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On Improved Portfolio Optimization: Alternative Approaches To Covariance Estimation

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Abstract

Forward looking information is employed in this paper to develop the first, second and third estimators for the covariance of the market returns.

In the performances of the three approaches, it is discovered that the performances show the clear picture of what value the covariance of the market return would have when certain conditions are put into consideration. The introduced constant p has value 0 throughout while constant k has value that ranges from-1 to 1. The three proposed values of k together with the constant value of P give difference values for covariance of market return.

The paper explains further the effect of the value of k at -1 on the covariance of the market return.

***Keywords*:** Optimal portfolio, systematic risk, covariance return, idiosyncratic variance, idiosyncratic error.

1.0 Introduction

In Finance [1 - 17], the most prominent problem is in selecting an optimal portfolio that is, selecting a set of share or investment that would produce or yield the best result in term of dividend. The establishment of the main-variance investor was well identified [13] of which the implementation remain a great task. With this more attention are concentrating on a global minimum variance portfolio (GMVP) which seems to be the only efficient portfolio that does not depend on expected returns. This global minimum variance portfolio (GMVP) gives a better out- of- sample performance than a mean-variance, optimized portfolio [11, 12].

This paper is used to show the true picture of the estimation covariance matrix especially when certain constraints are put in to consideration.

2.0 Implied Covariance Estimator

To determine implied covariance estimators, it is firstly assumed that asset returns follow a generalized version of market model [14] with time variance coefficient and the introduction of constant p and K to determine the estimated values. That is,

$$R_{(i+p)t} = \alpha_{(i+p)t} + k\beta_{(i+p)t}\, R_{(m+p)t} \qquad (1)$$

$$\forall\ \mathrm{i} = 1,2,\ldots\mathrm{N},$$
$$k = -1,\ 0,\ 1$$
$$p = 0$$

Where,

$R_{(i+p)t}$ = return of the $(\mathrm{i}+\mathrm{p})^{\mathrm{th}}$ asset

$R_{(m+p)t}$ = return of the $(\mathrm{m}+\mathrm{p})^{\mathrm{th}}$ market

$C_{(i+p)t}$ = a zero mean idiosyncratic error with positive variance and is independent of market return.

$\alpha_{(i+p)t}$ and $k\beta_{(i+p)t}$ are model coefficients index with k as the deterministic value for the return of $R_{(i+P)t}$ asset.

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k ranges from -1 to 1, i.e $-1 < k < 1$ for all $k \in \mathrm{R}$ and k is rational just like p. It shows that the coefficient can be varied depending on the time. It is also observed that in the market model the return covariances depend only on the beta coefficients and the variances of the market return, while C(i+p)t and C(j+p)t are independent for all $i \neq j$

$$Cov(R_{(i+p)t}, R_{(j+p)t} = K\beta_{(i+p)t}\beta_{(j+P)t}\mathrm{Var}R_{(m+p)t} \quad \forall \ \mathrm{i} \neq \mathrm{j} \tag{2}$$

It should be noted that this proportion can vary over time under the assumption

$\beta^2{}_{(i+P)t}\mathrm{Var}(R_{(m+p)t}) = C_t \ \mathrm{Var}(R_{(i+p)t})$ and $\mathrm{Var}(C_{(i+p)t}) = (1-C_t)\mathrm{Var}(R_{(i+p)t})$ with $\theta \le \mathrm{C_t} < 1...$ which

implies that stock with a higher total variance have both a higher beta and a higher idiosyncratic variance [10, 11] with the guarantee of the cross-sectional restriction on the positive nature of the return $(i+p)^{th}$ asset equals to

$$\mathrm{Var}R_{(i+p)t} = \beta^2{}_{(i+P)t}\mathrm{Var}(R_{(m+p)t}) + (1-C_t)\mathrm{Var}R_{(i+p)t} \tag{3}$$

2.1 Nature of the Return $(i+p)^{th}$ asset

$$\beta^2{}_{(i+P)t}\mathrm{Var}(R_{(m+p)t}) = \mathrm{Var}(R_{(i+p)t} - \mathrm{Var}(R_{(i+p)t} + C_t\mathrm{Var}(R_{(i+p)t} + C_t\mathrm{Var}(R_{(i+p)t}$$

$$\beta^2{}_{(i+P)t} = \frac{C_t\mathrm{Var}(R_{(i+p)t}}{\mathrm{Var}(R_{(m+p)t})}$$

$$\beta(i+p)t = Ct^{1/2}\left[\frac{\mathrm{Var}(R_{(i+p)t}}{\mathrm{Var}(R_{(m+p)t}}\right] \tag{4}$$

