Journal of the Nigerian Association of Mathematical Physics
***Volume* 25 (November, 2013), pp 467 – 470**
© J. of NAMP

On a Modified Variational Iteration Method for the Analytical Solution of Korteweg-de-Vries Equation

Olayiwola M. O.

Department of Mathematical & Physical Sciences
Faculty of Basic and Applied Sciences
College of Science, Engineering & Technology
Osun State University, Osogbo, Nigeria.

Abstract

In this paper, a new analytical technique known as Modified Variational Iteration Method (MVIM) for the solution of Korteweg-de-Vries equation is presented. Numerical examples are tested to illustrate the efficiency, reliability and pertinent feature of the proposed method.

***Keywords*:** Korteweg-de-Vries, modified variational iteration method, lagrange multiplier, Taylor's series, partial differential equation.

1.0 Introduction

The main objective of this paper is to present a numerical solution of Korteweg-De Vries equation (KdV);

$$u_t(x,t) + au(x,t)u_x(x,t) + u_{xxx}(x,t) = 0 \tag{1.0}$$

Subject to the initial condition

$$u(x,0) = f(x) \tag{1.1}$$

The KdV equation arises in the study of shallow water waves. In particular, the KdV equation is used to describe long waves travelling in canals. KdV equation satisfied the property that the nonlinear term uu_x and the dispersion u_{xxx} balance each other thereby generating wave solutions which propagate maintaining same form throughout. The term soliton was coined by Zabusky and Kruskal to describe this solitary wave, solution of the KdV equation [1,2,3].

The development of numerical techniques for obtaining approximate solutions of partial differential equations has very much increased in the last decades. Recently, the Wavelet-Petrov-Galerkin was employed by Jairo Villegas and Jorge [4] to find the solution of KdV equation

The variational iteration method was proposed by J.H He [5,6]. In this paper a Modified Variational Iteration Method proposed by Olayiwola [7-9] to the solution of nonlinear KdV equation is presented.

2.0 Modified Variational Iteration Method (MVIM)

To illustrate the basic concept of the MVIM, we consider the following general nonlinear partial differential equation:

$$Lu(x,t) + Ru(x,t) + Nu(x,t) = g(x,t) \tag{1.2}$$

where L is a linear time derivative operator, R is a linear operator which has partial derivative with respect to x, N is a nonlinear operator and g is an inhomogeneous term. According to MVIM, we can construct a correct functional as follows:

$$u_0(x,t) = u(x,0) + g_i(x)t^i \tag{1.3}$$

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda\left[Lu_n + R\tilde{u}_n + N\tilde{u}_n - g\right]d\tau \tag{1.4}$$

Corresponding author: ***Olayiwola M. O.,*** E-mail: Olayiwola.oyedunsi@uniosun.edu.ng, Tel.: +2348055218234

where $g_i(x)$ can be evaluated by substituting $u(x,0)$ in (1.3) and at limit $t = 0, i \leq 2$. .

λ is a Lagrange multiplier which can be identified optimally via variational iteration method. The subscript n denote the nth approximation, $\tilde{u}_n$ is considered as a restricted variation i.e, $\delta\tilde{u}_n = 0$.

3.0 MVIM for the Solution of KdV Equation

The analytical solution of the equation of the form (1.0) is presented.

$$u_t(x,t) + au(x,t)u_x(x,t) + u_{xxx}(x,t) = 0 \tag{1.5}$$

Applying MVIM in (1.5)

$$u_{n+1} = u_n + \int_0^t (u_{n,\tau}(x,\tau) + au_n(x.\tau)\, u_{n,x}(x,\tau) + u_{n,xxx}(x,\tau))\,d\tau \tag{1.6}$$

Making (1.6) stationary, this yields the following stationary condition

$$u_{n+1} = u_n + u\lambda - \tilde{u}\lambda' + \int \tilde{u}\lambda'' d\tau \tag{1.7}$$

$$\frac{\partial u_{n+1}}{\partial u_n} = 1 + \lambda\big|_{\tau=t} = 0 \tag{1.8}$$

$$\frac{\partial u_{n+1}}{\partial \tilde{u}_n} = -\lambda\big|_{\tau=t} \tag{1.9}$$

Solving (1.8-1.9) yields

$$\lambda(t,\tau) = -1 \tag{2.0}$$

4.0 Numerical Examples

Example 1:- Consider nonlinear KdV equation $u_t(x,t) + 6uu_x + u_{xxx} = 0$, $u(x,0) = x$.

Applying (1.3), (1.4), (1.7) we obtain

$$g_1 = x - 6xt \tag{2.1}$$

$$u_1 = u_0 + \left(\int_0^t -(u_{0,\tau}(x,\tau) + 6u_0(x,\tau)u_{n,x}(x,\tau) + u_{0,xxx}(x,\tau))\,d\tau \right) \tag{2.2}$$

$$u_1 = x - 6xt + 36xt^2 - 216xt^3 \tag{2.3}$$

$$u_2 = x - 6xt + 36xt^2 - 216xt^3 + 1296xt^4 \tag{2.4}$$

.
.
.

$$u_n = x(1 - 6t + 36t^2 - 216t^3 + 1296t^4 + ... + ..) = \frac{x}{1+6t} \tag{2.5}$$

Example 2:- Consider $u_t(x,t) - 6uu_x + u_{xxx} = 0$, $u(x,0) = \frac{2}{x^2}$.

