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Predicting The Compressive Strengths of Concrete Mixes Made With Washed Local Gravel Using Scheffe's (4,2) Lattice Polynomial

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Abstract

Most structural failures in Nigeria are due to inadequate strength of the construction materials, mainly concrete. This research seeks to use optimisation techniques to overcome the shortcomings of the laboratory trial mixes of determining concrete strengths. Washed local gravel from Abagana, eastern Nigeria, a major source for the construction industry was used. Based on a design matrix and using these aggregates and river sand, sixty concrete cubes of dimensions 150 mm X 150mm X 150 mm were made, cured and tested according to the procedures in BS 1881:1983. Scheffe's (4, 2) lattice polynomial with regression equation was used to develop a mathematical model for predicting the compressive strength characteristics of concretes made with these aggregates. A student's t-test was used to test the model's validity and the analysis of variance (ANOVA) carried out.

***Keywords*:** Concrete, Compressive Strength, Scheffe, Local gravel, Model

1.0 Introduction

1.1 Actual and Pseudo-Components

The requirement of the simplex that $x_1+ x_2+x_3 + x_4 = 1$ makes it impossible to use the normal mix ratios such as 1:1:2, etc., at a given water/cement ratio. Hence, a transformation of the actual components (normal mix ratios) to meet this condition is unavoidable. The design matrix is shown in Table 1. $x^{(i)}_1$, $x^{(i)}_2$, $x^{(i)}_3$ and $x^{(i)}_4$ are the pseudo-components for the *ith* experimental points. For any actual component Z, the pseudo-component (x) is given by

$$X = AZ \qquad (1)$$

Where A is the inverse of Z matrix and

$$Z = BX^T \qquad (2)$$

Where B is the inverse of Z matrix and X^T is the transpose of the matrix.

1.2 The Scheffe's (4, 2) Lattice Polynomial

Simplex is the structural representation of the line or planes joining the assumed positions of the constituent materials (atoms) of a mixture [1]. Scheffe [2] considered experiments with mixtures of which the property studied depended on the proportions of the components present but not on the quantity of the mixture. If a mixture has a total of q components and x_i be the proportion of the ith component in the mixture such that $x_i \geq 0$ (i = 1, 2… q), then

$$x_1+ x_2+x_3 +\ldots\ldots\ldots\ldots\ldots\ldots+ x_q = 1 \qquad (3)$$

Scheffe [2] described mixture properties by reduced polynomials obtainable from eqn (4):

$$\hat{Y} =b_0+\Sigma b_ix_i+\Sigma b_{ij}\, x_i\, x_j+\Sigma b_{i\,jk}\, x_i\, x_j\, x_k +\Sigma b_{i1,i\,2}\ldots i_n\, x_{i1}\, x_{i2}\, x_i n \qquad (4)$$

Where ($1\leq i \leq q$, $1\leq i \leq j \leq q$, $1 \leq i \leq k \leq q$) respectively and b is constant coefficient.

Multiplying eqn. (3) by b_0 and multiplying the outcome by x_1, x_2, x_3 and x_4 in turn and substituting into eqn. (4), we have:

$$\hat{Y} = b_0\, x_1+b_0\, x_2+ b_0\, x_3+ b_0\, x_3+ b_0\, x_4+ b_1\, x_1 + b_2\, x_2+ b_3x_3+ b_4x_4 + b_{12}\, x_1\, x_2+ b_{13}\, x_1\, x_3+b_{14}\, x_1\, x_4+b_{23}x_2x_3+b_{24}x_2x_4+b_{34}x_3x_4+b_{11}(x_1- x_1x_2 - x_1x_3 - x_1x_4)+b_{22}(x_2- x_1x_2 - x_2x_3 - x_2x_4)+b_{33}(x_3- x_1x_3 - x_2x_3 - x_3x_4) + b_{44}(x_4- x_1x_4 - x_2x_4 - x_3x_4) \qquad (5)$$

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Re-arranging eqn. (5), we have

$$\hat{Y} = \Sigma\propto_i x_i + \Sigma\propto_{ij} x_i x_j \qquad (6)$$

where $1 \le i \le q$, $1 \le i \le j \le q$, $1 \le i \le j \le q$ respectively and

$$\propto_i = b_0 + b_i + b_{ii} \text{ and } \propto_{ij} = b_{ij} + b_{i\,i} + b_{ii} \qquad (7)$$

