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Application of Softmultiset in Decision Making problems

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Abstract

The theories of softmultisets are important mathematical tools to handle uncertainties about vague concepts. In this paper we present basically the application of softmultiset theory in decision making problems with the help of the redefined softmultiset that maps the set of parameters to the power multiset and not to power whole multiset.

***Keywords*:** Softmultiset, Reduct-Softmultiset, Choice value, Multivalued Information System, tabular representation.

1.0 Introduction

Many complicated problems arising in a wide range of different disciplines such as engineering, economics, environment, medical sciences, social sciences, etc, involve data which are not always all crisp [1 - 15]. We cannot always use the classical methods because of various types of uncertainties present in these problems. Although a number of mathematical tools like probability theory, Fuzzy set ,rough sets and interval mathematics are well known and effective tools for dealing with uncertainties, each of them has distinguished advantages as well as certain inherent limitations. One major weakness shared by these theories is possibly the inadequacy of the parameterization tools as pointed out by Molodtsov[1].

Some theoretical study has been made on "Softmultiset theory"and its application in decision making problems, however, Roy and Maji [2] presented an application based on their definition of softmultiset, which restricted the mapping from the set of parameters to the powerwhole multiset. But in this paper, the decision is taken based on the new definition that maps the set of parameters to the power multiset, with the help of rough mathematics of Pawlak [3].

2.0 Preliminaries

In this section, we point out some deficiencies on the existing studies on softmultisets and present the redefined softmultiset that completely captures the notion of softset and without restriction on the mapping of the set of parameter.

Definition 1.3 Softmultiset [12]

Let $\{U_i{:}i \in I\}$ be a collection of universes such that $\cap\, U_i = \emptyset$ and let $\{EU_i{:}i \in I\}$ be a collection of sets of parameters. Let $U = \Pi_{i\in l}\, P\,(U_i)$where $P(U_i)$ denotes the power set of U_i,$E = \Pi_{i\in l} EU_i$ and $A \subseteq E$. Then a pair (F, A) is called a softmutiset over U, where F is a mapping given by$F{:}\, A \longrightarrow U$.

Remark 1

If $\cap\, U_i = \emptyset$, then $\cup\, U_i$ is just an ordinary set, hence the concept of multiset does not exist. Hence the remaining exercise in [11] collapse into soft set.

Definition 1.4 soft multisets [6]

Let U be universal mset and E be set of parameters. Then an ordered pair (F,E) is called a soft multiset where F is a mapping given by $F{:}\, A \longrightarrow PW$(U)

Remark 2

Because of the restriction on the submultiset, some of the submultisets are ignored hence the definition does not completely capture the notion of soft set.

Definition 2.1 Softmultiset

Let $\{U_i{:}\, i \in I\}$be a collection of universes such that there exists $U_j$and U_k and$U_j \cap U_k \neq \emptyset$. $Let\ U = \uplus_{i\in I}$P(U$_i$) where P(U$_i$) denotes the power set of U_i, and E be a set of parameters. Then a pair (F, A),where$A \subseteq E$, is called a soft multiset over U, where F is a mapping given by $F{:}\, A \rightarrow U$.

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That is, a softmultisetover U is a parameterized family of submultisets of U such that for $e \in A, F(e)$ is considered as the set of e- approximate element of the soft multiset (F, A).

As an illustration, let us consider the following: A Softmultiset (F, A) describes the condition of some states in a country that Mr. X with enough capital is considering for the location of his manufacturing industries.

Suppose

$U_1 = \{S_1, S_2, S_3\}$be a set of states with availability of land,

$U_2 = \{S_2, S_4, , S_6,\}$be a set of states with availability of labour

$U_3 = \{S_2, S_4, S_7,\}$be a set of state with availability of raw materials.

$$U = P(U_1) \uplus P(U_2) \uplus P(U_3) = \left\{ \begin{matrix} \{S_1\},\{S_2\},\{S_3\},\{S_1,S_2\},\{S_1,S_3\},\{S_2,S_3\},\{S_1,S_2,S_3\},\emptyset,\{S_2\},\{S_4\},\{S_6\},\{S_2,S_4\},\{S_2,S_6\},\{S_4,S_6\}, \\ \{S_2,S_4,S_6\},\emptyset,\{S_2\},\{S_4\},\{S_7\},\{S_2,S_4\},\{S_2,S_7\},\{S_4,S_7\},\{S_2,S_4,S_7\},\emptyset, \end{matrix} \right\}$$

$$= \left\{ \begin{matrix} \{S_1\},3\{S_2\},\{S_3\},2\{S_4\},3\emptyset,\{S_6\},\{S_7\},\{S_1,S_2\},\{S_1,S_3\},\{S_2,S_3\},2\{S_2,S_4\},\{S_1,S_2,S_3\},\{S_2,S_6\},\{S_4,S_6\}, \\ \{S_2,S_4,S_6\},\{S_2,S_7\},\{S_4,S_7\},\{S_2,S_4,S_7\} \end{matrix} \right\}$$

and this is clearly a multiset.

