty} \frac{1}{4k(n-k)} \lambda_8 \left[\sin M_1 + \sin M_2 + \sin M_3 + \sin M_4 - \sin M_5 - \sin M_6 - \sin_7 - \sin M_7 - \sin M_8\right]$$

$$+QRF \sum_{k=1}^{\infty} \frac{1}{k(n-k)} \lambda_9 [\cos L_{17} + \cos L_2 - 2\cos L_1 - \cos L_3 + \cos L_8 + \cos L_6 - \cos L_5 - \cos L_7$$

$$-\cos L_9 + \cos L_{11} \cos L_{13} - \cos L_8 + 2\cos l_6 - \cos l_5]$$

$$+QRF \sum_{k=1}^{\infty} \frac{1}{4k(n-k)} \lambda_{10} \left[\cos L_2 - \cos L_9 + \cos L_6 + \cos L_8 + \cos L_{12} - \cos L_{16} - \cos L_{10} - \cos L_3\right]$$

$$-QRF\sum_{k=1}^{\infty}\frac{1}{8k(n-k)}\lambda_7\left[\sin N_3-\sin N_4+\sin N_2+\sin N_6-\sin N_9+\sin N_{11}-\sin N_{10}-\sin N_7\right]$$

$$-QRF\sum_{k=1}^{\infty}\frac{1}{k(n-k)}\lambda_8\,[\,2\sin N_1-\sin N_2+\sin N_3+\sin N_4+\sin N_5-2\sin N_6+2\sin N_7+\sin N_8$$

$$-\sin N_9-\sin N_{10}+\sin_{11}-\sin N_{12}-\sin N_{13}\,]$$

$$+QRF\sum_{k=1}^{\infty}\frac{1}{k(n-k)}\lambda_8\left[\sin q_1\sin q_2+\sin q_3+\sin q_4-\sin q_5-\sin q_6-\sin q_7-\sin q_8\right]$$

$$-QRF\sum_{k=1}^{\infty}\frac{1}{4k(n-k)}\lambda_{13}\,[\,\sin N_2+\sin N_3+\sin N_4-\sin N_5+2\sin N_7+\sin N_8+\sin N_9-\sin N_{10}$$

$$-\sin N_{12}+\sin N_{13}-\sin N_{14}-2\sin N_1\,]$$

$$-QRF\sum_{k=1}^{\infty}\frac{1}{2k(n-k)}\lambda_{12}\left[\sin q_1-\sin q_2+\sin q_3-\sin q_4+\sin q_7+\sin q_6-\sin q_5-\sin q_8\right]$$

$$+QRF\sum_{k=1}^{\infty}\frac{1}{2k(n-k)}\lambda_{14}\,[\,\cos N_2-2\cos N_1+\cos N_{10}-\cos N_4+2\cos N_7+\cos N_6-\cos N_5-\cos N_8$$

$$-\cos N_{12}-\cos N_{13}+\cos N_9+\cos N_{11}\,]$$

$$+QRF\sum_{k=1}^{\infty}\frac{1}{8k(n-k)}\lambda_9\,[\,\cos N_4+\cos N_2-2\cos N_1-\cos N_3+\cos N_8+\cos N_{14}-\cos N_5-2\cos N_7$$

$$-\cos N_{11}+\cos N_9+\cos N_{10}-\cos N_{13}+\cos N_{12}+\cos N_6\,]$$

$$-QRF\sum_{k=1}^{\infty}\frac{1}{4k(n-k)}\lambda_{10}\left[\cos q_1+\cos q_2+\cos q_3+\cos q_4-\cos q_5-\cos q_6-\cos q_7-\cos q_8\right]$$

$$-QRF\sum_{k=1}^{\infty}\frac{1}{8k(n-k)}\lambda_{11}\left[\cos N_{11}+\cos N_8+\cos N_{12}-\cos N_1-\cos N_{13}-\cos N_{12}\right]$$