Let the response function to the pure components (x_i) be denoted by y_i and the response to a 1:1 binary mixture of components i and j be y_{ij}. From eqn (6), it can be written that

$$\Sigma\propto_i x_i = \Sigma y_i x_i \qquad (8)$$

Where (i = 1 … 4)

Evaluating y_i, for instance gives:

$$y_i = \propto_I \qquad (9)$$

Also evaluating y_{ij}, gives in general the equations of the form

$$\propto_{ij} = 4y_{ij} - 2y_i - 2y_j \qquad (10)$$

For the Scheffe's (4, 2) lattice polynomial, that is eqn. (6) becomes:

$$\hat{Y} = y_1 x_1 + y_2 x_2 + y_3 x_3 + y_4 x_4 + (4y_{12} - 2y_1 - 2y_2) x_1 x_2 + (4y_{13} - 2y_1 - 2y_3) x_1 x_3 + (4y_{14} - 2y_1 - 2y_4) x_1 x_4 + (4y_{23} - 2y_2 - 2y_3) x_2 x_3 + (4y_{24} - 2y_2 - 2y_4) x_2 x_4 + (4y_{34} - 2y_3 - 2y_4) x_3 x_4 \qquad (11)$$

1.2 The Student's T-Test

The unbiased estimate of the unknown variance $S_Y^{\,2}$ is given by Biyi [3]

$$S_Y^2 = \frac{\sum \left(y_i - \breve{Y}\right)^2}{n - 1} \qquad (12)$$

If $a_i = x_i(2x_i - 1)$, $a_{ij} = 4x_i x_j$; for ($1 \le i \le q$) and ($1 \le i \le j \le q$) respectively.

Then, $$\varepsilon = \Sigma a^2_i + \Sigma a^2_{ij} \qquad (13)$$

where ε is the error of the predicted values of the response.

The t-test statistic is given by Biyi [3]

$$t = \frac{\Delta Y}{S_Y} \frac{\sqrt{n}}{\sqrt{1 + \varepsilon}} \qquad (14)$$

where $\Delta Y = Y_0 - Y_t$; Y_0 = observed value, Y_t = theoretical value; n = number of replicate observations at every point; ε = as defined in eqn.(13).

2.0 Materials and Method

2.1 Preparation, Curing and Testing of Cube Samples

The aggregates were sampled in accordance with the methods prescribed in BS 812: Part 1:1975 [4]. The test sieves were selected according to BS 410:1986 [5]. The water absorption, the apparent specific gravity and the bulk density of the coarse aggregates were determined following the procedures prescribed in BS 812: Part 2: 1975 [6]. The Los Angeles abrasion test was carried out in accordance with ASTM. Standard C131: 1976 [7]. The sieve analyses of the fine and coarse aggregate samples satisfied BS 882:1992 [8]. The sieving was performed by a sieve shaker. The water used in preparing the experimental samples satisfied the conditions prescribed in BS 3148:1980 [9]. The required concrete specimens were made in threes in accordance with the method specified in BS 1881: 108:1983 [10].These specimens were cured for 28 days in accordance with BS 1881: Part 111: 1983 [11]. The testing was done in accordance with BS 1881: Part 116:1983 [12] using compressive strength testing machine.

2.2 Testing the Fit of the Quadratic Polynomials

The polynomial regression equation developed was tested to see if the model agreed with the actual experimental results. The null hypothesis (that there is no significant difference between the experimentally-observed values and the theoretically-obtained values) was denoted by H_0 and the alternative by H_1.

3.0 Results and Discussion

3.1 Physical and Mechanical Characterisation of the Aggregates

The maximum aggregate size for the local gravel was 53mm m and 2mm for the fine sand. The local gravel had water absorption of 4.55%, moisture content of 53.25%, apparent specific gravity of 1.88, Los Angeles abrasion value of 60% and bulk density of 1302.7 kg/m^3.

