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Transformation of a PDE Associated with the Pricing of Electricity Forward Contract into a Heat Equation

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Abstract

A coordinate transformation of an evolution partial differential equation into a heat equation via Lie symmetry method is presented in this paper.

Keywords: Transformation; PDE; Heat Equation; Lie Point Symmetry; Lie Algebra; Invertible Mapping

1.0 Introduction

Over the last few decades, there has been a great interest in the modeling and analysis of problems arising in commodity markets. Some of these problems are modeled in terms of evolution equations such as partial differential equations. Symmetry analysis is, however, one of the many methods of solution of a partial differential equation. A number of studies have been devoted to the use of symmetry techniques for partial differential equations arising in the field of financial mathematics [1–4].

The aim of this paper is to show that evolution equations with similar Lie algebra as the heat equation can be transformed to the heat equation. This may be achieved via appropriate isomorphism upon their algebras or by a coordinate transformation. Our approach exploits the later. It is well known that the main aim of transforming any equation into a diffusion equation is to find a form where a relatively easy solution exists.

A typical example of an evolution equation relevant to this study is given in Section 3 in order to appreciate the algorithm involved in finding solutions through the symmetry method. We start by defining the evolution partial differential equation.

Definition 1 (*Evolution equation*) An evolution partial differential equation is an equation involving an unknown function of several variables that includes time, *t*, as one of the independent variables.

In particular, a second-order evolution partial differential equation in one dependent and two independent variables is an equation of the form

$$F(x,t,u,u_x,u_t,u_{xx})=0, \qquad (x,t)\in D \tag{1}$$

where, as is indicated, the independent variables x and t lie in some given domain $D \in \mathbb{R}^2$. By this definition, u, is the dependent variable while the $x-t$ domain D in $\mathbb{R}^2$, where the problem is defined, is called the space-time domain. By a solution of (1) we mean a twice continuously differentiable function $u=u(x,t)$ defined on D which, when substituted into (1), reduces (1) to an identity on the domain D. The function $u=u(x,t)$ is assumed to belong to a set of all twice continuously differentiable functions on $\mathbb{R}$ that vanish at infinity so that calculating its first and second derivatives and substituting them into (1) is defined.

2.0 Preliminaries of Lie Group of Transformations

Consider $u=u(x,t)$, that is to say

$$f=f(x,t,u). \tag{2}$$

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If $f: X \rightarrow u$ is a smooth function from $X \simeq \mathbb{R}^p$ to $u \simeq \mathbb{R}^q$ so that $u = f(x) = (f'(x), ..., f^q(x))$ is a partial differential equation with q dependent variables (here u is the only dependent variable) and p independent variables. We let p = 2 such that x and t here represent the two independent variables. Then the general vector field on $X \times U \simeq \mathbb{R}^2 \times \mathbb{R}$. Here a point transformation is a diffeomorphism

$$\Gamma: (x,t,u) \mapsto (\hat{x}(x,t,u), \hat{t}(x,t,u), \hat{u}(x,t,u)). \tag{3}$$

This transformation maps the surface $u = u(x,t)$ to the surface parameterized by x and t as follows

$$\left.\begin{aligned} \hat{x} &= \hat{x}(x,t,u(x,t)) \\ \hat{t} &= \hat{t}(x,t,u(x,t)) \\ \hat{u} &= \hat{u}(x,t,u(x,t)) \end{aligned}\right\}. \tag{4}$$

The infinitesimal transformations of these variables in Taylor series for the Lie group action [5, 6] are

$$\left.\begin{aligned} \hat{x} &= x + \varepsilon\xi(x,t,u) + o(\varepsilon^2) = x + \varepsilon\Gamma x + \cdots \\ \hat{t} &= t + \varepsilon\tau(x,t,u) + o(\varepsilon^2) = t + \varepsilon\Gamma t + \cdots \\ \hat{u} &= u + \varepsilon\eta(x,t,u) + o(\varepsilon^2) = u + \varepsilon\Gamma u + \cdots \end{aligned}\right\} \tag{5}$$

with the vector fields which span the associated Lie algebra, called the *generators* of the i*nfinitesimal* transformation (5), Γ, so that

$$\Gamma = \xi(x,t,u)\frac{\partial}{\partial x} + \tau(x,t,u)\frac{\partial}{\partial t} + \eta(x,t,u)\frac{\partial}{\partial u} \tag{6}$$

and

$$\left.\begin{aligned} \hat{x} &= (1+\varepsilon\Gamma)x = x + \varepsilon\xi \\ \hat{t} &= (1+\varepsilon\Gamma)t = t + \varepsilon\tau \\ \hat{u} &= (1+\varepsilon\Gamma)u = u + \varepsilon\eta \end{aligned}\right\}. \tag{7}$$

