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Solving Abelian Differential Equation by Iterative Decomposition Method

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Abstract

In this paper, Abelian Differential Equation is considered. The numerical method based on the iterative decomposition technique is introduced to obtain its approximate solution. Comparison is made between the obtained results and those in the literature. The results reveal the simplicity and effectiveness of the recursive algorithm of the method.

Keywords: Iterative Decomposition Method, Abelian Differential Equation, Recursive algorithm.

1.0 Introduction

In scientific and engineering problems, many mathematical modeling which explain natural phenomena are widely formulated in terms of nonlinear differential equations, both ordinary and partial. However, most of the numerical methods developed in mathematics are full of restrictive assumptions, linearization, perturbation, discretization and consequent huge computation and complexity that are difficult for computer implementation. See [1 - 5]. The duo of Daftadar-Geji and Jafari [6] introduced the iterative decomposition method in solving functional nonlinear equation devoid of the shortcomings highlighted above. Taiwo et al [7], Taiwo and Odetunde [8], Taiwo and Osilagun [9, 10] have shown that this technique is robust and efficient in solving a class of initial/boundary value problems. For further details, see [11 - 16].

The purpose of this paper is to present an iterative decomposition method as a reliable tool in solving the Abelian Differential Equation of the form

$$\frac{du}{dx} = f_0(x) + f_1(x)u(x) + f_2(x)\big(u(x)\big)^2 + f_3(x)\big(u(x)\big)^3 \qquad (1)$$

subject to the initial condition

$$u(0) = \alpha \qquad (2)$$

where $f_i(x),\ i = 1,2,3$ are given continuous linear or nonlinear real value functions and α is a real constant.
It follows that if $f_3(x) = 0$, equation (1) reduces to Riccati equation. The importance of this equation arises in the optimal control problems.

2.0 Analysis of the Method

In order to elucidate the solution procedure for the Abelian differential equation, we give a review of the method of solution. Daftadar – Geji and Jafari in their paper [6] consider a general functional equation

$$u = N(u) + g(x) \qquad (3)$$

where N is a nonlinear operator from a Banach space $B \to B$ and $g(x)$ is a known function. We seek a solution u of equation (3) having the series form

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$$u = \sum_{k=0}^{\infty} u_k \tag{4}$$

The nonlinear operator N can be decomposed as

$$N = \left(\sum_{k=0}^{\infty} u_k\right) = N(u_0) + N(u_1) + \sum_{k=1}^{\infty} \left\{ N\left(\sum_{j=0}^{k} u_j\right) - N\left(\sum_{j=0}^{k-1} u_j\right) \right\} \tag{5}$$

So, equation (3) becomes

$$\sum_{k=1}^{\infty} u_k = g(x) + N(u_0) + \sum_{k=3}^{\infty} \left\{ N\left(\sum_{j=0}^{k} u_j\right) - N\left(\sum_{j=0}^{k-1} u_j\right) \right\} \tag{6}$$

Thus, a recursive formula is obtained as follows

$$\left.\begin{aligned} u_0 &= g(x) \\ u_1 &= L^{-1}[N(u_0)] \\ u_{n+1} &= L^{-1}\left[N\left(\sum_{k=0}^{n} u_k\right)\right] - L^{-1}\left[N\left(\sum_{k=0}^{n-1} u_k\right)\right], \qquad n \geq 1 \end{aligned}\right\} \tag{7}$$

So $u(x) = u_0 + u_1 + u_2 + \cdots$ and the series $\sum_{k=0}^{\infty} u_k$ absolutely and uniformly converges to a solution of equation (3), which is unique in view of the Banach fixed point theorem.

3.0 Application and Numerical examples

In this section, we presented some numerical and analytical solution for Abelian differential equation.
To apply the iterative decomposition method, equation (1) is expressed in operator form as

$$Lu = f_0(x) + f_1(x)u + f_2(x)N_1(u) + f_3(x)N_2(u) \tag{8}$$

where $L = \frac{d}{dx},\ N_1(u) = u^2(x),\ N_2(u) = u^3(x)$

Since L is in invertible, L^{-1} exists and is defined as the single fold definite integral

$$L^{-1}(\cdot) = \int_0^X (\cdot)ds \tag{9}$$

On applying equation (9) to equation (8) and simplifications, yields the recursive formula

$$\left.\begin{aligned} u_0 &= \alpha + L^{-1}[f_0(x)] \\ u_1 &= L^{-1}[f_1(x).u_0(x)] + L^{-1}[f_2(x)N_1(u_0) + L^{-1}[f_3(x)N_2(u_0) \\ u_{n+1} &= L^{-1}[f_2(x)N_1(u_n)] + L^{-1}[f_3(x)N_2(u_n)] + L^{-1}[f_n(x)u_n], \qquad n \geq 1 \end{aligned}\right\} \tag{10}$$

Example 1: Consider the Abelian differential equation

$$\frac{du}{dx} = 1 - u + u^3 \tag{11}$$

subject to the initial condition

$$u(0) = 0 \tag{12}$$

Using the proposed method in solving the above example, on expressing the problem in operator from and applying the recursive formula (10), yields