Table 1 Design Matrix for Experiment based on Scheffe's (4, 2) Lattice Polynomial

Pseudo-components					Actual components			
S/N	x_1	x_2	x_3	x_4	z_1	z_2	z_3	z_4
1	1	0	0	0	0.6	1	1.5	2
2	0	1	0	0	0.5	1	1	2
3	0	0	1	0	0.55	1	2	5
4	0	0	0	1	0.65	1	3	6
5	½	½	0	0	0.55	1	1.25	2
6	½	0	½	0	0.575	1	1.75	3.5
7	½	0	0	½	0.625	1	2.25	4
8	0	½	½	0	0.525	1	1.5	3.5
9	0	½	0	½	0.575	1	2	4
10	0	0	½	½	0.6	1	2.5	5.5
Control								
11	½	¼	¼	0	0.5625	1	1.5	2.75
12	½	0	¼	¼	0.6	1	2.0	3.75
13	0	½	¼	¼	0.55	1	1.75	3.75
14	¼	¼	¼	¼	0.575	1	1.875	3.75
15	¾	¼	0	0	0.575	1	1.375	2
16	¾	0	¼	0	0.5875	1	1.625	2.75
17	¾	0	0	¼	0.6125	1	1.875	3.0
18	0	¾	¼	0	0.5125	1	1.25	2.75
19	0	¾	0	¼	0.5375	1	1.5	3.0
20	0	0	¾	¼	0.5850	1	2.25	5.25

Legend: z_1= water/cement ratio; z_2=Cement; z_3=Fine aggregate; z_4=Coarse aggregate

3.2 The Regression Equation for the Compressive Strength Tests Results

Applying the responses (average compressive strengths) in determining the coefficients of the (4, 2) lattice polynomial to eqns. (9) and (10), we had α_1= 23.46, α_2= 25.01, α_3 =14.83, α_4=9.41, α_{12}=2.06, α_{13} = -0.78, α_{14}=- -4.38, α_{23}= -2.32, α_{24}= -6.44, α_{34}= 9.28. Thus, from eqn.(11): $\hat{Y} = 23.46\, x_1 + 25.01\, x_2 + 14.83\, x_3 + 9.41\, x_4 + 2.06\, x_1 x_2 – 0.78 x_1 x_3 – 4.38\, x_1 x_4 - 2.32\, x_2 x_3 – 6.44\, x_2 x_4 + 9.28\, x_3 x_4$. This is the mathematical model for predicting the compressive strength characteristics of the washed local gravel concrete, based on Scheffe's (4, 2) polynomial. $\hat{Y}$ represents the compressive strength of the concrete.

Table 2 Compressive Strength Tests Results and Sample Variances, S_i^2, for Washed Local - Gravel Concrete, based on Scheffe's (4, 2) Simplex Lattices

S/NO	**Replication**	**Responses y_i(N/mm^2)**	**Response symbol**	**Σy_i**	**Σy_i^2**	**$\breve{y}$**	**$(\Sigma y_i)^2$**	**S_i^2**
1	1A 1B 1C	23.85 23.39 23.14	y_1	70.38	1651.37	23.46	4953.34	0.128
2	2A 2B 2C	24.00 25.20 25.83	y_2	75.03	1878.23	25.01	5629.50	0.865
3	3A 3B 3C	15.00 14.82 14.67	y_3	44.49	659.84	14.83	1979.36	0.0267
4	4A 4B 4C	8.95 9.85 9.43	y_4	28.23	266.05	9.41	796.93	0.203
5	5A 5B 5C	25.00 24.82 24.43	y_{12}	74.25	1837.86	24.75	5513.06	0.087
6	6A 6B 6C	18.55 19.00 19.30	y_{13}	56.85	1077.59	18.95	3231.92	0.142
7	7A 7B 7C	15.80 15.40 14.82	y_{14}	46.02	706.43	15.34	2117.84	0.242
8	8A 8B 8C	19.56 19.90 18.56	y_{23}	58.02	1123.08	19.34	3366.32	0.487
9	9A 9B 9C	15.20 16.00 15.60	y_{24}	46.8	730.4	15.6	2190.24	0.16
10	10A 10B 10C	14.85 15.02 13.45	y_{34}	43.32	627.03	14.44	1876.62	0.745

