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Supergravity Coupling, Field Breaking and Restoration of Gauge Invariance

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Abstract

In this paper $d = 10, N = 1$ ***Yang-Mills system is coupled to*** $d = 10, N = 1$ ***supergravity. We give a critical analysis of the current, low dimension auxiliary fields, and reveal the existence of two ordinary axial. Maxwell-Einstein current agrees with the Noether coupling. The coupling of the photon*** A_μ ***to anti-symmetric tensor is consistent following Maxwell transformation*** $\partial A_\mu = \partial_\mu \wedge$ ***extended to*** $\partial A_{\mu\nu} = K \wedge F_{\mu\nu}$.

***Keywords*:** supergravity, gauge invariance, gauge algebra, supersymmetry, auxiliary fields

AMS subject classifications: 83E50, 70S15, 81T13, 53C07, 58E15, 81T60

1.0 Introduction

A central problem in supergravity is to find sets of auxiliary fields, which when added to the physical fields, lead to closed gauge algebra. It has been deeply studied [1 – 8]. The issue of auxiliary fields has two separate aspects, (i) finding a set of fields with closed gauge algebra, (ii) constructing actions for such a field representation which are invariant under the gauge transformations of the algebra. This paper deals with the first aspect, but it should be noted that a solution of the representation problem does not imply existence of meaningful actions. Our motivation is that the higher the value of d the simpler the model becomes. Since beyond $d = 10$ no matter exists [9], only $N = 1$ gauge action exists in $d = 11$ [10], whereas in $d = 10$, only $N = 1$ supersymmetry yang-mills matter exists [11]. We increase the dimension because we obtained multiplet which is larger than the set of fields of $d = 4, N = 4$ conformal supergravity. In this way $d = 4$ revealed the existence of an axial auxiliary field [12, 13]. Similarly, the multiplet of $N \leq 4$ conformal supergravity is from the multiplet of currents [5, 14]. In general, one can recover ordinary supergravity with closed gauge algebra by eliminating extra symmetries and by fixing certain fields [14, 16]. Part of our work is the $d = 10$ counterpart of the analysis of Howe and Lindstrom [15]. The result of [15] is somewhat puzzling since there are arguments [17, 18]. Our multiplet is a Poincare multiplet, since in $d = 10$ no simple superconformal algebra seems to exist [18]. We are further inspired by antisymmetric tensor fields coupling [21] which were latter modified in [20] and reconstructed by Chamsedine [19].

But our results in sect.2 disagree with Neother coupling in [21]. We also present our results on Weyl invariance in $d = 10$ and derived the associated differential identities in $d = 10$.

2.1 Supergravity Theory and Dimensional formulation

In this section, we study the action and showed how the $N = 1, d = 10$ supergravity theory may be derived from $d = 11$ dimensions.

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2.1.1 The Action: In $d=11$, the physical supergravity fields are the elbein E_μ^m, one gravitino $\psi_\mu^a(a=1,..,32)$, and antisymmetric tensor $A_{\mu\nu\rho}$. The gravitino is a majorana spinor

$$\bar{\psi}_\mu = \psi_\mu^T c,\ \ c\Gamma_\mu c^{-1} = -\Gamma_\mu^T\ ,\ \ 1 \le \mu \le 11 \tag{2.1}$$

Since all the fields are gauge fields, a dimensional argument and others [12] state that the action must be polynomial in all fields except E_μ^m. Putting $K=1$ always, the $d=11$ action reads

$$\ell(d=11) = -\tfrac{1}{2}ER(E,\Omega) - \tfrac{1}{2}E\overline{\psi}_\mu\Gamma^{\mu\rho\sigma}D_\rho\left(\tfrac{\Omega+\hat{\Omega}}{2}\right)\psi_\sigma - \tfrac{1}{48}EF_{\mu\nu\rho\sigma}^2 - \tfrac{1}{384}\sqrt{2}E\left(\overline{\psi}_\mu\Gamma^{\mu\alpha\beta\gamma\delta\nu}\psi_\nu + 12\overline{\psi}^\alpha\Gamma^{\beta\gamma}\psi^\delta\right)\left(F+\hat{F}\right)_{\alpha\beta\gamma\delta}$$

$$-\tfrac{1}{36\times 96}i\sqrt{2}\varepsilon^{\mu_1\ldots\mu_{11}}F_{\mu_1\ldots\mu_4}F_{\mu_5\ldots\mu_8}A_{\mu_9\ldots\mu_{11}} \tag{2.2}$$

$$F_{\mu\nu\rho\sigma} = \partial_\mu A_{\nu\rho\sigma} + 23\text{terms}$$

$$\Gamma^{\mu\nu\rho} = \Gamma^{[\mu}\Gamma^\nu\Gamma^{\rho]}$$

$$\hat{\Omega}_{\mu mn} = \Omega_{\mu mn}(E)$$

But $\Omega_{\mu mn}(E,\psi)$ is the solution of the Ω field equation which differs from $\hat{\Omega}$ such that

$$\Omega_{\mu mn} = \hat{\Omega}_{\mu mn} - \tfrac{1}{8}\bar{\psi}^\alpha\Gamma_{\alpha\mu mn\beta}\psi^\beta \tag{2.3}$$

The transformation rules read

$$\partial E_\mu^m = \tfrac{1}{2}\bar{\varepsilon}\Gamma^m\psi_\mu$$

$$\partial A_{\mu\nu p} = -\tfrac{1}{8}\sqrt{2}\bar{\varepsilon}\Gamma_{[\mu\nu}\psi_{\rho]}$$

$$\partial\psi_\mu = D_\mu(\hat{\Omega})\varepsilon + \tfrac{1}{288}\sqrt{2}(\Gamma_\mu^{\alpha\beta\gamma\delta} - 8\partial_\mu^\alpha\Gamma^{\beta\gamma\delta})\varepsilon\hat{F}_{\alpha\beta\gamma\delta} \tag{2.4}$$

$$E_{\hat{\mu}}^{\hat{m}} = \begin{pmatrix} E_\mu^m & 0 \\ 0 & E_{\dot{1}1}^{11} \end{pmatrix}$$

$$A_{\mu\nu\rho} = 0$$

$$\psi_\mu^R = \tfrac{1}{2}(1-\Gamma_{11})\psi_\mu = 0$$

$$\psi_{\dot{1}1}^L = \tfrac{1}{2}(1-\Gamma_{11})\psi_{\dot{1}1} = 0 \tag{2.5}$$

$$\hat{\Omega}_{11mn} = \hat{\Omega}_{\mu 11n} = F_{\alpha\beta\gamma\delta} = \hat{F}_{\alpha\beta\gamma\delta} = 0 \tag{2.6}$$

Theorem 1: If the Einstein action has an extra factor $E_{\dot{1}1}^{11}$ due to E, and we redefine $E_\mu^m = e_\mu^m\phi^\gamma$, with $\phi = E_{\dot{1}1}^{11}$, then the ϕ factors in the leading part cancel in $d=10$ dimension if $\gamma = -(d-2)^{-1}$.