To find the parameter Ct, use the weight $w_{t(i+p)(m+p)}$ of the different assets $i = 1...N$ in the market portfolio, that is

$$\sum_{i=1}^{N} w_{t(i+p)(m+p)}\beta_{(i+p)t} = \sum_{i=1}^{N} w_{t(i+p)(m+p)}Ct^{1/2}\left[\frac{\mathrm{Var}_{(\mathrm{var}\,(}R_{(i+p)t}}{\mathrm{Var}(R_{(m+p)t}}\right]^{1/2} = 1$$

$$\sum_{i=1}^{N} w_{t(i+p)(m+p)}Ct^{1/2}Var\left[\frac{R_{(i+p)t}}{\mathrm{Var}(R_{(m+p)t}}\right]^{1/2} = 1$$

$$Ct^{1/2} = \left[\frac{\mathrm{Var}(R_{(m+p)}{}^{1/2}}{w_{t(i+p)(m+p)}Var(R_{(i+p)t}}\right]$$

$$C_t = \frac{\mathrm{Var}(R_{(m+p)t}}{\sum_{i=1}^{N} w_{t(i+p)(m+p)}Var(R_{(i+p)t})^{1/2})^2} \qquad \mathrm{V\ i} \neq \mathrm{j} \tag{5}$$

2.2 Substitute $Ct^{1/2}$ in equation (4)

$$\beta_{(i+p)t} = \frac{Var(R_{(m+p)t^{1/2}}Var(R_{(i+p)t})^{1/2}}{\sum_{i=1}^{N} w_{t(i+p)(m+p)}Var(R_{(i+p)t})^{1/2})Var(R_{(m+p)t})^{1/2}}$$

$$= \frac{Var(R_{(i+p)t})^{1/2}}{\sum_{i=1}^{N} w_{t(i+p)(m+p)}Var(R_{(i+p)t})^{1/2}}$$

Using equation (2) and substituting for $\beta_{(i+p)t}$ we have

$$Cov(R_{(i+p).}R_{(j+p).}) = \frac{KVar(R_{(i+p)t})^{1/2}.Var(R_{(j+p)t}{}^{1/2}Var(R_{(m+p)t}}{\sum_{i=1}^{N} w_{t(i+p)(m+p)}Var(R_{(i+p)t})^{1/2})^2} \qquad \forall\, i \neq j \qquad (6)$$

2.3 Values of Market Covariance of the returns $R_{(i+p)}$ and $R_{(j+p)}$

For the purpose of this work we are interesting in finding the values of market covariance of the returns $R_{(i+p)}$ and $R_{(j+p)}$ market at which the value of p is zero and k attains the values -1 to 1, that is the range of k is $-1 \le k \le 1$ which implies that k can be -1, 0 and 1.

<u>Case One:</u>

When p = 0, and k = -1 we have

$$Cov(Rit, Rjt) = -1\left[\frac{Var.R_{(jt)}VarR_{mt}}{\sum_{i=1}^{N} w_{t(m)}}\right] = -\left[\frac{VarR_{jt}{}^{1/2}\ .\ VarR_{it}\ .\ VarR_{mt}}{\sum_{i=1}^{N} w(_{im})tVarR_{it}{}^{1/2})^2}\right]$$

This result shows a negative relationship between return Rit and Rjt, thus the covariance would be negative indicating deficit or loss.

<u>Case two:</u> When k has value 0 and p is also 0

$$Cov(Rit, Rjt) = 0\left[\frac{Var.R_{(jt)}VarR_{mt}}{\sum_{i=1}^{N} w_{tim}}\right] = 0\left[\frac{VarR_{jt}{}^{1/2}\ .\ VarR_{it}\ .\ VarR_{mt}}{\sum_{i=1}^{N} w_{tim}Var(R_{(it)}{}^{1/2})^2}\right]$$

i.e Cov (Rit, Rjt)=0

This result shows that there is no relationship between market returns Rit and Rjt. In this situation covariance would be zero which shows that there is no deficit, no surplus or no gain, no loss in the market returns. Thus we have a return at par.

<u>Case three:</u>

When p = 0 and k = 1

$$Cov(Rit, Rjt) = 1\left[\frac{Var(R_{(jt)}VarR_{mt}}{\sum_{i=1}^{N} w_{t(im)}}\right]$$

$$= \frac{Var(R_{(jt)}VarR_{mt}}{\sum_{i=1}^{N} w_{t(im)}}$$

The obtained result shows that there is a positive relationship between the two market returns R_{it} and R_{jt}, thus the Covariance will be positive which indicates excess of revenue over the capital invested.