In particular the components of Γ at (x,t,u) are

$$\xi(x,t,u) = \left.\frac{\partial\hat{x}}{\partial\varepsilon}\right|_{\varepsilon=0}, \quad \tau(x,t,u) = \left.\frac{\partial\hat{t}}{\partial\varepsilon}\right|_{\varepsilon=0}, \quad \eta(x,t,u) = \left.\frac{\partial\hat{u}}{\partial\varepsilon}\right|_{\varepsilon=0}. \tag{8}$$

Notice that ξ, τ and η occur in the definition of Γ called *coefficient functions*.

3.0 The Lie Point Symmetries of given PDE

The general form of the model of interest given in [7] is

$$\frac{1}{2}\sigma^2\nu_t\frac{\partial^2 y(t,\nu_t)}{\partial\nu_t^2} + (\kappa\theta - (\kappa+\lambda_\nu)\nu_t)\frac{\partial y(t,\nu_t)}{\partial\nu_t} + \frac{\partial y(t,\nu_t)}{\partial t} = k_1\nu_t y(t,\nu_t) \tag{9}$$

with boundary condition for the value at maturity

$$y(T,\nu_t) = exp(k_1,\nu_T). \tag{10}$$

For simplicity and without loss in generality, we let $\lambda_\nu = \lambda,\ \nu_t = x, k_1 = k$ so that $y(\nu_t,t) = u(x,t)$, hence (9) becomes

$$\frac{\partial u}{\partial t} + \frac{1}{2}\sigma^2 x\frac{\partial^2 u}{\partial x^2} + (\kappa\theta - (\kappa+\lambda)x)\frac{\partial u}{\partial x} - kxu = 0. \tag{11}$$

The computation of Lie point symmetries is algorithmic [8] but for this work we present results obtained through the package SYM [9, 10] which was used interactively with MATHEMATICA 6.0 for the analysis which results we present below.

As we saw in Section 2, determining the Lie point symmetries means finding the functions $\xi(x,t,u)$, $\tau(x,t,u)$ and $\eta(x,t,u)$ such that the symmetry conditions are met. The symmetry conditions lead to a system of linear partial differential equations for the independent and dependent variables which eventually split into many more equations since they (independent and dependent variables) are independent of the derivatives of the dependent variable. However, the coefficients of these variables do depend on these derivatives.

The basis operators of the Lie algebra under this sub-case when $\alpha = \kappa + \lambda$ and $\sigma^2 = 4\theta\kappa/3$ are given as

$$\Gamma_1 = \frac{\partial}{\partial t} \tag{12a}$$

$$\Gamma_2 = u\frac{\partial}{\partial u} \tag{12b}$$

$$\Gamma_{3,4} = e^{\pm\frac{1}{2}\phi t}\left[\sqrt{x}\frac{\partial}{\partial x} - \frac{1}{4\theta\kappa\sqrt{x}}\left(2\theta\kappa \pm 3x(\alpha\pm\phi)\right)u\frac{\partial}{\partial u}\right] \tag{12c,d}$$

$$\Gamma_{5,6} = e^{\pm\phi t}\left[x\frac{\partial}{\partial x} \pm \frac{1}{\phi}\frac{\partial}{\partial t} + \frac{3}{4\theta\kappa\phi}(\phi x - \theta\kappa)(\alpha\pm\phi)u\frac{\partial}{\partial u}\right] \tag{12e,f}$$

and

$$\Gamma_\infty = f(x,t)\frac{\partial}{\partial u}. \tag{12g}$$

where $\phi = \sqrt{\kappa^2 + \kappa^2 + 2\kappa\lambda + 2k\sigma^2}$ and $f(x,t)$ is any solution of (11). The basis symmetries in (12) may be represented for convenience as

$$\Gamma_1 = \frac{\partial}{\partial t} \tag{13a}$$
$$\Gamma_2 = u\frac{\partial}{\partial u} \tag{13b}$$

$$\Gamma_3 = e^{\frac{1}{2}\phi t}\left[\sqrt{x}\frac{\partial}{\partial x} - \left(\frac{1}{2\sqrt{x}} + \mathcal{A}\sqrt{x}\right)u\frac{\partial}{\partial u}\right] \tag{13c}$$
$$\Gamma_4 = e^{-\frac{1}{2}\phi t}\left[\sqrt{x}\frac{\partial}{\partial x} - \left(\frac{1}{2\sqrt{x}} - \mathcal{B}\sqrt{x}\right)u\frac{\partial}{\partial u}\right] \tag{13d}$$
$$\Gamma_5 = e^{\phi t}\left[x\frac{\partial}{\partial x} + \frac{1}{\phi}\frac{\partial}{\partial t} + (\mathcal{A}x - \mathcal{C})u\frac{\partial}{\partial u}\right] \tag{13e}$$
$$\Gamma_6 = e^{-\phi t}\left[x\frac{\partial}{\partial x} - \frac{1}{\phi}\frac{\partial}{\partial t} + (\mathcal{B}x - \mathcal{D})u\frac{\partial}{\partial u}\right] \tag{13f}$$