Proof:

$$\text{Let}\ \ E_{\hat{\mu}}^{\hat{m}} = \begin{pmatrix} \phi^{-(d-2)^{-1}}e_\mu^m & 0 \\ 0 & \phi \end{pmatrix} \tag{2.7}$$

Using the well known Palatini identity

$$\Omega_{\mu mn}(E) = \omega_{\mu mn}(e) - \tfrac{1}{d-2}\left(e_{m\mu}e_n^{\nu} - e_{n\mu}e_m^{\nu}\right)\partial_\nu\phi/\phi \qquad (2.8)$$

$$-\tfrac{1}{2}eR(\omega(e)+\tau) = -\tfrac{1}{2}eR(\omega(e) + \tfrac{1}{2}e(\tau^{mcn}\tau_{ncm} - \tau^2_{mmc}) \qquad (2.9)$$

$$\Omega_{\dot{1}111n} = \phi^{1/(d-2)}e_n^{\mu}\partial_\mu^{\phi} \qquad (2.10)$$

$$E_{\hat{\mu}}^{\hat{m}} = EE_n^{\mu}D_\mu(\Omega(E))\Omega_{\dot{1}111n} \qquad (2.11)$$

$$-\tfrac{1}{2}ER(E,\Omega(E)) = -\tfrac{1}{2}eR(e,\omega(e)) - \tfrac{1}{2}e\left(\tfrac{d-1}{d-2}\right)\left(\tfrac{\partial_\mu\phi}{\phi}\right)^2 \qquad (2.12)$$

Lemma 1: The scalar field ϕ has a kinetic term satisfied under the locally scale-invariant action with e on the r.h.s. expressed in terms of e' and ϕ.

$$-e'R(e') = -e\phi^2R(e) + 4e\tfrac{(d-1)}{(d-2)}(\partial_\mu\phi)^2 \rightarrow \text{Locally scale-invariant action}$$

$$-e\phi^2R(e) = -e'R(e') - 4e\tfrac{(d-1)}{(d-2)}(\partial_\mu\phi)^2 \rightarrow \text{Reverse relation}$$

Lemma 2: There are also cross terms between ψ_μ and $\psi_{\dot{1}1}$ but no terms quadratic in $\psi_{\dot{1}1}$, therefore one arrives at the following result:

$$-\tfrac{1}{2}E\overline{\psi}_\mu\Gamma^{\mu\rho\sigma}D_\rho(\Omega(E))\psi_\sigma = -\tfrac{1}{2}e\overline{\psi}_\mu\Gamma^{\mu\rho\sigma}D_\rho(\omega(e))\psi_\sigma$$

$$-\tfrac{1}{2}e\overline{\lambda}\Gamma^{\mu}D_\mu(\omega(e))\lambda - \tfrac{3}{8}\sqrt{2}e\overline{\psi}_\mu\tfrac{\partial\phi}{\phi}\Gamma^{\mu}\lambda,$$

$$\psi_\mu^L = \phi^{-1/16}\left(\psi_\mu + \tfrac{1}{12}\sqrt{2}\Gamma_\mu\lambda\right)$$

$$\psi_{\dot{1}1}^R = \tfrac{2}{3}\sqrt{2}\phi^{17/16}\lambda \qquad (2.13)$$

Lemma 3: In addition to (2.6), one finds easily the reduction of the photon kinetic term. Defining (2.14), hence the result (2.15):

$$A_{\mu\nu} = 6A_{\mu\nu\dot{1}1}\,, \qquad F_{\mu\nu\rho} = \partial_{[\mu}A_{\nu\rho]} \qquad (2.14)$$

$$\hat{F}_{\alpha\beta\gamma\dot{1}1} = 3\hat{F}_{\alpha\beta\gamma} - \tfrac{7}{96}\sqrt{2}\phi^{3/4}\bar{\lambda}\Gamma_{\alpha\beta\gamma}\lambda\,,$$

$$\hat{F}_{\alpha\beta\gamma} = F_{\alpha\beta\gamma} - \tfrac{1}{4}\sqrt{2}\phi^{3/4}\bar{\psi}_{[\alpha}\Gamma_\beta\psi_\gamma + \tfrac{1}{4}\phi^{3/4}\bar{\psi}_{[\alpha}\Gamma_{\beta\gamma}\lambda \qquad (2.15)$$

Lemma 4: The action, except the four fermions couplings which will be given in (2.28) reads

$$\mathcal{L}(N=1,d=10) = -\tfrac{1}{2}eR(e,\omega(e)) - \tfrac{1}{2}e\bar{\psi}_\mu\Gamma^{\mu\rho\sigma}D_\rho(\omega(e))\psi_\sigma - \tfrac{3}{4}e\phi^{-3/2}F^2_{\mu\nu\rho}$$

$$-\tfrac{1}{2}e\bar{\lambda}\Gamma^{\mu}D_\mu(\omega(e))\boldsymbol{\lambda} - \tfrac{9}{16}e(\partial_\mu\phi/\phi)^2 - \tfrac{3}{8}\sqrt{2}e\bar{\psi}_\mu(\partial\phi/\phi)\Gamma^{\mu}\lambda$$

$$+\tfrac{1}{16}\sqrt{2}e\phi^{3/4}F_{\alpha\beta\gamma}\left(\bar{\psi}_\mu\Gamma^{\mu\alpha\beta\gamma\nu}\psi_\nu + 6\bar{\psi}^{\alpha}\Gamma^{\beta}\psi^{\gamma} - \sqrt{2}\bar{\psi}_\mu\Gamma^{\alpha\beta\gamma}\Gamma^{\mu}\lambda\right) \qquad (2.16)$$

Remarks: The consistency of (2.5) follows further from (2.6). After dimensional reduction to $d = 10$ and truncation as in (2.5), the kinetic terms are cast in canonical form by a suitable Weyl rescaling of the zehnbein and field redefinitions of the other fields. These follow easily from (2.2). The ϕ-dependent terms in the Einstein action are only due to the torsion terms in Ω noted in (2.6) and (2.8). The contributions coming from (2.10) yield a vanishing result in terms of zehnbein E_μ^m which is a total derivative in (2.11). The final result is that one finds the canonical Einstein action plus a physical scalar. In the Rarita-Schwinger action, the factor ϕ due to the rescaling of the zehnbein in (2.7), were removed by rescaling ψ_μ by a factor $\phi^{-[2(d-2)]^{-1}} = \phi^{-1/16}$. We do not introduced the $\partial_\mu\phi$ term by the rescaling because the gravitino is a Majorana spinor. The $\partial_\mu\phi$ terms due to the spin connection canceled. We do not shift $\psi_{1\dot{1}} \to \psi_{1\dot{1}} + A\Gamma.\psi$, in order to avoid a $D\varepsilon$ term in$\partial\psi_{1\dot{1}}$, this leads to the result in (2.13). The term $\partial\phi$ as in (2.13) is due to (2.8) and the rescaling of $\psi_{1\dot{1}}$. The coupling involving $F_{\mu\nu\rho}$ does not contain $F\lambda^2$, but only $F\psi^2$ and $F\psi\boldsymbol{\lambda}$ terms. This we will discuss in subsection 2.3. It may be noted that most of these results agree with [19], but instead of the Noether coupling $\bar{\psi}_\mu\partial\phi\Gamma^\mu\lambda$, a coupling $\bar{\psi}_\mu\partial^\mu\phi\boldsymbol{\lambda}$ appears in [19].