3.0 Estimator Based on higher moments

Let consider estimator based on higher moments, i.e a skewness based estimator and a Kurtosis base estimator using the same frame work. This is done by replacing the assumption concerning the proportion of systematic variance with a corresponding assumption about how higher moments is being affected by systematic risk. The demonstration is shown on the third and fourth moments skewness based Estimator of covariance.

The assumption in this approach is that the proportion of systematic return skewness is equal for all assets.

This proportion is denoted by C_t^{skew} then the return skewness of $(i+p)^{th}$ asset is

$$Skew(R_{(i+p)t}) = \beta^3_{(i+p)t}\ Skew(R_{m+p)}) + (1 - C_t^{skew})\ Skew(R_{(j+p)t}) \tag{7}$$

$$Solving \text{ for } \beta_{(j+p)t}\text{, we have}$$

$$\beta^3_{(i+p)t}\ Skew(R_{m+p)t}) = Skew(R_{(i+p)t}) = -(1 - C_t^{skew})\ Skew(R_{(j+p)t})$$

$$\beta^3_{(i+p)t} = \frac{Skew(R_{(i+p)t}) - (1 - C_t^{skew})\ Skew(R_{(j+p)t})}{Skew(R_{(m+p)t})}$$

$$\beta^3_{(i+p)t} = \frac{\beta^3_{(i+p)t} = Skew(R_{(i+p)t}) - (1 - C_t^{skew})\ Skew(R_{(j+p)t})}{Skew(R_{(m+p)t})}$$

$$\beta^3_{(i+p)t} = C_t^{skew}\ \frac{Skew(R_{(j+p)t})}{Skew(R_{(m+p)t})}$$

$$\beta^3_{(i+p)t} = \left(C_t^{skew}\right)^{1/3} \left[\frac{Skew(R_{(j+p)t})}{Skew(R_{(m+p)t})}\right]^{1/3} \tag{8}$$

Notice the condition that says that the market beta is equals to one and by using the different asset weights $w_{t(i+p)(m+p)}$, i =1………. N in the different portfolio to solve for C_t^{skew} and substituting the result in equation (8) gives

$$\sum_{i=1}^{N} w_{t(i+p)(m+p)}\beta_{(i+p)t} = \sum_{i=1}^{N} w_{t(i+p)(m+p)}\beta_{(i+p)} + \left(C_t^{skew}\right)^{1/3} \left[\frac{Skew(R_{(j+p)t})}{Skew(R_{(m+p)t})}\right]^{1/3} = 1$$

$$\left(C_t^{skew}\right)^{1/3} = \frac{1}{\sum_{i=1}^{N} w_{t(i+p)(m+p)} \dfrac{(Skew(\mathrm{R}_{(j+p)t}))^{1/3}}{(Skew(\mathrm{R}_{(m+p)t}))^{1/3}}}$$

$$\left(C_t^{skew}\right)^{1/3} = \frac{(Skew(\mathrm{R}_{(m+p)t}))^{1/3}}{\sum_{i=1}^{N} w_{t(i+p)(m+p)} (Skew(\mathrm{R}_{(j+p)t}))^{1/3}}$$

From equation 8

$$\beta_{(i+p)t} = \frac{(Skew(\mathrm{R}_{(m+p)t}))^{1/3}}{\sum_{i=0}^{N} w_{t(i+p)(m+p)} (Skew(\mathrm{R}_{(j+p)t}))^{1/3}} \left[\frac{Skew(\mathrm{R}_{(j+p)t})}{Skew(\mathrm{R}_{(m+p)t})}\right]^{1/3}$$

3.1 Market Model.

In the market model, the return covariance depend only on the beta coefficients and the variance of the market return, i.e

$$Cov(R_{(i+p)t}, R_{(j+p)t.}) = K\beta_{(i+p)t} + \beta_{(i+p)} \mathrm{V}ar(R_{(m+p)t})$$

$\forall\, i \neq \mathrm{j}$

$$Cov(R_{(i+p)t.}R_{(j+p)t.}) = K\left[\frac{(Skew(\mathrm{R}_{(m+p)t}))^{1/3}\, Skew(\mathrm{R}_{(i+p)t})\, {}^{1}\!/_{3}\, \mathrm{V}ar(R_{(m+p)t})}{\sum_{i=1}^{N} w_{t(i+p)(m+p)} Skew(\mathrm{R}_{(i+p)t})^{1/3}\, Skew(\mathrm{R}_{(m+p)t})^{1/3}}\right]$$