and

$$\Gamma_\infty = f(x,t)\frac{\partial}{\partial u}. \tag{13g}$$

$$\mathcal{A} = \frac{3}{4\theta\kappa}(\alpha+\phi), \mathcal{B} = \frac{3}{4\theta\kappa}(\alpha-\phi), \mathcal{C} = \frac{3}{4\phi}(\alpha+\phi), \text{and } \mathcal{D} = \frac{3}{4\phi}(\alpha-\phi).$$

The nongeneric Lie point symmetries $\Gamma_1, \Gamma_3 - \Gamma_6$ comprise of groups: the first group of symmetries Γ_1, Γ_5 and Γ_6 constitute the Lie algebra $sl(2,R)$ and the second group Γ_3 and Γ_4 correspond to the solution symmetries of the one–dimensional free particle. The former is characteristic of an equation arising from finance. It is important to note that the Lie algebra of infinitesimal symmetries for (13) spanned by the vectors Γ_1(translation in t), Γ_2 (dilatation in *u*), Γ_3 and Γ_4 (Galilean boost), Γ_5 and Γ_6 (local symmetries), and Γ_∞ is an additional infinite–dimensional subalgebra in which *f*(*x*, *t*) is the solution of (11) and reflects its linearity. The associated Lie algebra of the above six–parameter Lie group of infinitesimal operators is $\{sl(2,R) \oplus W_3\} \oplus \infty A_1$, where W_3 is the three-dimensional Heisenberg-Weyl algebra implied by the commutation relations given in Table 1, leaving out the solution symmetry. Note that the symbol $\oplus$ is used when all elements of the first subalgebra have zero lie brackets with all elements of the second. This Lie algebra, when certain special subcases are considered is equivalent to that of the heat equation given in [11] as $sl(2,R) \oplus W_3$.

Table 1: Lie Bracket for Lie point symmetries (13)–(13f)

$[.,.]_{LB}$	Γ_1	Γ_2	Γ_3	Γ_4	Γ_5	Γ_6
Γ_1	0	0	$\frac{\emptyset}{2}\Gamma_3$	$-\frac{\emptyset}{2}\Gamma_4$	$\phi\Gamma_5$	$-\phi\Gamma_6$
Γ_2	0	0	0	0	0	0
Γ_3	$-\frac{\emptyset}{2}\Gamma_3$	0	0	$-\frac{3\emptyset}{\theta\kappa}\Gamma_2$	0	$-\Gamma_4$
Γ_4	$\frac{\emptyset}{2}\Gamma_4$	0	$\frac{3\emptyset}{\theta\kappa}\Gamma_2$	0	$-\Gamma_3$	0
Γ_5	$-\phi\Gamma_5$	0	0	Γ_3	0	$\frac{32}{\phi}\Gamma_1 - \frac{24\alpha}{\phi}\Gamma_2$
Γ_6	$\phi\Gamma_6$	0	Γ_4	0	$-\frac{32}{\phi}\Gamma_1 + \frac{24\alpha}{\phi}\Gamma_2$	0

4.0 Construction of the transformation of (11) into the heat equation

It has been proved in [12, 13, 14] that the heat equation is the only polynomial partial differential equation of the second order in two independent variables invariant under the finite group of the heat equate itself. Bluman and Kumei [15] provide the framework for the existence and construction of a transformation between two (linear or nonlinear) partial differential equations.

Theorem 1 [15] *In the case of one dependent variable, u, a mapping* m *defines an invertible mapping from* $(x, u, u_{(1)}, \ldots, u_{(p)})$*-space to* $(z, w, w_{(1)}, \ldots, w_{(p)})$*-space for any fixed p if and only if* m *is a* ***one-to-one contact transformation*** *of the form*

$$z = \phi(x, u, u_{(1)}), \tag{14}$$

$$w = \psi(x, u, u_{(1)}), \tag{15}$$

$$w_{(1)} = \psi_{(1)}(x, u, u_{(1)}). \tag{16}$$

Note that, if ϕ *and* ψ *are independent of* $u_{(1)}$*, then* (14)–(16) *defines a point transformation.*
We apply this theorem in the construction of invertible point mapping by finding the point transformation which relates our given equation to the heat equation. Since the generators Γ_4 and Γ_6 of (13) commute, we use them for the construction of a transformation that will map (11) invertibly into the heat equation.