2.2. The Transformation Laws

It may be noted that the action obtained is in canonical form on the basis $\left(e_n^m, \psi_\mu, \lambda, A_{\mu\nu}, \phi\right)$, but the transformation rules of e_n^m and the $\partial\psi_\mu = \partial_\mu\varepsilon$ part are no longer canonical. We remedy this by redefining the supersymmetry parameter and by adding a field-dependent Lorentz rotation to the supersymmetry transformations:

$$\partial_Q(\eta, d = 10) = \partial_Q(\varepsilon, d = 11) + \partial_L\left(-\tfrac{1}{24}\sqrt{2}\bar{\eta}\Gamma^{mn}\lambda\right)$$

$$\eta = \varepsilon\phi^{1/16} \qquad (2.17)$$

Hence, $\partial e_\mu^m = \frac{1}{2}\bar{\eta}\Gamma^m\psi_\mu$ (2.18)

Next, we reduced the super-covariant spin connections$\hat{\Omega}$. The result follows

$$\hat{\Omega}_{1\dot{1}11n} = \phi^{1/8}\hat{D}_n\phi, \qquad \hat{\Omega}_{\mu 11n} = \hat{\Omega}_{1\dot{1}mn} = 0$$

$$\hat{\Omega}_{\mu mn} = \hat{w}_{\mu mn}(e,\psi) + \tfrac{1}{24}\sqrt{2}\bar{\psi}_\mu\Gamma_{mn}\lambda$$

$$-\tfrac{1}{288}\bar{\lambda}\Gamma_{\mu mn}\lambda - \tfrac{1}{8}\left(e_{m\mu}\hat{D}_n\phi - e_{n\mu}\hat{D}_m\phi\right)\phi^{-1} \qquad (2.19)$$

The final result for the transformation rules reads:

$$\partial e_\mu^m = \tfrac{1}{2}\bar{\eta}\Gamma^m\psi_\mu \ , \qquad \partial\phi = -\tfrac{1}{3}\sqrt{2}\bar{\eta}\lambda\phi \ ,$$

$$\partial A_{\mu\nu} = \tfrac{1}{4}\sqrt{2}\phi^{3/4}\left(\bar{\eta}\Gamma_\mu\psi_\nu - \bar{\eta}\Gamma_\mu\psi_\nu - \tfrac{1}{2}\sqrt{2}\bar{\eta}\Gamma_{\mu\nu}\lambda\right)$$

$$\partial\lambda = -\tfrac{3}{8}\sqrt{2}\left(\hat{D}\phi/\phi\right)\eta + \tfrac{1}{8}\phi^{-3/4}\Gamma^{\alpha\beta\gamma}\eta\hat{F}_{\alpha\beta\gamma} \ ,$$

$$\partial\psi_\mu = D_\mu\left(\hat{\omega}(e,\psi)\right)\eta + \tfrac{1}{32}\sqrt{2}\phi^{-3/4}\left(\Gamma_\mu^{\alpha\beta\gamma} - 9\partial_\mu^\alpha\Gamma^{\beta\gamma}\right)\eta\hat{F}_{\alpha\beta\gamma}$$

$$-\tfrac{1}{16\times 32}\left(\Gamma_\mu^{\alpha\beta\gamma} - 5\partial_\mu^\alpha\Gamma^{\beta\gamma}\right)\eta\bar{\lambda}\Gamma_{\alpha\beta\gamma}\lambda$$

$$+\tfrac{1}{96}\sqrt{2}\left[\left(\bar{\psi}_\mu\Gamma_{mn}\lambda\right)\Gamma^{mn}\eta + \left(\bar{\lambda}\Gamma_{mn}\eta\right)\Gamma^{mn}\psi_\mu\right]$$

$$+\tfrac{1}{96}\left[2\left(\bar{\psi}_\mu\lambda\right)\eta - 2\left(\bar{\lambda}\eta\right)\psi_\mu + 4\left(\bar{\psi}_\mu\Gamma_m\eta\right)\Gamma^m\lambda\right] \qquad (2.20)$$

Remarks: The presence of $\bar{\psi}_\mu\Gamma_{mn}\lambda$ terms in $\hat{\Omega}_{\mu mn}$ noted in (2.19) does not mean, of course, that an 11-dimensional super-covariant tensor would not be super-covariant in $d = 10$. It may be noted that undoing the Lorentz rotation in (2.17) the $\partial_\mu\varepsilon$ terms from $\partial_L\hat{\omega}_{\mu mn}$ canceled those from $\partial\psi_\mu \to \partial_\mu\varepsilon$. We noticed that this subtlety can only occur for quantities that are not covariant under local Lorentz transformations. This explains why $F_{\mu\nu\rho 1\dot{1}}$ in (2.15) is super-covariant. It is now obvious how to obtain $\partial\psi_\mu$ and $\partial\lambda$. From (2.4), we replaced ε by η, and $\left(\Psi_\mu, \Psi_{1\dot{1}}\right)$ by (ψ_μ, λ), and used (2.19). For $\partial\lambda$; all $\lambda^2\varepsilon$ terms canceled. In $\partial\psi_\mu$ we find $\lambda^2\varepsilon$ terms. We will explain these remarkable results in the next subsection.