S/NO	Replication	Responses y_i(N/mm^2	Response symbol	Σy_i	Σy_i^2	Ȳ	$(\Sigma y_i)^2$	S_i^2
CONTROL								
11	11A 11B 11C	21.75 22.45 21.80	C_1	66	1452.31	22	4356	0.155
12	12A 12B 12C	17.50 17.25 17.42	C_2	52.17	907.27	17.39	2721.71	0.017
13	13A 13B 13C	18.00 18.50 17.77	C_3	54.27	982.02	18.09	2945.23	0.138
14	14A 14B 14C	18.00 18.60 18.00	C_4	54.60	993.96	18.2	2981.16	0.12
15	15A 15B 15C	24.75 23.95 23.60	C_5	72.30	1743.13	24.1	5227.29	0.35
16	16A 16B 16C	20.80 21.32 20.88	C_6	63	1323.16	21	3969	0.08
17	17A 17B 17C	19.04 19.86 18.34	C_7	57.24	1093.30	19.08	3276.42	0.58
18	18A 18B 18C	21.90 22.45 21.95	C_8	66.30	1465.42	22.1	4395.69	0.095
19	19A 19B 19C	20.00 19.46 19.31	C_9	58.77	1151.57	19.59	3453.91	0.133
20	20A 20B 20C	15.02 14.95 15.00	C_{10}	44.97	674.10	14.99	2022.30	0.00

Table 3 Regression Analysis of the Compressive Strength Tests Results

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.989537875
R Square	0.979185205
Adjusted R Square	0.802111141
Standard Error	0.474268237
Observations	10

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	4	63.48805783	15.87201446	94.0855	6.82527E-05
Residual	6	1.349582166	0.224930361		
Total	10	64.83764			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*
Intercept	7.177898089	1.107201131	6.482921564	0.00064	4.468674525
x1	15.96178344	1.240826623	12.86383057	1.36E-05	12.92559008
x2	16.79974522	1.325676311	12.67258461	1.48E-05	13.55593215
x3	10.08687898	1.534083278	6.575183449	0.000594	6.333112434
x4	0	0	65535	#NUM!	0

RESIDUAL OUTPUT

Observation	*Predicted Y*	*Residuals*	*Standard Residuals*
1	21.88044586	0.11955414	0.325435522
2	17.68050955	-0.290509554	-0.790789246
3	18.09949045	-0.009490446	-0.02583372
4	17.89	0.31	0.843843731
5	23.34917197	0.750828025	2.043811364
6	21.67095541	-0.670955414	-1.826392001
7	19.14923567	-0.069235669	-0.188464791
8	22.29942675	-0.199426752	-0.542854885
9	19.77770701	-0.187707006	-0.510952841
10	14.74305732	0.246942675	0.672196866

Legend df = degree of freedom, SS = sum of squares, MS = mean of squares, F = F-statistic, #N/A = insignificant value, ANOVA = analysis of variance.

3.3 Regression Analysis of the Compressive Strength Tests Results for the Washed Local Gravel Concrete

Table 3 shows the summary output of the regression analysis of the compressive strength tests results of the washed local gravel concrete concrete. The coefficient of determination, $r^2 = 0.9788$ shows a very strong relationship between the independent variables (x_1, x_2, x_3, x_4) and the dependent variable, $\hat{Y}$. From the F distribution Table [13], F critical is 3.3. Since the F –observed value of 92.41813 is much higher than 3.3, it is extremely unlikely that an F value this high occurred by chance. The extremely small, significance F = 7.13188E-05 means that the observed F value of 92.41813 is unlikely to have occurred by chance. From the Student's t distribution Table [13], t critical is 3.69. The absolute values of the t stat are greater than this t critical. This shows that all the variables used in the regression equation are useful in predicting the response. The P-values being very small means that the experimentally-obtained values and the predicted values of $\hat{Y}$ have variances that are not significantly different. Thus, the regression equation for the prediction of the compressive strength characteristics of the washed-local gravel concrete is valid.

3.3 Fit of the Polynomial

The polynomial regression equation developed i.e., $\hat{Y} = 23.46\,x_1 + 25.01\,x_2 + 14.83\,x_3 + 9.41\,x_4 + 2.06\,x_1 x_2 - 0.78x_1 x_3 - 4.38\,x_1 x_4 - 2.32\,x_2 x_3 - 6.44\,x_2 x_4 + 9.28\,x_3 x_4$, was tested to see if the model agreed with the actual experimental results. There was no significant difference between the experimental and the theoretically expected results. The null hypothesis, H_0 was therefore satisfied.