$$-QRF\sum_{k=1}^{\infty}\frac{1}{4k(n-k)}\lambda_{12}\left[\sin L_2-\sin L_3+\sin L_4+\sin L_5+\sin L_8-\sin L_9+\sin L_{10}+\sin L_{12}+\sin L_{13}-\sin L_{14}\right]$$

$$-QRF\sum_{k=1}^{\infty}\frac{1}{2k(n-k)}\lambda_{18}\,[\,\sin L_3+\sin L_4+\sin L_2-2\sin L_1+\sin L_8+2\sin L_6-\sin L_5-\sin L_{10}-\sin L_{14}+\sin L_9$$

$$\sin L_{13}-\sin L_7-\sin L_{12}\square\sin L_{15}\,]$$

$$+QRF\sum_{k=1}^{\infty}\frac{1}{2k(n-k)}\lambda_{12}\left[\sin M_1-\sin M_2+\sin M_3-\sin M_4-\sin M_5+\sin M_6+\sin M_7-\sin M_8\right]$$

$$-QRF\sum_{k=1}^{\infty}\frac{1}{4k(n-k)}\lambda_{14}\,[\,\cos L_3-\cos L_2-2\cos L_1-\cos L_4+2\cos L_7+2\cos L_6-\cos L_5-\cos L_8$$

$$-\cos L_{10}+\cos L_9\cos L_{14}-\cos L_{12}-\cos L_{13}\,]$$

$$+QRF\sum_{k=1}^{\infty}\frac{1}{4k(n-k)}\lambda_{15}\,[\,\cos L_3+\cos L_2-2\cos L_1-\cos L_4+2\cos L_7+2\cos L_6-\cos L_5-\cos L_8-\cos L_{10}\,]$$

$$+QRF\sum_{k=1}^{\infty}\frac{1}{2k(n-k)}\lambda_{16}\,[\cos M_2-\cos M_1\square\cos M_4-\cos M_3\square\cos M_3-\cos M_8-\cos M_6\square\cos M_5\square\cos M_9]$$

$$+QRF\sum_{k=1}^{\infty}\frac{1}{4k(n+k)}\lambda_{12}$$

$$\left[\sin N_2-\sin N_3+\sin N_4-\sin N_5+\sin N_8+\sin N_{12}+\sin N_{13}-\sin N_9+\sin N_{10}-\sin N_{11}\right]\quad(11)$$

Substituting equation (8) and Equation (11) into Equation (5) gives

the analytical solution for the displacement of the structure in the form

$$V(x,t)=\sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}\times\frac{1}{w_n}\int_0^{\bar{t}} p_n(\tau)\exp[-\zeta\omega_n(t-\tau)\sin w_d(t-\tau)d\tau] \qquad (12)$$

5.0 Numerical Solution and Graphical Illustration

A computer package, MATLAB, is used to implement the scheme in equations (12) in orders to illustrate the analysis in this paper numerically and graphically. The effects of the individual case of the damping ratio (ζ>1,ζ=1, and ζ<1) on the dynamic response of the beam are discussed. Various values of the amplitude of the deflection V(x,t) of the beam for various values of dimensionless time $\bar{t}$ with a fixed value of dimensionless velocity $\bar{U}$ and various value of damping ratio are shown respectively in Table 1 ,Table 2, and Table3 below. The corresponding graphical representations are as well shown in Figure 2, Figure 3 and Figure 4 respectively.