2.3 The Four-Fermions Coupling

We noted in $d = 4$, a dimensional argument [10] that there cannot be six or more fermions coupling; otherwise the action in (2.16) was complete up to four fermions coupling I^4. Finding I^4 requires that the terms ψ_μ and λ field equations be super-covariant. We now review this argument. Gauge invariance of the action implies $(\partial I/\partial\phi^j)(\partial\partial^j) = 0$; hence, under a second gauge variation ∂' one has

$$[\partial'(\partial I/\partial\phi^j)]\partial\phi^j = -(\partial I/\partial\phi^j)[\partial'(\partial\phi^j)] \quad (2.21)$$

The field equation is obtained from (2.16). Using left-derivatives, it reads

$$\partial I/\partial\bar{\lambda} = -D(\omega(e))\lambda - \tfrac{3}{8}\sqrt{2}\Gamma^\mu\Gamma^\nu\psi_\mu(\partial_\nu\phi/\phi)$$

$$+\tfrac{1}{8}\Gamma^\mu\Gamma^{\alpha\beta\gamma}\psi_\mu F_{\alpha\beta\gamma}\phi^{-3/4} + \partial I^4/\partial\bar{\lambda} \quad (2.22)$$

$$\partial I^{(4)}/\partial\bar{\lambda} = -\tfrac{1}{4}\Gamma^\mu\Gamma^{mn}\lambda\left(\widehat{\omega}_{\mu mn}(e,\psi) - \omega_{\mu mn}(e)\right) - \tfrac{1}{4}\Gamma^\mu\Gamma^\nu\psi_\mu(\bar{\psi}_\nu\lambda)$$

$$+\tfrac{1}{32}\Gamma^\mu\Gamma^{\alpha\beta\gamma}\psi_\mu\left(\bar{\psi}_\alpha\Gamma_{\beta\gamma}\lambda\right) - \tfrac{1}{32}\sqrt{2}\Gamma^\mu\Gamma^{\alpha\beta\gamma}\psi_\mu\left(\bar{\psi}_\alpha\Gamma_\beta\psi_\gamma\right) \quad (2.23)$$

$$\partial I/\partial\bar{\lambda} = -\Gamma^{\mu\rho\sigma}D_\rho\left(\omega(e)\right)\psi_\sigma + \tfrac{1}{8}\sqrt{2}\phi^{-3/4}F^{\alpha\beta\gamma}\left(\Gamma^{\mu\alpha\beta\gamma\nu}\psi_\mu + 6g^{\mu\alpha}\Gamma^\beta\psi^\gamma\right)$$

$$-\tfrac{3}{8}\sqrt{2}(\partial\phi/\phi)\Gamma^\mu\lambda - \tfrac{1}{8}\Gamma^{\alpha\beta\gamma}\Gamma^\mu\lambda F_{\alpha\beta\gamma}\phi^{-3/4} + \partial I^{(4)}/\partial\bar{\lambda} \quad (2.24)$$

$$\partial I^{(4)}(\psi^4\text{terms})/\partial\bar{\psi}_\mu = -\tfrac{1}{4}\Gamma^{\mu\rho\sigma}\Gamma^{mn}\psi_\sigma\left(\widehat{\omega}_{\rho mn}(e,\psi) - \omega_{\rho mn}(e)\right)$$

$$-\tfrac{1}{16}\left(\Gamma^{\mu\alpha\beta\gamma\nu}\psi_\mu + 6g^{\mu\alpha}\Gamma^\beta\psi^\gamma\right)\left(\bar{\psi}_{[\alpha}\Gamma_\beta\psi_{\gamma]}\right) \quad (2.25)$$

$$I^{(4)}(\psi^4\text{terms in Noether}) = -\tfrac{1}{64}e\bar{\psi}_\rho\Gamma_m\psi_n\left(\bar{\psi}_\alpha\Gamma^{\alpha\rho mn\beta}\psi_\beta + 6\bar{\psi}^{[\rho}\Gamma^m\psi^{n]}\right) \quad (2.26)$$

$$I^{(4)}(\psi^4\text{terms in E + RS}) = \tfrac{1}{8}e(\bar{\psi}.\Gamma\psi_m)^2 - \tfrac{1}{32}e\left(\bar{\psi}_\mu\Gamma_\nu\psi_\rho\right)^2$$

$$-\tfrac{1}{16}e\left(\bar{\psi}_\mu\Gamma_\nu\psi_\rho\right)(\bar{\psi}^\mu\Gamma^\rho\psi^\nu) - \tfrac{1}{64}e\left(\bar{\psi}_\rho\Gamma_m\psi_n\right)\left(\bar{\psi}_\alpha\Gamma^{\alpha\rho mn\beta}\psi_\beta\right) \quad (2.27)$$

Theorem 2: The variation of (2.26), (2.27) lead to (2.25) by using gauge identity.

Proof: From the four fermion coupling in the $N = 1$ gauge action, we note that;

$$\tfrac{1}{16}\left(\Gamma^{\tau\alpha\beta\gamma\nu}\psi_\nu\right)\left(\bar{\psi}_\alpha\Gamma_\beta\psi_\gamma\right) - 16\left(\bar{\psi}_\mu\Gamma^{\mu\tau\alpha\beta\gamma\nu}\psi_\nu\right)\left(\Gamma_\beta\psi_\gamma\right)$$

$$+\tfrac{1}{2}g^{\beta[\tau}\Gamma^{\alpha\nu\gamma]}\psi_\nu\left(\bar{\psi}_\alpha\Gamma_\beta\psi_\gamma\right) = 0 \quad - \quad \text{gauge identity}^*$$

Now, final result;

$$e^{-1}I^{(4)} = \tfrac{1}{8}e(\bar{\psi}.\Gamma\psi_m)^2 - \tfrac{1}{32}e\left(\bar{\psi}_\mu\Gamma_\nu\psi_\rho\right)^2 - \tfrac{1}{16}e\left(\bar{\psi}_\mu\Gamma_\nu\psi_\rho\right)(\bar{\psi}^\mu\Gamma^\rho\psi^\nu)$$

$$-\tfrac{1}{64}\left(\bar{\lambda}\Gamma^{\alpha\beta\gamma}\lambda\right)\left(\tfrac{1}{12}\bar{\psi}^\rho\Gamma_{\alpha\beta\gamma}\psi_\rho + \tfrac{1}{2}\bar{\psi}^\rho\Gamma_{\rho\alpha\beta}\psi_\gamma - \bar{\psi}_\alpha\Gamma_\beta\psi_\gamma\right)$$

$$-\sqrt{2}\left(\bar{\lambda}\Gamma^\mu\Gamma^{\alpha\beta\gamma}\psi_\mu\right)\left(\bar{\psi}_\alpha\Gamma_\beta\psi_\gamma\right) - \tfrac{1}{32}\left(\bar{\psi}_\rho\Gamma_m\psi_n\right)\left(\bar{\psi}_\alpha\Gamma^{\alpha\rho mn\beta}\psi_\beta\right)$$

$$-\tfrac{3}{32}\left(\bar{\psi}_\rho\Gamma_m\psi_n\right)\left(\bar{\psi}^{[\rho}\Gamma^m\psi^{n]}\right) \quad (2.28)$$

Q.E.D

Lemma 5: The Noether couplings $\frac{1}{2}(F+\hat{F})$ and $\frac{1}{2}(\partial\phi+\hat{D}\phi)$ accounts for all terms in I^4. The next simplest terms are the $\psi^3\lambda$ arising from the Noether terms with $\frac{1}{2}(F+\hat{F})$. This Noether coupling gives

$$\frac{1}{128}\sqrt{2}\left(\bar{\psi}_{[\alpha}\Gamma_{\beta\gamma]}\lambda\right)\left(\bar{\psi}_\mu\Gamma^{\mu\alpha\beta\gamma v}\psi_v+6\bar{\psi}^\alpha\Gamma^\beta\psi^\gamma\right)$$

$$-\frac{1}{64}\sqrt{2}\left(\bar{\psi}_\alpha\Gamma_\beta\psi_\gamma\right)\left(\bar{\lambda}\Gamma^\mu\Gamma^{\alpha\beta\gamma}\psi_\mu\right) \quad (2.29)$$

Remarks: The question now arises whether all these four-fermion terms are super-covariantizations. It now obvious from this paper that (2.29) does not agree with (2.28). Hence the four-fermion terms cannot be rewritten in terms of covariantizations mentioned above.