Applying (1.3), (1.4), (1.7) we have

$$g_1 = 0 \tag{2.6}$$

$$u_1 = u_0 + \left(\int_0^t -(u_{0,\tau}(x,\tau) - 6u_0(x,\tau)u_{0,x}(x,\tau) + u_{0,xxx}(x,\tau))\,d\tau \right) \tag{2.7}$$

$$u_1 = \frac{2}{x^2} \tag{2.8}$$

$$u_2 = \frac{2}{x^2} \tag{2.9}$$

.
.
.

$$u_n = \frac{2}{x^2} \tag{3.0}$$

MVIM admits that

$$u = \lim_{n \to \alpha} u_n \tag{3.1}$$

Which gives the exact solution

$$u = \frac{2}{x^2} \tag{3.2}$$

Example 3:- Consider $u_t(x,t) - 6uu_x + u_{xxx} = 0$, $u(x,0) = \frac{x-2}{12}$.

$$g_1 = \frac{x-2}{24} \tag{3.3}$$

$$u_1 = u_0 + \left(\int_0^t -(u_{0,\tau}(x,\tau) - 6u_0(x,\tau)u_{0,x}(x,\tau) + u_{0,xxx}(x,\tau))d\tau \right) \tag{3.4}$$

$$u_1 = \frac{x}{12} - \frac{1}{6} + (\frac{x}{24} - \frac{1}{12})t + (\frac{x}{48} - \frac{1}{24})t^2 + (\frac{x}{96} - \frac{1}{48})t^3 \tag{3.5}$$

$$u_2 = \frac{x}{12} - \frac{1}{6} + (\frac{x}{24} - \frac{1}{12})t + (\frac{x}{48} - \frac{1}{24})t^2 + (\frac{x}{96} - \frac{1}{48})t^3 + (\frac{x}{192} - \frac{1}{96})t^4 \tag{3.6}$$

$$u_n = \frac{1}{6}(x-2)(2-t)^{-1} \tag{3.7}$$

MVIM admits that

$$u = \lim_{n \to \alpha} u_n \tag{3.8}$$

Which gives the exact solution

$$u = \frac{1}{6}\frac{(x-2)}{(2-t)} \tag{3.9}$$

Numerical Example 4:- Consider $u_t(x,t) - 6uu_x + u_{xx} = 0$, $u(x,0) = \frac{x-4}{18}$.

Using the method, we obtain

$$g_1 = \frac{x-4}{54} \tag{4.0}$$

$$u_1 = u_0 + \left(\int_0^t -(u_{0,\tau}(x,\tau) - 6u_0(x,\tau)u_{0,x}(x,\tau) + u_{0,xxx}(x,\tau))d\tau \right) \tag{4.1}$$

$$u_1 = \frac{x}{18} - \frac{2}{9} + (\frac{x}{54} - \frac{2}{27})t + (\frac{x}{162} - \frac{2}{81})t^2 + (\frac{x}{486} - \frac{2}{243})t^3 \tag{4.2}$$

This converges to

$$u = \frac{1}{6}\frac{(x-4)}{(3-t)} \tag{4.3}$$

4.0 Conclusion

We have successfully applied the Modified Variational Iteration method to some nonlinear partial differential equations of KdV class. It is observed that the method produces the analytical solution rapidly. The method is elegant and can be further applied to the general KdV equation.

References

[1]. A.Biswas and S.Konar. (2007): Soliton perturbation theory fort the compound KdVequation .International Journal of Theoretical Physics,Vol. 46, No.2, 237-243.

[2]. F.G.Drazin, R.S.Johnson.(1989) :Solitons: A Introduction. Second Edition, Cambridge University Press, NewYork.

[3]. L.Debnath. Nonlinear Partial Differential Equations for Scientists and Engineers.Second Edition, Boston 2005.Publiser: Springer

[4]. Jairo Villegas G and Jorge C.B. (2012): Wavelet-Petrov-Galerkin Method for the Numerical Solution of the KdV Equation. Applied Mathematical Sciences,Vol.6,,no.69, 3411-3423

[5]. Ji-Huan He (1999): Variational Iteration method: a kind of non-linear analytical technique: Some example. Int. Journal of Non-linear mechanics, vol. 3494 ,pp699-708.

[6]. He. J.H (2000) : Variational iteration method for autonomous ordinary differential system. App. Maths and Computation, vol.114, pp115-123.

[7]. M.O. Olayiwola, A .W. Gbolagade , A .O .Adesanya (2010): An Efficient Algorithm for Solving the Telegraph Equation. Journal of the Nigerian Association of Mathematical Physics, vol. 16, pp199-204.

[8]. M.O. Olayiwola, A .W. Gbolagade and A .O .Adesanya (2010): Solving Variable Coefficient Fourth-Order Parabolic Equation by Modified initial guess Variational Iteration Method. Journal of the Nigerian Association of Mathematical Physics Vol. 16,pp 205-210

[9]. M.O. Olayiwola, A .W. Gbolagade , F. O Akinpelu (2011) : An Efficient Algorithm for Solving the Nonlinear PDE, International Journal of Scientific and Engineering Research, vol. 2. issue 10, pp1-10.