$u_0 = x$

$u_1 = -\frac{x^2}{2} + \frac{x^4}{4} = -\frac{1}{2}x^2 + \frac{1}{4}x^4$

$u_2 = \frac{1}{6}x^3 - \frac{7}{20}x^5 + \frac{1}{8}x^6 + \frac{5}{56}x^7 - \frac{3}{32}x^8 + \frac{1}{48}x^9 + \frac{3}{160}x^{10} - \frac{3}{252}x^{11} + \frac{1}{832}x^{12}$

$$\begin{aligned} u_3 = &-\frac{1}{24}x^4 + \frac{17}{120}x^6 - \frac{5}{56}x^7 - \frac{391}{3360}x^8 + \frac{829}{4320}x^9 - \frac{1627}{18900}x^{10} - \frac{2279}{36960}x^{11} + \frac{8353}{79200}x^{12} - \frac{46913}{873600}x^{13} \\ &- \frac{438043}{33633600}x^{14} + \frac{421159}{11088000}x^{15} - \frac{185227709}{8072064000}x^{16} - \frac{2998421}{4574116900}x^{17} + \frac{1023351239}{9686476800}x^{18} \\ &- \frac{28204091}{4089845760}x^{19} + \frac{331473}{1025024000}x^{20} + \frac{539532613}{226017792000}x^{21} - \frac{1522378469}{994478284800}x^{22} \\ &+ \frac{482059}{7616716800}x^{23} + \frac{512394479}{1136546611200}x^{24} - \frac{1160214941}{4735610880000}x^{25} - \frac{108654761}{12312588288000}x^{26} \\ &+ \frac{1810343}{27060633600}x^{27} - \frac{632017453}{24822199988608}x^{28} - \frac{2335181}{485802803200}x^{29} + \frac{15983811}{2345254912000}x^{30} \\ &- \frac{12436857}{90878622784000}x^{31} - \frac{1984531}{2814305894400}x^{32} + \frac{14749}{36854005760}x^{33} - \frac{631833}{87712533708800}x^{34} \\ &- \frac{1899}{42640998400}x^{35} + \frac{59}{6030655468}x^{36} + \frac{9}{4097966080}x^{37} - \frac{9}{9259188224}x^{38} + \frac{x^{40}}{23037214720} \end{aligned}$$

Other components are determined similarly. The fourth term approximate solution is given by

$$\begin{aligned} u(x) = x &- \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{5}{24}x^4 - \frac{7}{20}x^5 + \frac{4}{15}x^6 - \frac{353}{1680}x^8 + \frac{919}{4320}x^9 - \frac{10181}{151200}x^{10} - \frac{1297}{18480}x^{11} + \frac{8353}{79200}x^{12} \\ &- \frac{45863}{873600}x^{13} - \frac{438043}{33633600}x^{14} + \frac{421159}{11088000}x^{15} - \cdots \end{aligned}$$

Example 2

Consider the following Abelian differential equation

$$\frac{du}{dx} = 4 + 4xu + 2xu^2 + x^2u^3 \qquad (13)$$

subject to the initial condition

$$u(0) = 0 \qquad (14)$$

In similar approach, expressing equation (13) in operator form and the use of recursive formula (10), gives

$u_0 = 4x$

$u_1 = \frac{16}{3}x^3 + 8x^4 + \frac{32}{3}x^6$

$$\begin{aligned} u_2 = &\frac{64}{15}x^5 + \frac{176}{9}x^6 + \frac{128}{7}x^7 + \frac{400}{9}x^8 + \frac{2176}{27}x^9 + \frac{704}{15}x^{10} + \frac{15872}{99}x^{11} + \frac{8512}{81}x^{12} + \frac{2048}{13}x^{13} + \frac{14848}{63}x^{14} \\ &+ \frac{2560}{27}x^{15} + 256x^{16} + \frac{2048}{17}x^{17} + \frac{8192}{81}x^{18} + \frac{8192}{57}x^{19} + \frac{32768}{567}x^{21} \end{aligned}$$

$$\begin{aligned} u_3 = &\frac{256}{105}x^7 + \frac{824}{45}x^8 + \frac{24320}{567}x^9 + \frac{13408}{175}x^{10} + \frac{37888}{165}x^{11} + \frac{4669888}{14175}x^{12} + \frac{18496768}{27027}x^{13} + \frac{1448898688}{1091475}x^{14} \\ &+ \frac{964462592}{552825}x^{15} + \frac{54532505408}{14189175}x^{16} + \frac{499238912}{98175}x^{17} + \frac{138653527808}{16372125}x^{18} + \frac{856485100544}{59520825}x^{19} \\ &+ \frac{197337928452352}{10854718875}x^{20} + \frac{1680816413433856}{50953327425}x^{21} + \cdots \end{aligned}$$

So,

$$\begin{aligned} u(x) &= u_0 + u_1 + u_2 + u_3 + \cdots \\ &= 4x + \frac{16}{3x^3} + 8x^4 + \frac{64}{15}x^5 + \frac{272}{9}x^6 + \frac{2176}{105}x^7 + \frac{2824}{45}x^8 + \frac{70016}{567}x^9 + \frac{64864}{525}x^{10} + \frac{193024}{495}x^{11} \\ &\quad + \frac{22754560}{27027}x^{13} + \frac{1706140288}{1091475}x^{14} + \frac{1016878592}{552825}x^{15} + \frac{58164934208}{14189175}x^{16} + \cdots \end{aligned}$$

4.0 Conclusion

It has been shown in this paper; based on the approximate solution of Abelian differential equation that the iterative decomposition method is reliable, powerful and efficient. The method has been successfully applied and results obtained is better that Adomian Decomposition Method [16] and is in excellent agreement with variational iteration method [11].

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