3.0. Maxwell-Einstein Supergravity in $d=10$

In this section we coupled the $d=10$ Maxwell system to the $N=1,\ d=10$ gauge action which we derived in sect. 2.

Theorem 3: Suppose ten dimensions is the highest dimension where matter exists, then $d=10,\ N=1$ global supersymmetric Maxwell action

$$\mathcal{L}^{(0)}=-\tfrac{1}{4}F_{\mu v}^2-\tfrac{1}{2}\bar{\chi}\partial\chi\,, \qquad F_{\mu v}=\partial_\mu A_v-\partial_v A_\mu \quad (3.1)$$

Is invariant under the following transformation rules;

$$\partial A_\mu=\tfrac{1}{2}\bar{\varepsilon}\Gamma_\mu\chi\,,\quad \partial\chi=\tfrac{1}{4}\Gamma.F\varepsilon\,,\quad \Gamma.F\equiv\Gamma^{mn}F_{mn} \quad (3.2)$$

Proof: It follows from (3.2) that χ and ε must have same chirality, whereas χ and λ have opposite chirality. Let χ be a majorana spinor, $\bar{\chi}\gamma^\mu\chi=0,\ A_\mu\Longrightarrow gh=1$, $[g=f^{-1}\ \&\ h=f\,]$ and $f=1$ normalization. Now

$$e^{-1}\mathcal{L}^{(0)}=\tfrac{1}{4}F_{\mu v}^2f^2(\phi)-\tfrac{1}{2}\bar{\chi}\Gamma^\mu D_\mu\big(\omega(e)\big)\chi$$

$$\partial A_\mu=\tfrac{1}{2}\bar{\varepsilon}\Gamma_\mu\chi g(\phi),\quad \partial\chi=-\tfrac{1}{4}\Gamma.F\varepsilon h(\phi) \quad (3.3)$$

$$\mathcal{L}^{(N)}=\tfrac{1}{4}ek\bar{\psi}_\mu\Gamma.F\Gamma^\mu\chi h\,,\quad h=f^2g \quad (3.4)$$

$$\partial\mathcal{L}^{(0)}=\tfrac{1}{4}e\bar{\chi}\Gamma.F\partial f\varepsilon \quad (3.5)$$

$$\partial\mathcal{L}=\tfrac{1}{16}ekF_{\alpha\beta}F_{\rho\sigma}\bar{\psi}_\mu\Gamma^{\alpha\beta\mu\rho\sigma}\varepsilon f^2$$

$$=\tfrac{1}{8}ekA_\beta F_{\rho\alpha}f^2\begin{pmatrix}(D_\alpha\bar{\varepsilon})\Gamma^{\alpha\beta\mu\rho\sigma}\psi_\mu\\ +\bar{\varepsilon}\Gamma^{\alpha\beta\mu\rho\sigma}D_\alpha\psi_\mu\end{pmatrix}+O(k^2) \quad (3.6)$$

We imposed the conditions;

$$R^\mu=\Gamma^{\mu\rho\sigma}D_\rho\big(\omega(e)\big)\psi_\alpha$$

$$\Gamma.R=8\Gamma^{\rho\sigma}D_\rho\psi_\sigma$$

$$\Gamma^\sigma(D_\sigma\psi_\alpha-D_\alpha\psi_\sigma)=R_\alpha-\tfrac{1}{8}\Gamma_\alpha\Gamma.R \quad (3.7)$$

NB: The theory solves this problem in another manner such that the variation of $F_{\alpha\beta\gamma}$ is also proportional to $\bar{\varepsilon}\Gamma_{[\alpha}D_\beta\psi_{\gamma]}$ **.** In this way all order $kF^2\psi\varepsilon$ variations are cancelled if one takes

$$e^{-1}\mathcal{L}=-\tfrac{1}{4}F_{\mu v}^2f^2-\tfrac{1}{2}\bar{\chi}D\big(\omega(e)\big)\chi-\tfrac{1}{4}k\bar{\psi}_\mu\Gamma.F\Gamma^\mu\chi f$$

$$-\tfrac{1}{16}k^2A_vF_{\alpha\beta}\bar{\psi}_\mu\Gamma^{\mu v\alpha\beta\rho}\psi_\rho f^2+\tfrac{3}{4}\sqrt{2}kA_\mu F_{v\rho}F^{\mu v\rho}\phi^{-3/4}f^2,$$

$$\partial\bar{\psi}_\mu(extra) = \tfrac{1}{32}kA_{[v}F_{\alpha\beta]}\bar{\varepsilon}\left(\Gamma_\mu^{v\alpha\beta} + 9\partial_\mu^\alpha\Gamma^{v\beta}\right)f^2 \quad (3.8)$$

$$e^{-1}\partial\mathcal{L} = -\tfrac{1}{4}k\bar{\varepsilon}\Gamma.\psi\bar{\chi}D\chi - \tfrac{1}{8}\bar{\chi}\Gamma^\mu\Gamma^{mn}\chi\partial\omega_{\mu mn}(e)$$

$$+\tfrac{1}{4}k(\bar{\varepsilon}\Gamma^\mu\psi_v)\left(\bar{\chi}\Gamma^v D_\mu\chi\right) - \tfrac{1}{4}k\left(\bar{\psi}_\mu\Gamma^{\alpha\beta}\Gamma^\mu\chi\right)fD_\alpha\left(\bar{\varepsilon}\Gamma_\beta\chi f^{-1}\right) \quad (3.9)$$

$$\partial\widehat{\omega}_{\mu mn}(e,\psi) = \tfrac{1}{4}k\bar{\varepsilon}\left(\Gamma_n\psi_{\mu m} - \Gamma_m\psi_{\mu n} - \Gamma_\mu\psi_{mn}\right)$$

$$\psi_{\mu v} \equiv D_\mu\psi_v - D_v\psi_\mu \quad (3.10)$$

$$\mathcal{L}(extra) = -\tfrac{1}{16}\sqrt{2}ek\bar{\chi}\Gamma^{\alpha\beta\gamma}\chi F_{\alpha\beta\gamma}\phi^{-3/4} \quad (3.11)$$