3.4 t -value from table

The t-student's test had a significance level, $\alpha = 0.05$ and $t_{\alpha/l(ve)} = t_{0.005(9)} = 3.69$. This was greater than any of the t values calculated in Table 4. Therefore, the regression equation for the washed local gravelconcrete was adequate.

Table 4 t –Statistic for the controlled Points, washed local gravel concrete compressive test, based on Scheffe's (4, 2) polynomial

Response Symbol	i	j	a_i	a_{ij}	a_i^2	a_{ij}^2	ε	$\check{Y}$ (N/mm²)	$\hat{Y}$ (N/mm²)	ΔY	t
C_1	1	2	0	0.5	0	0.25	0.609	22	21.70	0.3	0.635
	1	3	0	0.5	0	0.25					
	1	4	0	0	0	0					
	2	3	-0.125	0.25	0.0156	0.0625					
	2	4	-0.125	0	0.0156	0					
	3	4	-0.125	0	0.0156	0					
	4	—	0	—	0	—					
				Σ	0.0468	0.5625					
	Similarly										
C_2	—	—	—	—	—	—	0.484	17.39	17.725	-0.33	-0.781
C_3	—	—	—	—	—	—	0.734	18.09	18.05	0.04	0.079
C_4	—	—	—	—	—	—	0.593	18.2	18.01	0.19	0.399
C_5	—	—	—	—	—	—	0.289	24.1	24.23	-0.13	-0.359
C_6	—	—	—	—	—	—	0.859	21	21.15	-0.15	-0.291
C_7	—	—	—	—	—	—	0.593	19.08	19.12	-0.04	-0.100
C_8	—	—	—	—	—	—	0.483	22.1	22.03	0.07	0.163
C_9	—	—	—	—	—	—	0.640	19.59	19.90	-0.31	-0.659
C_{10}	—	—	—	—	—	—	0.469	14.99	15.21	-0.22	-0.530

Legend: C_i =response; $a_i = x_i (2x_i - 1)$; $a_{ij} = 4 x_i x_j$; $\varepsilon = \Sigma a^2_i + \Sigma a^2_{ij}$; $\check{y}$ = experimentally-observed value; $\hat{Y}$= theoretical value; t = t-test statistic.

Conclusion

The strengths (responses) of the concretes were a function of the proportions of its ingredients: water, cement, fine aggregate and coarse aggregates. Since the predicted strengths by the model were in total agreement with the corresponding experimentally -observed values, the null hypothesis was satisfied. This meant that the model equation was valid.

References

[1] Jackson, N. and Dhir, R. K., *Civil Engineering Material*, Macmillan ELBS, Hampshire RG21 2XS, England, 1988.
[2] Scheffe, H., *Experiments with mixtures*, Royal Statistical Society Journal, Ser. B, Vol. 20, 1958, pp340-60.

[3] Biyi, A., *Introductory Statistics*, Abiprint & Pak Ltd., Ibadan, 1975.
[4] British Standard 812: Part 1 *Sampling, shape, size and classification*. Methods for sampling and testing of mineral aggregates, sands and fillers. British Standards Institution Publication, London, 1975.
[5] British Standard 410 *Specification for test sieves.* British Standards Institution Publication, London, 1986.

[6] British Standard 812: Part 2 Methods for sampling and testing of mineral aggregates, sands and fillers. *Physical properties*. British Standards Institution Publication, London, 1975.

[7] ASTM. Standard C 131 *Tests for Resistance to Abrasion of Small Size Coarse Aggregate by Use of the Los Angeles Machine.* American Society for Testing and Materials Publication, New York, 1976.

[8] British Standard 882 *Specification for aggregates from natural sources for concrete*. British Standards Institution Publication, London, 1992.

[9] British Standard 3148 *Tests for water for making concrete.* British Standards Institution Publication, London, 1980.

[10] British Standard 1881: Part 108 *Method for making test cubes from fresh concrete.* British Standards Institution Publication, London, 1983.

[11] British Standard 1881: Part 111 *Method of normal curing of test specimens (20 ^{0}C).* British Standards Institution Publication, London, 1983.

[12] British Standard 1881: Part 116. *Method for determination of compressive strength.* British Standards Institution Publication, London, 1983.

[13] Nwaogazie, I.L., *Probability and Statistics for Science and Engineering Practice*, University of Port Harcourt Press, Port Harcourt, 2006.