Let E be a set of decision parameters related to the above universes, where

E = {e_1 = peaceful, e_2 = kidnapping, e_3 = Armed robbery, e_4= Accessibility, e_5 = market}.

Let A = {e_1 = peaceful, e_2 = kidnapping, e_3 = Armed robbery, e_4 = Accessibility}.

The Softmultiset (F, A) is a parametrized family $\{F(e_i), i = 1, 2, \dots, 4\}$of subsets of the set U and gives us a collection of approximate description of the conditions of some states in a country favourable to Mr. X for the location of his manufacturing industries.

In this case, to define a softmultiset means to point out the peaceful states, Kidnapping states, armed robbery, Accessible states, states of market. It is worth noting that the set $F(e)$ may be empty for some $e \in E$.

3.0 Softmultiset theory as multi-value information system

Molodtsov [1], presented some applications of soft set theory in several directions, which includes: the study of smoothness of functions, game theory, operations research, Riemann-integration, perron integration, probability, theory of measurement, etc. It has been shown that there is compact connection between softsets and information system. From the concept and the example of soft multisets given in the foregoing section, it can be seen that a soft multi set is a multi-valued information system.

Definition. A multi-valued information system is a quadruple $I = (X, A, f, V)$ where X is a non empty finite set of objects, A is a non empty finite set of attribute, $V = \cup_{a \in A} V_a$ where V is the domain (value set) set of attribute a which has multi value $(/Va/ \geq 3)$ and f: $U \times A \twoheadrightarrow V$ is a total function such that f $(U, a) \in V_a$ for every (U, a) $\in X \times A$.

Proposition. If (F, A) is a softmultiset over universe U then (F, A) is a multi-value information system.

Proof: Let (F, A) be a softmultiset over U. We define a mapping f where

$f: U \times A \twoheadrightarrow V \ as \ f(U, a) = C_{F(a)}(U) \ where \ C$ is the count of element U in the multiset F(a).

Hence, $V = U_{a \in A} V_a \ where \ V_a$ is the set of all counts of U in $F(a)$.Then the multi-valued information system(U, A, f, V)represents the softmultiset (F, A)

Recall the case of Mr. X with enough capital who is considering some states for the location of his manufacturing industries.

Suppose

$U_1 = \{S_1, S_2, S_3\}$be a set of states with availability of land,

$U_2 = \{S_2, S_4, , S_6,\}$be a set of states with availability of labour

$U_3 = \{S_2, S_4, S_7,\}$be a set of state with availability of raw materials.

$$U = P(U_1) \uplus P(U_2) \uplus P(U_3) = \left\{ \begin{matrix} \{S_1\},\{S_2\},\{S_3\},\{S_1,S_2\},\{S_1,S_3\},\{S_2,S_3\},\{S_1,S_2,S_3\},\emptyset,\{S_2\},\{S_4\},\{S_6\},\{S_2,S_4\},\{S_2,S_6\},\{S_4,S_6\}, \\ \{S_2,S_4,S_6\},\emptyset,\{S_2\},\{S_4\},\{S_7\},\{S_2,S_4\},\{S_2,S_7\},\{S_4,S_7\},\{S_2,S_4,S_7\},\emptyset, \end{matrix} \right\}$$

Let $A = \{Peaceful, Accessible, Market\}$

Then$F\ (Peaceful)$=$\left\{ {}^{C_2}/_{S_2}, {}^{C_4}/_{S_4}, {}^{C_6}/_{S_6} \right\}$.

$F\ (Accessible) = \left\{ {}^{C_1}/_{S_1}, {}^{C_3}/_{S_3} \right\}$.

$$F\ (Market)\ = \left\{ {}^{C_2}/_{S_2}, {}^{C_4}/_{S_4} \right\}$$

Then the softmultiset defined above describes the conditions of some states in a country. Then the quadruple I= (X, A, f, V) corresponding to the softmultiset given above is a multivalue information system.

Where $X = U$ and A is the set of parameters in the softmultiset and $V_{peaceful} = \{C_2, C_4, C_6\}$, $V_{accessible} = \{C_1, C_3\}$ and $V_{market} = \{C_2, C_4\}$. For the pair ($S_2$, Peaceful) we have f($S_2$, peaceful) = C_2, for (S_2, market), we have f(S_2,, market) = C_2,. Continuing in this way we obtain the values of other pairs. Therefore, according to the result above, it is seen that soft multisets are multi-valued information systems. Nevertheless, it is obvious that multi-valued information systems are not necessarily soft multisets.