The last term in (3.9) yields

$$= -\tfrac{1}{4}k\left(\bar{\psi}.\Gamma\Gamma^{\alpha\beta}\chi\right)D_\alpha\left(\bar{\psi}\Gamma_\beta\chi\right) - \tfrac{1}{2}k\left(\psi^\beta\Gamma^\alpha\chi\right)D_\sigma\left(\bar{\varepsilon}\Gamma_\beta\chi\right)$$

$$+\tfrac{1}{2}k\left(\bar{\psi}^\mu\Gamma^\beta\chi\right)D_\mu\left(\bar{\varepsilon}\Gamma_\beta\chi\right) \quad (3.12)$$

$$e^{-1}\partial\mathcal{L} = \frac{1}{4}k\left(\bar{\varepsilon}\Gamma_\mu\psi_v\right)\bar{\chi}(\Gamma^v D^\mu - \Gamma^\mu D^v)\chi$$

$$= \tfrac{1}{8}k\bar{\chi}\Gamma^{v\mu\lambda}\chi D_\lambda\left(\bar{\varepsilon}\Gamma_\mu\psi_v\right) + \partial\chi \quad (3.13)$$

$$e^{-1}\partial\mathcal{L} = \tfrac{1}{4}k\left(\bar{\psi}.\overleftarrow{D}\Gamma_\beta\chi\right)\left(\bar{\varepsilon}\Gamma^\beta\chi\right) + \tfrac{1}{48}k\left(\bar{\varepsilon}\Gamma^{(3)}\psi^\mu\right)\left(\bar{\chi}\Gamma^{(3)}D_\mu\chi\right) \quad (3.14)$$

Final result; $\partial\mathcal{L}\sim kD_\mu\chi^2\psi\varepsilon$ terms yield

$$\partial\chi = -\tfrac{1}{4}k(\bar{\varepsilon}\Gamma.\psi)\chi + \tfrac{1}{4}k\Gamma^\mu\Gamma_\beta\psi_\mu\left(\bar{\varepsilon}\Gamma^\beta\chi\right) - \tfrac{1}{4}k(\bar{\psi}.\Gamma\chi)\varepsilon + \tfrac{1}{4}k\left(\bar{\varepsilon}\Gamma_\mu\psi_v\right)\Gamma^{\mu v}\chi\ ,$$

$$\partial\psi_\mu = -\tfrac{k}{8\times 32}\left(\Gamma_\mu^{\alpha\beta\gamma} - 5\partial_\mu^\alpha\Gamma^{\beta\gamma}\right)\varepsilon\bar{\chi}\Gamma_{\alpha\beta\gamma}\chi\ ,$$

$$e^{-1}\mathcal{L}^{(4)} = \tfrac{1}{16}\sqrt{2}k\bar{\chi}\Gamma^{\alpha\beta\gamma}\chi F_{\alpha\beta\gamma}\phi^{-3/4} + \tfrac{1}{8}k^2(\bar{\psi}.\Gamma\chi)^2$$

$$-\tfrac{1}{4}k^2\left(\bar{\psi}_\mu\Gamma_\beta\chi\right)\left(\bar{\psi}^\mu\Gamma^\beta\chi\right) + \tfrac{1}{16}k^2\bar{\chi}\Gamma^{\alpha\beta\gamma}\chi\left(\bar{\psi}_\alpha\Gamma_\beta\psi_\gamma\right) \quad (3.15)$$

Remarks: The Maxwell action is invariant under $\partial A_\mu = \partial_\mu\Lambda$ and supersymmetry is broken. In order k, a coupling $\mathcal{L} = \tfrac{3}{4}\sqrt{2}keA_\mu F_{v\rho}F^{\mu v\rho}\phi^{-3/4}$ was found in (3.8) which violate this invariance by an amount proportional to $\Lambda F_{v\rho}\partial_\mu F^{\mu v\rho}$. It may be noted that Maxwell-gauge invariance can be restored using Noether method. This leads to a Maxwell transformation rule of $A_{\mu v}$: $\partial_M A_{\mu v} = \tfrac{1}{2}\sqrt{2}k\Lambda F_{\mu v}$. Introduction of a Maxwell-covariant $A_{\mu v}$ curl; $F'_{\mu v\rho} = F_{\mu v\rho} - \tfrac{1}{2}\sqrt{2}kA_{[\mu}F_{v\rho]}$ implies that the field A_μ can only occur through its field strength $F_{\mu v}$ or through the covariantizations in $F'_{\mu v\rho}$. It remain to investigates whether the replacement of $F_{\mu v\rho}$ by $F'_{\mu v\rho}$ causes simplifications. In the gauge action, this replacement absorbs the $A_\mu F_{v\rho}F^{\mu v\rho}$ coupling, while the exact expression for the $\left(A_\mu F_{v\rho}\right)^2$ predicted;

$$\mathcal{L}(extra) = -\tfrac{3}{8}ek^2 A_{[\mu}F_{v\rho]}(A^\mu F^{v\rho})\phi^{-3/2}\ .$$

Lemma 6: As suggested by $d = 4$ Maxwell-Einstein system [20], we rewrite the action as follows:

$$\mathcal{L} = \mathcal{L}\left(N = 1 \text{ gauge, but with } F'_{\mu v\rho}\right) - \tfrac{1}{4}e\phi^{-3/4}F_{\mu v}^2 - \tfrac{1}{2}e\bar{\chi}D(\widehat{\omega})\chi$$

$$-\tfrac{1}{8}ke\phi^{-3/8}\bar{\chi}\Gamma^\mu\Gamma^{\rho\sigma}\left(F_{\rho\sigma} + \widehat{F}_{\rho\sigma}\right)\left(\psi_\mu + \tfrac{1}{12}\sqrt{2}\Gamma_\mu\lambda\right)$$

$$+\frac{1}{16}\sqrt{2}ke\phi^{-3/4}\bar{\chi}\Gamma^{\alpha\beta\gamma}\chi\widehat{F}'_{\alpha\beta\gamma}$$

$$-\tfrac{1}{16\times 96}\sqrt{2}k^2 e\bar{\chi}\Gamma_{\alpha\beta\gamma}\chi\bar{\psi}_\mu\left(4\Gamma^{\alpha\beta\gamma}\Gamma^\mu + 3\Gamma^\mu\Gamma^{\alpha\beta\gamma}\right)\lambda$$

$$-\tfrac{1}{512}k^2 e\bar{\chi}\Gamma_{\alpha\beta\gamma}\chi\bar{\lambda}\Gamma^{\alpha\beta\gamma}\lambda \tag{3.16}$$

Remarks: The transformation rules under which (3.16) is invariant will be discussed in sect. 4.

4.0. The gauge algebra

We derived the $d = 10$ algebra and then compare the $d = 11$ case. The transformation rules of the pure gauge theory were obtained in subsection 2.2 and given in (2.20).