Let $X_1 \equiv \Gamma_4$ and $X_2 \equiv \Gamma_6$. From the vectors of the infinitesimal symmetries of (11),

$$X_1 = \xi_{11}(x,t)\frac{\partial}{\partial x} + \xi_{12}(x,t)\frac{\partial}{\partial t} + f_1(x,t)u\frac{\partial}{\partial u} \tag{17}$$

$$X_2 = \xi_{21}(x,t)\frac{\partial}{\partial x} + \xi_{22}(x,t)\frac{\partial}{\partial t} + f_2(x,t)u\frac{\partial}{\partial u} \tag{18}$$

where $\xi_{11} = \sqrt{x}e^{\frac{1}{2}\phi t}$ $\quad \xi_{12} = 0$ $\quad f_1 = \left(\frac{1}{2\sqrt{x}} - \mathcal{B}\sqrt{x}\right)ue^{\frac{1}{2}\phi t}$

$\xi_{21} = xe^{-\phi t}$ $\quad \xi_{22} = \frac{1}{\phi}e^{-\phi t}$ $\quad f_2 = -(\mathcal{B}x - \mathcal{D})e^{-\phi t}$

To verify that the Jacobian of the transformation is nonzero, we find the determinant of

$$J = \begin{vmatrix} \xi_{11} & \xi_{12} \\ \xi_{21} & \xi_{22} \end{vmatrix} = \frac{\sqrt{x}}{\phi}e^{-\frac{3}{2}\phi t} \neq 0, \quad \phi \neq 0$$

Since $J \neq 0$ then the existence of an invertible mapping of the form

$$z = \alpha(x,t) \qquad \tau = \varphi(x,t) \qquad \omega = \nu(x,t)u \tag{19}$$

is a guarantee for mapping (11) into a constant partial differential equation. The mapping (19) must satisfy the following necessary conditions

$$\xi_{11}\alpha_x + \xi_{12}\alpha_t = 1; \quad \xi_{21}\alpha_x + \xi_{22}\alpha_t = 0 \tag{20}$$

$$\xi_{11}\varphi_x + \xi_{12}\varphi_t = 0; \quad \xi_{21}\varphi_x + \xi_{22}\varphi_t = 1 \tag{21}$$

$$\xi_{11}\nu_x + \xi_{12}\nu_t + f_1\nu = 0; \quad \xi_{21}\nu_x + \xi_{22}\nu_t + f_2\nu = 0. \tag{22}$$

Substituting the values of the $\xi_{ij's}$ in (20)–(22) and solving the resulting simultaneous systems yields

$$\left.\begin{aligned} \sqrt{x}e^{-\frac{1}{2}\phi t}\alpha_x + 0 \quad &= 1 \\ xe^{-\phi t}\alpha_x - \frac{1}{\phi}e^{-\phi t}\alpha_t &= 0 \end{aligned}\right\} \begin{aligned} \alpha_x &= \frac{1}{\sqrt{x}}e^{\frac{1}{2}\phi t} \\ \alpha_t &= -\phi\sqrt{x}e^{\frac{1}{2}\phi t} \end{aligned} \tag{23}$$

$$\left.\begin{aligned} \sqrt{x}e^{-\frac{1}{2}\phi t}\varphi_x + 0 \quad &= 0 \\ xe^{-\phi t}\varphi_x - \frac{1}{\phi}e^{-\phi t}\varphi_t &= 1 \end{aligned}\right\} \begin{aligned} \varphi_x &= 0 \\ \varphi_t &= -\phi e^{\phi t} \end{aligned} \tag{24}$$

$$\left.\begin{aligned} \sqrt{x}e^{-\frac{1}{2}\phi t}\nu_x + 0 &= \left(\frac{1}{2\sqrt{x}} - \mathcal{B}\sqrt{x}\right)e^{-\frac{1}{2}\phi t}u \\ xe^{-\phi t}\nu_x - \frac{1}{\phi}e^{-\phi t}\nu_t &= (\mathcal{B}x - \mathcal{D})e^{-\phi t} \end{aligned}\right\} \begin{aligned} \nu_x &= -\frac{1}{\phi}(2\mathcal{B}x - 1)u \\ \nu_t &= \frac{\phi}{2}(\mathcal{D} - \mathcal{B}x + 1)u \end{aligned} \tag{25}$$