We can construct an information table representing softmultiset (F, A) defined above as follows

Table 1

'States'	*'Peaceful'*	*'Accessible'*	*'Market'*
S_1	C_1	C_1	0
S_2	0	0	C_2
S_3	C_3	0	0
S_4	0	0	C_4
S_5	0	C_5	0
S_6	C_6	0	0

4.0 Application of softmultisets theory in Decision Making Problems

The problem of decision making in an imprecise environment has been found very important in the recent years. Maji*et al [11]* introduce the definition of reduct-softset and describe the use of softset theory to a decision-making problem using rough set approach. Babitha and Sunil [6] extended the Maji*et al*[11] approach to softmultiset theory with restriction on the mapping of the set of the parameters. In this section we present our decision making based on the redefined softmultiset that does not restrict the mapping of the set of parameters.

Consider the problem, for Mr. X with enough capital who wants to make a choice of a state for the location of his manufacturing Industry.

Suppose,

$U_1 = \{S_1, S_2, S_3\}$be a set of states with availability of land,

$U_2 = \{S_2, S_4, , S_6,\}$be a set of states with availability of labour

$U_3 = \{S_2, S_4, S_7,\}$be a set of state with availability of raw materials.

U=

$$P(U_1) \uplus P(U_2) \uplus P(U_3) = \left\{ \begin{matrix} \{S_1\}, \{S_2\}, \{S_3\}, \{S_1, S_2\}, \{S_1, S_3\}, \{S_2, S_3\}, \{S_1, S_2, S_3\}, \emptyset, \{S_2\}, \{S_4\}, \{S_6\}, \{S_2, S_4\}, \{S_2, S_6\}, \{S_4, S_6\}, \\ \{S_2, S_4, S_6\}, \emptyset, \{S_2\}, \{S_4\}, \{S_7\}, \{S_2, S_4\}, \{S_2, S_7\}, \{S_4, S_7\}, \{S_2, S_4, S_7\}, \emptyset, \end{matrix} \right\}$$

$$= \left\{ \begin{matrix} \{S_1\}, 3\{S_2\}, \{S_3\}, 2\{S_4\}, 3\emptyset, \{S_6\}, \{S_7\}, \{S_1, S_2\}, \{S_1, S_3\}, \{S_2, S_3\}, 2\{S_2, S_4\}, \{S_1, S_2, S_3\}, \{S_2, S_6\}, \{S_4, S_6\}, \\ \{S_2, S_4, S_6\}, \{S_2, S_7\}, \{S_4, S_7\}, \{S_2, S_4, S_7\} \end{matrix} \right\}$$

A = {Peaceful, Accessible, Market, raw materials} be a set of Parameters. Consider a soft multiset describing the conditions of some states under consideration and is given by

F (Peaceful) = { S_2, S_4, S_6 }

F (Accessible) = {$2S_2$, $2S_4$}

F (Market) = { S_2, S_6 }

Suppose that Mr. X would like to choose a state according to the choice of parameters

B= {Peaceful, Accessible, Market}. To solve the problem, we consider the following theoretical characterizations of softset theory.

Definition.Reduct Softmultiset. If (F, A) is a softmultiset andlet$B \subseteq A$. If B is a reduct of A, then the softmultiset (F, B) is called the reduct softmultiset of the softmultiset (F, A).

Definition. If (F, A) is a softmultiset over U, the weighted choice value of an object $u_i \in U is$ given $by\ c_i = \sum j \times d_{ij}$ where$d_{ij} = w_i \times u_{ij}$. Here w_i is weights imposed on to the parameters by the entrepreneur Mr. X and u_{ij} is the count of u_i in $F(e_j)$.

The revised algorithms for the choice of states are:

Step1: Input the softmultiset (F, A)

Step 2: Input the set B of choice parameters of Mr. X which is a subset of A

Step 3: Find all reduct soft multiset of (F, B)
Step 4: Choose reduct softmultiset suppose (F, Q) of (F, B)
Step 5: Find weighted table of the softmultiset (F, Q) according to the weights decided by Mr. X
Step 6: Find K, for whichC_k, = $\max_i C_i$
To solve the given problem using the above revised algorithm. The reduct of B could be B itself.
Suppose Mr. X put the weights as follows: $Peaceful = 0.5$, $Accessible = 0.9$ and Market $= 0.6$

Table 2

'States'	***'Peaceful'***	***'Accessible'***	***'Market'***	***Choice value***
S_1	0	0	0	0
S_2	1	2	1	2.9
S_3	0	0	0	0
S_4	1	2	0	2.3
S_5	0	0	0	0
S_6	1	0	1	1.1
S_7	0	0	0	0

From the weighted Table 2, the maximum choice value is 2.9 and therefore, Mr. X can choose state S_2 in the country. If more than one state is needed, the next choice is S_4, with a choice value of 2.3.

5.0 Conclusion

The softset theory proposed by Molodtsov offers a general mathematical tool for dealing with uncertain, fuzzy, or vague objects. Molodtsov [1] has presented several possible applications of softset theory. In this present paper, we give an application of softmultiset theory in a decision making problem by using the rough mathematics of Pawlak [14].

The algorithm and the procedure for making the choice is similar to that of [6]. But in this paper the restriction to only power whole multiset is striped off. And this allows one to have advantages of making a lot of choices.

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