$$Cov(R_{(i+p)t.}R_{(j+p)t.}) = K\left[\frac{(Skew(\mathrm{R}_{(i+p)t})\, {}^{1}\!/_{3}\, Skew(\mathrm{R}_{(j+p)t})\, {}^{1}\!/_{3}\, \mathrm{V}ar(R_{(m+p)t})}{\sum_{i=0}^{N} w_{t(i+p)(m+p)} Skew(\mathrm{R}_{(i+p)t})\, {}^{1}\!/_{3}}\right]$$

$$Cov(R_{(i+p)t.}R_{(j+p)t.}) = K\left[\frac{(Skew(\mathrm{R}_{(i+p)t})\, {}^{1}\!/_{3}\, \mathrm{V}ar(R_{(m+p)t})}{\sum_{i=1}^{N} w_{t(i+p)(m+p)}}\right] \qquad \forall\, i \neq \mathrm{j} \qquad (9)$$

This is the first alternative implied estimator of covariance in which the value solely depends on the numeric nature of K and P

Case one: At P=0, and K=-1 the covariance of market returns $R_{(i+p)t}$ and $R_{(m+p)t}$ becomes

$$Cov(Rit, Rjt) = -\left[Skew \frac{(R_{(jt)}\, {}^{1}\!/_{3} VarR_{mt}}{\sum_{i=1}^{N} w_{t(im)}}\right]$$

With the above value of returns R_{it} could be above its average return while Rjt could be below its average return and vice versa. Thus a deficit or loss is incurred.

Case two: when p = 0, k = 0, the covariance of the returns $R_{(i+p)t.}$ and $R_{(j+p)t.}$ becomes

$$Cov(R_{it}, R_{jt}) = 0\frac{(Skew(R_{(it)})^{1/3}VarR_{mt}}{\sum_{i=0}^{N} w_{t(im)}}$$

$$Cov(R_{it}, R_{jt}) = 0$$

The above result shows that there is no relationship, thus R_{it} and R_{jt} show no pattern which indicate that there is no loss nor profit

Case three: When P = 0 and K is 1, the covariance of the returns gives the following values. i.e

$$Cov(R_{it}, R_{jt}) = 1\left[\frac{Skew(R_{it})^{1/3}VarR_{mt}}{\sum_{i=1}^{N} w_{t(im)}}\right]$$

$$= \frac{Skew(R_{it})^{1/3}VarR_{mt}}{\sum_{i=1}^{N} w_{t(im)}}$$

The obtained value shows that there is a positive relationship between the two returns $R_{(i+p)t.}$ and $R_{(j+p)t.}$. This implies that the value of covariance would be positive and moreover there would be excess of revenue over capital invested

The third method of covariance estimation make use of first moments on the assumption that the proportion of systematic kurtosis is equal for all N assets and this is denoted by C_t^{kurt}

Now

$$Kurt(\mathrm{R}_{(i+p)t}) = \beta^4{}_{(i+p)t} kurt(\mathrm{R}_{m+p)}) - (1 - C_t^{kurt})\, Kurt(\mathrm{R}_{(j+p)t}) \qquad (10)$$

Solving for $\beta_{(i+p)t}$ we get

$$\beta^4{}_{(i+p)t} Kurt(\mathrm{R}_{(m+p)t}) = Kurt(\mathrm{R}_{i+p)}) - (1 - C_t^{kurt})\, Kurt(\mathrm{R}_{(j+p)t})$$

$$\beta^4{}_{(i+p)t} = \frac{Kurt(\mathrm{R}_{i+p)}) - Kurt(\mathrm{R}_{i+p)}) + C_t^{kurt}\ Kurt(\mathrm{R}_{(i+p)t})}{Kurt(\mathrm{R}_{(m+p)t})}$$

$$\beta_{(i+p)t} = \left(C_t^{kurt}\right)^{1/4}\left[\frac{\left(kurt(\mathrm{R}_{(i+p)t})\right)^{1/4}}{\left(kurt(\mathrm{R}_{(m+p)t})\right)^{1/4}}\right]$$

Using the market weight value

$\sum_{i=1}^{N} w_{t(i+p)(m+p)}\beta_{(i+p)t}$ for substituting for $\beta_{(i+p)t}$, we have

$$\sum_{i=1}^{N} w_{t(i+p)(m+p)}\beta_{(i+p)t} = \sum_{i=1}^{N} w_{t(i+p)(m+p)}\beta_{(i+p)t}\ \left(C_t^{kurt}\right)^{1/4}\left[\frac{kurt(\mathrm{R}_{(i+p)t})}{kurt(\mathrm{R}_{(m+p)t})}\right]^{1/4} = 1 \qquad (12)$$