From (23),

$$\alpha(x,t) = \int \alpha_x dx = 2\sqrt{x}e^{\frac{1}{2}\phi t} + a(t)$$

$$\therefore \ a(t) = A_1 \text{ and hence } a(x,t) = \mathcal{K} + \frac{2\sqrt{x}}{\phi}(2\phi - 1)e^{\frac{1}{2}\phi t}.$$

From (24),

$$\varphi(x,t) = \int \varphi_x dx = b(t)$$

$\frac{\partial}{\partial t}\varphi(x,t) = \varphi_t \Rightarrow \dot{b}(t) = -\phi e^{\phi t}$, $b(t) = \mathcal{L} - e^{\phi t}$

Therefore $\varphi(x,t) = \mathcal{L} - e^{\phi t}$.

From (25),

$$v(x,t) = \int v_x dx = -u\mathcal{B} + \frac{1}{2}u\ln x + c(t)$$

$\frac{\partial}{\partial t}v(x,t) = v_t \Rightarrow 0 + \dot{c}(t) = \frac{1}{2}(\mathcal{D} - \mathcal{B}x + 1)u$ and

$c(t) = \frac{\phi}{2}(\mathcal{D} - \mathcal{B}x + 1)ut + \mathcal{M}$

therefore, $v(x,t) = \mathcal{M}\exp\left\{\frac{\phi}{2}(\mathcal{D} - \mathcal{B}x + 1)t - \mathcal{B}x + \frac{1}{2}\ln x\right\}$

where $\mathcal{K}, \mathcal{L}$ *and* $\mathcal{M}$ are arbitrary constants. Without loss in generality, we may let $\mathcal{K} = \mathcal{L} = 0$ and $\mathcal{M} = 1$ to have (26) – (28).

$$z = -\frac{2}{\phi}\sqrt{x}(2\phi - 1)e^{-\frac{1}{2}\phi t}, \tag{26}$$

$$\tau = -e^{\phi t}, \text{ and} \tag{27}$$

$$w(z,\tau) = u(x,t)\exp\left\{\frac{\phi}{2}(\mathcal{D} - \mathcal{B}x + 1)t - \mathcal{B}x + \frac{1}{2}\ln x\right\}. \tag{28}$$

We now map (11) invertibly to the heat equation by using transformations (26)–(28).
Using calculus and especially the principle of chain rule,

$$\frac{\partial u}{\partial t} = \left\{\frac{x^{-\frac{1}{2}}}{\phi}(2\phi - 1)e^{\frac{1}{2}\phi t}\frac{\partial w}{\partial z} - \phi e^{\phi t}\frac{\partial w}{\partial \tau} - \frac{\phi}{2}\sqrt{x}(\mathcal{B}x - \mathcal{D} - 1)w\right\}e^{\psi} \tag{29}$$

$$\frac{\partial u}{\partial x} = \left\{\frac{x^{-\frac{1}{2}}}{\phi}(2\phi - 1)e^{\frac{1}{2}\phi t}\frac{\partial w}{\partial z} - \frac{x^{-\frac{1}{2}}}{2}(\phi\mathcal{B}tx + 2\mathcal{B}x - 1)w\right\}e^{\psi} \tag{30}$$

and

$$\frac{\partial^2 u}{\partial x^2} = \left\{\begin{matrix} \frac{x^{-1}}{\phi^2}(2\phi - 1)e^{\phi t}\frac{\partial^2 w}{\partial z^2} - \frac{x^{-\frac{3}{2}}}{2\phi}(2\phi - 1)e^{\frac{1}{2}\phi t}\frac{\partial w}{\partial z} \\ +\frac{1}{4}\left((\phi\mathcal{B}tx + 2\mathcal{B}x - 1)x^{-1} - x^{-3/2}\right)(\phi\mathcal{B}tx + 2\mathcal{B}x - 1)w \end{matrix}\right\}e^{\psi} \tag{31}$$

where $\psi = \frac{\phi}{2}(\mathcal{D} - \mathcal{B}x + 1)t - \mathcal{B}x$.
Substituting (29) – (31) into (11) we obtain the linear heat equation

$$\frac{\partial w}{\partial \tau} - \mathcal{H}\frac{\partial^2 w}{\partial z^2} = 0 \tag{32}$$

where

$$\mathcal{H} = \frac{2\theta\kappa}{3\phi^3}(2\phi - 1)^2$$

5.0 Conclusion

We have been able to connect the partial differential equation associated with the pricing of electricity futures contract with the well-known and widely studied heat equation and therefore one can use the results for the later to obtain solutions of the former. Practitioners in the financial institutions can also have direct method to obtain solutions important to their needs.

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