Table 1:The Amplitude of displacement of the beam for damping ratio greater than one.

t	d = 1.5	d = 2.5	d = 4.0
0	2.46E+08	7.73E+07	2.11E+09
2	4.83E+09	3.84E+07	2.37E+09
4	1.06E+11	9.07E+10	6.77E+10
6	1.71E+10	1.41E+10	9.65E+09
8	5.81E+10	4.97E+10	3.72E+10
10	5.83E+09	4.82E+09	3.30E+09
12	-2.76E+07	-3.03E+07	-3.43E+07

Table2: The Amplitude of displacement of the beam for damping ratio equal to one.

t	0	2	4	6	8	10	12	14	16
d	2.79E+08	-8.82E+11	-9.71E+11	-1.97E+12	-1.46E+11	-6.77E+11	4.01E+10	5.22E+11	1.67E+11

Table3:The Amplitude of displacement of the beam for damping ratio less than one.

t	**d=-0.12**	**d = -0.08**	**d = -0.04**	**d = 0.00**	**d = 0.04**	**d = 0.08**	**d = 0.12**
0	**-6.84E+00**	**-1.08E+00**	**2.99E+01**	**-9.44E+04**	**2.92E+01**	**-1.45E+01**	**-7.09E+00**
4	**3.79E+07**	**1.46E+07**	**3.39E+08**	**-7.79E+11**	**3.39E+08**	**8.49E+07**	**3.79E+07**
8	**1.00E+08**	**2.26E+09**	**9.03E+08**	**-2.07E+12**	**9.03E+08**	**2.26E+08**	**1.00E+08**
12	**-6.53E+08**	**-1.47E+07**	**-5.87E+08**	**1.35E+12**	**-5.87E+07**	**-1.47E+07**	**-6.52E+06**
16	**-8.87E+09**	**-1.99E+08**	**-7.97E+08**	**1.83E+12**	**-7.97E+08**	**1.99E+08**	**-8.86E+07**
20	**-1.02E+07**	**-2.28E+07**	**-9.06E+07**	**1.35E+12**	**-9.06E+08**	**-2.28E+07**	**-1.02E+07**
24	**-3.18E+06**	**-7.17E+10**	**-2.88E+07**	**2.08E+11**	**-2.87E+07**	**-7.17E+06**	**-3.18E+06**
28	**-4.65E+06**	**-1.07E+07**	**-4.35E+07**	**6.61E+10**	**-4.35E+07**	**-1.07E+06**	**-4.65E+06**