$$\left(C_t^{kurt}\right)^{1/4} = \frac{1}{\sum_{i=1}^{N} w_{t(i+p)(m+p)} \left[\frac{kurt(\mathrm{R}_{(i+p)t})}{kurt(\mathrm{R}_{(m+p)t})} \right]^{1/4}}$$

$$\left(C_t^{kurt}\right)^{1/4} = \frac{(kurt(\mathrm{R}_{(m+p)t})}{\sum_{i=1}^{N} w_{t(i+p)(m+p)} (kurt(\mathrm{R}_{(j+p)t})^{1/3}} \qquad (13)$$

$$\beta_{(i+p)t} = \frac{kurt(\mathrm{R}_{(m+p)t} \quad kurt(\mathrm{R}_{(i+p)t})_{1/4}}{\sum_{i=0}^{N} w_{t(i+p)(m+p)} (Skew(\mathrm{R}_{(j+p)t})_{1/4} \mathrm{kurt}(\mathrm{R}_{(m+p)t})}$$

$$Co\,\mathrm{var}(R_{(i+p)t}, R_{(j+p)t.}) = K\beta_{(i+p)t}\beta_{(i+p)} \mathrm{V}ar(R_{(m+p)t}) \qquad \forall\, i \neq \mathrm{j} \qquad (14)$$

This gives the covariance of the market returns by substituting for $\beta_{(i+p)t}$.

$$Cov(R_{(i+p)t.}, R_{(j+p)t.}) = K\left[\frac{k\mathrm{urt}(\mathrm{R}_{(m+p)t})_{1/4}\, k\mathrm{urt}(\mathrm{R}_{(i+p)t})_{1/4} \mathrm{V}ar(R_{(m+p)t})}{\sum_{i=1}^{N} w_{t(i+p)(m+p)} kurt(\mathrm{R}_{(i+p)t})^{1/4} kurt(\mathrm{R}_{(m+p)t})^{1/4}} \right]$$

$$= \frac{K(kurt(\mathrm{R}_{(j+p)t})^{1/4} \mathrm{V}ar(R_{(m+p)t})}{\sum_{i=1}^{N} w_{t(i+p)(m+p)}}$$

$$= Cov(R_{(i+p)t.}, R_{(j+p)t.}) = \frac{K(kurt(\mathrm{R}_{(j+p)t})^{1/4} \mathrm{V}ar(R_{(m+p)t})}{\sum_{i=1}^{N} w_{t(i+p)(m+p)}}$$

$$\forall\, i \neq \mathrm{j}$$

The value of $Cov(R_{(i+p)t.}, R_{(j+p)t.})$ depends on the values of constants p and k as these values are used to determine the exact value of covariance of returns $R_{(i+p)t.}$ and $R_{(j+p)t.}$

K has the range -1 to 1 thus the values of k includes, -1,0,1. These values are used to determine the exact value of covariance of returns $R_{(i+p)t.}$ and $R_{(j+p)t.}$ while p has values 0 throughout. The outcome of the results can be discussed just as in the implied covariance estimator earlier mentioned above.

4.0 Conclusion

In this research paper, an improved portfolio optimization through covariance estimator was developed. Deterministic constants p and k was introduced to help the researcher obtain the exact value of covariance of market return especially when certain condition which p and k represent are put in to consideration. Example of these conditions are: ignorant of the market nature by the investor, the volume of capital invested especially the beginner, over-optimistic about the value of returns, none-transparency on the part of the market operators. False information as regard the true value of the market returns e.t.c

In the work of [1] in which first fully implied estimator of covariance return was developed and that of the present researcher's the basic objective is to obtain estimates of the covariance. The difference being in the introduction of deterministic constants P and K. These help to determine the exact value of covariance of returns. Note that these returns are expected return, which is the sum of the product of each outcome return and its associate probability. It is observed that return on a security consist of two parts, the dividend and the capital gain bearing it in mind that risk and return concepts are basic to the understanding of the valuation of assets or securities and an average investor is extremely risk-averse.

The market model of this research followed a generalized version of [14] with time varying coefficients, together with the introduction of deterministic constants P and K. The enlargement of the market returns coupled with various mind set of the market operators before and after operation constitutes the difference between the present work and that of [1]. When P attains value 0 and k has value 1, one arrives at the same result as in [1]. When the value of P is zero, and K attains -1 and O the exact values of market covariances are clearly established, this gives the operator the clear knowledge of the return expectation.

It is clear that P and K represent those conditions that cannot be controlled and neither can they be accounted for when investing, and a portfolio being a bundle of combination of individual assets or securities each of these assets will have its own uncontrollable conditions and these would definitely affect the value of covariance of market returns, effects of which this research paper has been able to elucidate.

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