The $\partial\psi_\mu = D_\mu(\hat{\omega})\varepsilon$ terms give the same algebra as $d = 4$,

$$[\partial_Q(\varepsilon_1), \partial_Q(\varepsilon_2)] = \partial_{gc}(\xi^\mu) + \partial_Q(-\xi^\mu\psi_\mu) + \partial_L(\xi^\mu\hat{\omega}_{\mu mn}) \tag{4.1}$$

$$\lambda_{12,mn} = \xi^\mu\hat{\omega}_{\mu mn} + (\bar{\varepsilon}_2\Gamma_{mn}^{\alpha\beta\gamma}\varepsilon_1)\left(\tfrac{1}{32}\sqrt{2}\hat{F}_{\alpha\beta\gamma}\phi^{-3/4} - \tfrac{1}{16\times 32}\bar{\lambda}\Gamma_{\alpha\beta\gamma}\lambda\right)$$

$$+\xi^\mu\left(\tfrac{9}{8}\sqrt{2}\hat{F}_{mn\mu}\phi^{-3/4} - \tfrac{5}{8\times 32}\bar{\lambda}\Gamma_{mn\mu}\lambda\right) \tag{4.2}$$

$$\varepsilon_{12} = -\xi^\mu\psi_\mu + \tfrac{1}{96\times 160}\sqrt{2}(\bar{\varepsilon}_2\Gamma^{(5)}\varepsilon_1)\Gamma^{(5)}\lambda - \tfrac{7}{64}\sqrt{2}(\bar{\varepsilon}_2\Gamma^{\alpha}\varepsilon_1)\Gamma_\alpha\lambda \tag{4.3}$$

$$\partial_M^{(2)}A_{\mu v} = \partial_\mu\Lambda_v - \partial_v\Lambda_\mu \tag{4.4}$$

$$\Lambda_{12,\mu} = -\tfrac{1}{2}\sqrt{2}\xi_\mu{}^{3/4} - \xi^v A_{v\mu} \tag{4.5}$$

The complete gauge commutator for $N = 1, d = 10$ supergravity reads

$$[\partial_Q(\varepsilon_1), \partial_Q(\varepsilon_2)] = \partial_{gc}(\xi^\mu) + \partial_Q(\varepsilon_{12}\ in\ (4.3))$$

$$+\partial_L\left(\lambda_{12,mn}\ in\ (4.2)\right) + \partial_M^{(2)}\left(\Lambda_{12,\mu}\ in\ (4.5)\right) \tag{4.6}$$

Comparing the $d = 11$ case (Lorentz parameter);

$$\Lambda_{12,\mu v}(d = 11) = -\tfrac{1}{24}\sqrt{2}\bar{\varepsilon}_2\Gamma_{\mu v}\varepsilon_1 - \xi^\sigma A_{\sigma\mu v} \tag{4.7}$$

$$\lambda^{mn}(d = 11) = \xi^\mu\hat{\Omega}_\mu^{mn} + \tfrac{1}{8\times 36}\sqrt{2}\bar{\varepsilon}_2\left(\Gamma^{mn\alpha\beta\gamma\delta} + 24e^{m\alpha}e^{n\beta}\Gamma^{\gamma\delta}\right)\varepsilon_1\hat{F}_{\alpha\beta\gamma\delta} \tag{4.8}$$

$$\hat{\Omega}_\mu^{mn} = \hat{\omega}_\mu^{mn}(e,\psi) - \tfrac{1}{8}\left(e_\mu^m e^{nv} - e_\mu^n e^{mv}\right)\hat{D}_v\phi/\phi$$

$$-\tfrac{1}{288}\bar{\lambda}\Gamma_\mu^{mn}\lambda + \tfrac{1}{24}\sqrt{2}\bar{\psi}_\mu\Gamma^{mn}\lambda \tag{4.9}$$

Next, we turn to the gauge algebra by adding the following matter contributions;

$$\partial A_{\mu v}(matter) = \tfrac{1}{2}k\phi^{3/8}\bar{\varepsilon}\Gamma_{[\mu}\chi A_{v]} \tag{4.10}$$

$$\partial\lambda(matter) = \tfrac{1}{12\times 36}\sqrt{2}k(\bar{\chi}\Gamma^{\alpha\beta\gamma}\chi)\Gamma_{\alpha\beta\gamma}\varepsilon \tag{4.11}$$

$$\partial\psi_\mu(matter) = -\tfrac{1}{256}k(\bar{\chi}\Gamma^{\alpha\beta\gamma}\chi)(\Gamma_{\mu\alpha\beta\gamma} - 5g_{\mu\alpha}\Gamma_{\beta\gamma})\varepsilon \tag{4.12}$$

$$\partial A_\mu = \tfrac{1}{2}k\phi^{3/8}\bar{\varepsilon}\Gamma_\mu\chi \tag{4.13}$$

$$\partial\chi = \frac{1}{4}\phi^{-3/8}\Gamma.\hat{F}\varepsilon + \frac{1}{64}\sqrt{2}k(3(\bar{\lambda}\chi)\varepsilon$$

$$-\tfrac{3}{2}(\bar{\lambda}\Gamma^{\alpha\beta}\chi)\Gamma_{\alpha\beta}\varepsilon - \tfrac{1}{24}(\bar{\lambda}\Gamma^{\alpha\beta\gamma\delta}\chi)\Gamma_{\alpha\beta\gamma\delta}\varepsilon) \tag{4.14}$$

$$\therefore [\partial_Q(\varepsilon_1), \partial_Q(\varepsilon_2)]A_{\mu v} = e_\mu^m + \tfrac{1}{2}\sqrt{2}k\Lambda_{12}F_{\mu v} \tag{4.15}$$

Note that $\Lambda_{12} = -\xi^\mu A_\mu$.

Now,

$$\left[\partial_Q(\varepsilon_1),\partial_Q(\varepsilon_2)\right]=\partial_{gc}(\xi^\mu)+\partial_Q\left(-\xi^\mu\psi_\mu\right)+\partial_L(\xi.\widehat{\omega}^{mn})$$

$$+\,\partial_Q\left(\tfrac{1}{96\times160}\sqrt{2}(\bar{\varepsilon}_2\Gamma^{(5)}\varepsilon_1)\Gamma^{(5)}\lambda-\tfrac{7}{32}\sqrt{2}\xi^\alpha\Gamma_\alpha\lambda\right)$$

$$+\partial_M^{(1)}\left(-\xi^\mu A_\mu\right)+\partial_M^{(2)}\left(-\tfrac{1}{2}\sqrt{2}\phi^{3/4}\xi_\mu-\xi^v A_{v\mu}\right)$$

$$\partial_L\begin{Bmatrix}\bar{\varepsilon}_2\Gamma^{mn\alpha\beta\gamma}\varepsilon_1\left(\frac{1}{32}\sqrt{2}\phi^{-3/4}\hat{F}'_{\alpha\beta\gamma}-\frac{1}{16\times32}\left(\bar{\lambda}\Gamma_{\alpha\beta\gamma}\lambda+2\bar{\chi}\Gamma_{\alpha\beta\gamma}\chi\right)\right)\\ +\xi^\mu\left(\frac{1}{32}\sqrt{2}\phi^{-3/4}\hat{F}'_{mn\mu}-\frac{5}{8\times32}\left(\bar{\lambda}\Gamma_{mn\mu}\lambda+2\bar{\chi}\Gamma_{mn\mu}\chi\right)\right)\end{Bmatrix}\tag{4.16}$$

Note that $\partial_M^1(\Lambda)A_\mu=\partial_\mu\Lambda$ and $\partial_M^1 A_{\mu v}=\sqrt{\frac{1}{2}}k\Lambda F_{\mu v}$.