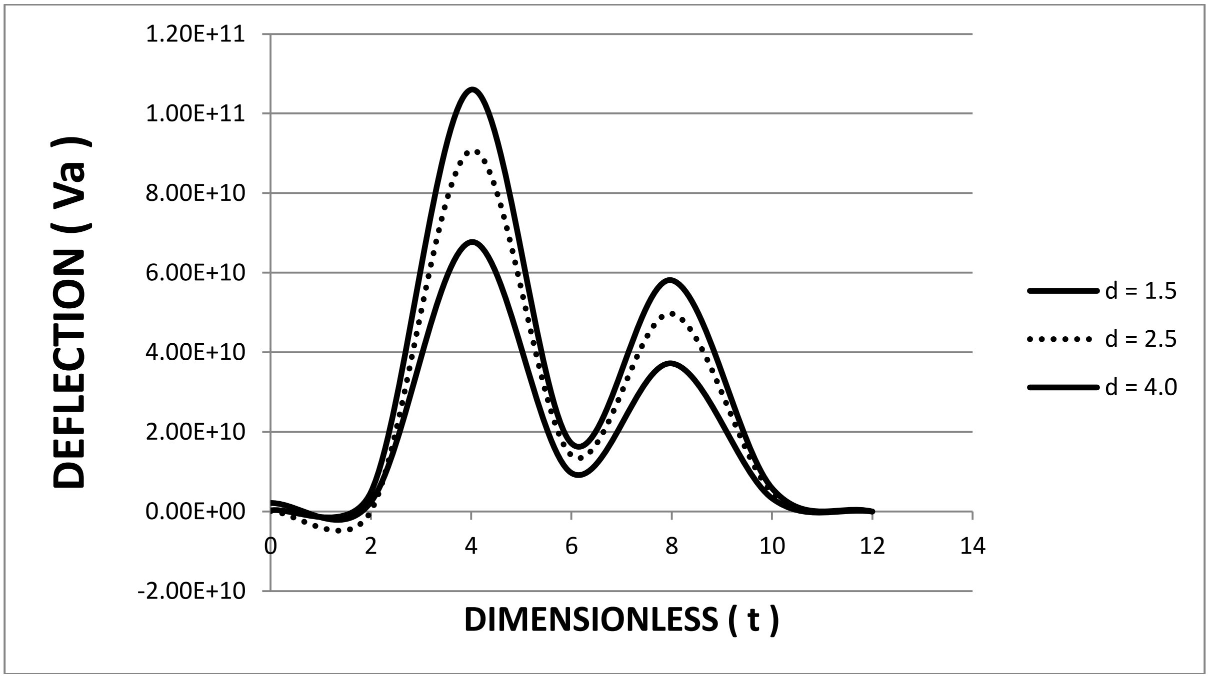

Figure 2: The deflection curves of the beam for d>1

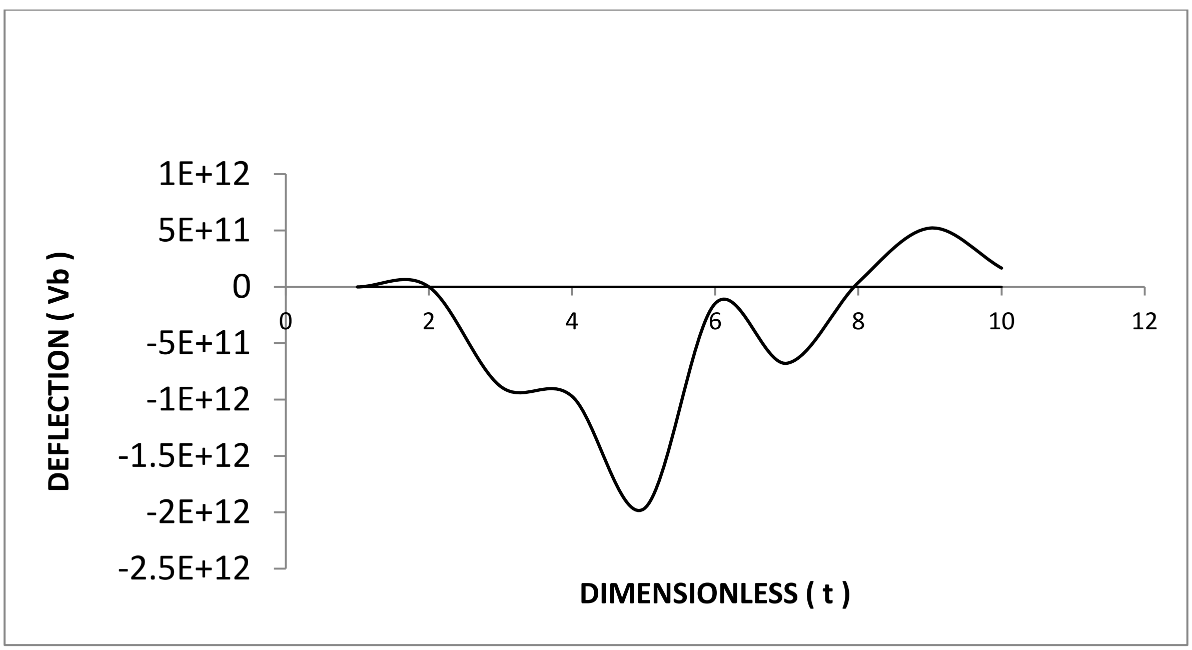

Figure 3: The deflection curves of the beam for d=1

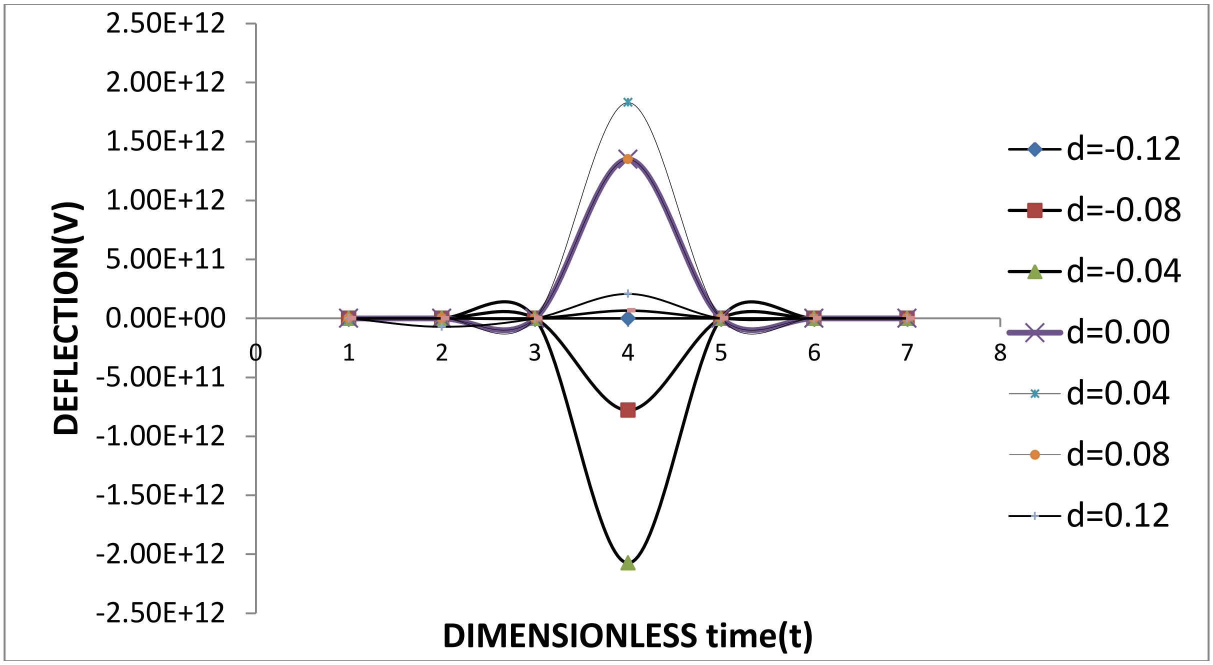

Figure 4: The deflection curves of the beam for d<1

6.0 Discussion of the Result

Figure.2, 3 and 4 depict the amplitude displacement of the beam for ζ>1, $\zeta = 1$ and ζ<1 respectively, where d represent the damping term ζ. It was observed that when ζ>1 the deflection curves exhibit maximum amplitude at dimensionless time $\bar{t}$=4.00 for $\zeta = 1.5$ as shown in Figure. 2. Also for ζ =1 the deflection curves exhibit maximum amplitude at $\bar{t}$= 6.0 as shown in Figure. 3. And when ζ<1 the deflection curves exhibit maximum amplitude at dimensionless time $\bar{t}$= 8.0 for $\zeta = 0.0$ as shown in Figure. 4.

Similarly, From the Table 1 it is noted that for ζ >1, as the damping ratio increases the absolute values of the deflection decreases except for dimensionless $\bar{t}$.= 12.0 and14.0. So also Table 3 shows that as the damping ratio increases for $\zeta > 0$ the absolute values of the deflection decreases.

7.0 Summary of the Result

The main findings from the investigations carried out on the effects of damping on the response of the beam subjected to moving loads with boundary conditions is summarized as follows

It was observed that the time at which the curve exhibits the maximum amplitude increases as damping ratio ζ decreases.

8.0 Conclusion

The importance and practical application of this analysis is seen in moving loads as they transverse along suspended bridges and railways.

Engineers, applied mathematician and applied physicist who are concerned in designing structures such as railway and highway bridges must take into consideration the determination of the deflection curves and natural frequencies of the beam. Important attention may be paid to affect these effects to avoid road-rail disaster.

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