Remarks: We recovered the modified Maxwell transformation rules by comparing (4.15) with (3.22). Of course there remains the independent Maxwell transformation $\partial A_{\mu v}=\partial_\mu\Lambda_v-\partial_v\Lambda_\mu$ in the commutator, with A_μ given in (4.5). Both commutators of the two local supersymmetry transformations for Maxwell-Einstein system summarized in (4.16) vanish when the Maxwell and Einstein systems are decoupled, but one of them becomes non-vanishing in the presence of coupling:

$$\left[\partial_Q(\varepsilon),\partial_M^1(\Lambda)\right]=\partial_M^{(2)}\left(\tfrac{1}{2}\sqrt{2}k\Lambda\phi^{3/8}\bar{\varepsilon}\Gamma_\mu\chi\right),\quad\left[\partial_Q(\varepsilon),\partial_M^{(2)}\left(\Lambda_\mu\right)\right]=0.$$

5.0. The Multiplet of Currents and Noether Couplings for auxiliary Fields

The Maxwell theory contains the photon A_μ , and a Majorana spinor χ .

Theorem 4: The energy-momentum tensor $\theta_{\mu v}$ and the supersymmetry current J_μ field equations are satisfied by A_μ and χ .

$$\theta_{\mu v}=4F_{\mu\alpha}F_{v\alpha}-F^2\delta_{\mu v}+\bar{\chi}\left(\gamma_\mu\partial_v+\gamma_v\partial_\mu\right)\chi,$$

$$J_\mu=\tfrac{1}{2}\sigma.F\gamma_\mu\chi\ .\tag{5.1}$$

$$\partial J^\mu=-\tfrac{1}{2}\gamma^\lambda\varepsilon\theta_{\mu\lambda}-\tfrac{3}{8}i\gamma_5\left(\sigma_{\mu\lambda}\gamma_\rho+\tfrac{1}{3}\gamma_\rho\sigma_{\mu\lambda}\right)\partial_\lambda J_\rho^{(5)}\varepsilon\tag{5.2}$$

$$J_\mu^{(5)}=i\bar{\chi}\gamma_\mu\gamma_5\chi\tag{5.3}$$

$$\partial J_\mu^{(5)}=-2i\bar{\varepsilon}\gamma_5 J_\mu\tag{5.4}$$

Final Remarks:

The multiplet of currents contains only three currents: $\theta_{\mu v}$, J_μ and $J_\mu^{(5)}$. However, the transformation rules of its fields $(A_\mu\,,\chi)$ are again Weyl covariant, whether or not the coupling has taking place. It may be noted that the current of the $d=10$ Maxwell system only couple to a subset of the supergravity fields. Therefore one expects the extra local symmetries at present in the coupling of the Maxwell system to supergravity. The Weyl invariance announced above indicates that these extra symmetries have conformal echos.

Conclusion

There are two axial vector auxiliary fields in ten-dimensional supergravity. It will be difficult if not impossible to write down an action for this theory which includes the complete set of auxiliary fields. This point appears most clearly in the fact that the extent of the multiplet of currents implies the existence of fields with too low a dimension to appear in a convectional action. It follows that a new rescaling is needed $(d=10\geq N)$. The higher the values, the better the fields description.

References

[1] P. Van Nieuwenhuizen, Phys. Report **68**(1981)189, sects. (1.9), (1.11), (6.2), (6.5)
[2] F. A Berends, J. W Van Holten, B. de wit and P. Van Nieuwenhuizen, J. Phys. **A13** (1980) 1643
[3] S. Ferrara and P. Van Nieuwenhuizen, Phys. Lett. **74B** (1978) 333

[4] M. Kaku, P.K Townsend and P. Van Nieuwenhuizen, Phys. Rev. **D17** (1978) 3179
[5] E. S Frank and M. A Vasiliev, Nuovo Cim. Lett. 25 (1979); Phys. Lett. **85B** (1979) 47

[6] E. Bergshoeff, M. de Roo and B. de Wit, Nucl. Phys. **B182** (1981)173

[7] W. Nahm, Nucl. Phys. **B135** (1978) 149

[8] E. Cremmer, B. Julia and J. Scherk, Phys Latt. **78B** (1978) 409

[9] L. Brink, J. Scherk and J. H. Scharz, Nucl. Phys. **B121** (1977) 77

[10] S. Ferrara, F. Gliozzi, J. Scherk and P. Van Nieuwenhuizen, Nucl. Phys. **B117** (1976) 333

[11] S. Ferrara, and B. Zumino, Nucl. Phys. **B87** (1975) 207

[12] P. Howe, K.S. Stelle and P.K Townsend, Nucl. Phys. **B192** (1981) 332

[13] M. Kaku and P.K Townsend, Phys. Lett. **76B** (1978) 54

[14] B. de wit, J.W Van Holten and Van Proeyen, Nucl. Phys. **B184** (1981) 77

[15] P. Howe and U. Lindstrom, Phys. Lett. **103B** (1981) 422

[16] J. H Scharz, Nucl. Phys. **B185** (1981) 221; B. S de Wit and P. Van Nieuwenhuizen, ITP-SB- 81-16

[17] W. Siegel, Phys. Lett. **93B** (1980) 170; D. Z Freedman and P. K Townsend, Nucl. Phys. **B177** (1981) 282

[18] H. Nicolai and P. K Townsend, Phys. Lett. **98B** (1981) 257

[19] A. H Chamsedine, Nucl. Phys. **B185** (1981) 403

[20] S. Ferrara, J. Scherk and P. Van Nieuwenhuizen, Phys. Rev. Lett. **37** (1976) 1035

[21] B.E.W Nilson, Nucl. Phys. **B188** (1981) 176; D. Z Freedman, Phys. Rev. **D15** (1977) 1007; B. de Wit and P. Van Nieuwenhuizen, Nucl. Phys. Rev. **B139** (1978) 216