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Simulation of Positron Beam Driven Plasma Wakefield Acceleration Using the Particle-In-Cell Code, Oopic

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Abstract

We use a 2-d object oriented particle-in-cell simulation code (OOPIC) to model the interaction of positron beam with plasma. A 28.5 GeV positron beam passes through 1 metre of pre-ionized lithium plasma. The interaction regime is in the magnetically self –focused (beam radius is much less than the plasma wavelength). The plasma electrons are blown out creating a large amplitude wake of about 3 x 10^{-4} m, driven by a strong axial electric field E_Z of order 1Gv/m, which is used in accelerating the electrons to high energies. This is in good agreement with the Stanford linear accelerator centre (SLAC) E-157 experiment.

Keywords: plasma acceleration, wake field, positron beam, particle-in-cell (PIC), perturbation.

1.0 Introduction

Studies of charged particle acceleration processes remain one of the most important areas of research in laboratory, space and astrophysical plasmas. Plasma acceleration is a technique for accelerating charged particles, such as electrons, ions or protons, using an electric field associated with an electron plasma wave. The wave is created by the passage of an intense laser beam or electron pulse through the plasma [1].

The beam driven plasma wakefield accelerator (PWFA) are also capable of providing high acceleration gradients, and thus may lead to next generation of smaller and cheaper high energy accelerators [2]. The technique appears to offer a way to build high performance particle accelerators of much smaller size than conventional devices at the expense of coherency. Current experimental devices show accelerating gradients several orders of magnitude than current particle accelerators.

In a plasma accelerator, the role of the accelerating structure is played by an ionized gas. Instead of being a problem, electrical breakdown is part of the design because the gas is broken down. The power source is either a laser or a charged particle beam [3]. Different methods exist for generating the plasma wave. The common are laser beat wave, laser wakefield, particle beam driven wakefield [4]

2.0 The Positron beam driven PWFAs

The interaction between a positively charged beam and plasma is different from an electron beam-plasma interaction in the magnetically self focused regime. A high charge-density electron beam will expel plasma electrons from a small region surrounding the beam. A positively charged beam will do the reverse, drawing plasma into the core of the beam. This process leads to a complicated non-linear wake structure following the beam that appears to be less favorable for accelerating a witness beam when compared to the electron driver scenario. However, when an ultra relativistic, highly focused positron beam enters an under-densed plasma (density of the beam greater than the density of plasma), the in-vacuum balance between the beam’s space-charge defocusing field and self-magnetic focusing field is modified by the highly mobile plasma electrons that are pulled in neutralizing the excess space charge of the positron beam. The degree of neutralization depends not only on

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the plasma density, but also on the longitudinal position along the positron bunch. As plasma electrons from various radii arrive on the axis of the beam at various times and overshoot, they create a wake field structure that has complex longitudinal and transverse electric field components [5]. However, there are a significant number of particles in the tail of the beam where the wake field has changed sign and are therefore accelerated as indicated below:

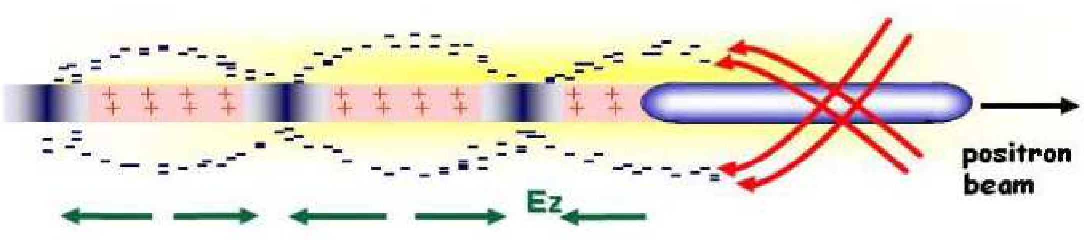

Fig. 1 Wakefield generated by a positron beam.

3.0 Basic Equations

Although the interaction of a dense, short positron beam with plasma is inherently nonlinear, we can use the linear theory which predicts the plasma response to either a short electron or positron beam as a guide to interpret the PIC code (OOPIC) results. Here the beam density, plasma density, beam radius, beam longitudinal characteristic length, and the wavelength are donated by $n_{b,}$ n_p, σ_r, σ_z and λ_p respectively. The beam induces a small perturbation in the plasma whose response can be calculated, assuming that the beam is infinitely short.

The interaction regime is magnetically self-focused and the important feature of the regime is that the transport of the beam can be theoretically approximated by a linear theory in which the beam is assumed to be a point like ($\sigma_r << \lambda_p$). The plasma is described by eqns (1) – (7):

$$\nabla . \mathbf{E}_1 = 4\pi\rho - 4\pi e n_1 \quad (1)$$

$$\nabla . \mathbf{B}_1 = 0 \quad (2)$$

$$\nabla \times \boldsymbol{E}_1 = -\frac{1}{C}\frac{\partial \boldsymbol{B}_1}{\partial t} \quad (3)$$

$$\nabla \times \mathbf{B}_1 = \frac{4\pi}{C}\boldsymbol{J}_1 + \frac{\partial \boldsymbol{E}_1}{C\partial t} \quad (4)$$

$$\frac{\partial \boldsymbol{V}_1}{\partial t} = -\frac{e\boldsymbol{E}_1}{M_e} \quad (5)$$

$$\frac{dn_1}{dt} + n_o \nabla . \mathbf{V}_1 = 0 \quad (6)$$

$$\mathbf{J}_1 = q\, n_o \mathbf{V}_1 \quad (7)$$

where $\mathbf{E_1}$, $\mathbf{B_1}$, $\mathbf{J_1}$, $\mathbf{V_1}$, n_1, q, e, M_e, n_0 and c are electric field, magnetic field, current density, electron velocity, density, electronic charge, mass of electron, unperturbed plasma density and speed of light. The equations are exact in the limit of non relativistic motion.

The driver beam is approximated as a delta function; this will allow the calculation of impulse response of the plasma. Thus,

$$\rho_{beam} = \frac{q}{c}\delta(r)\delta\left[t - \frac{z}{c}\right] \quad (8)$$

Now, our goal is to solve for the density perturbation as a function of the impulse response. Taking the time derivative of the linearized continuity equation (6)

$$\frac{\partial}{\partial t}\left(\frac{\partial n_1}{\partial t} + n_0 \nabla . \mathrm{V}_1 = 0\right)$$

$$\frac{\partial^2 n_1}{\partial t^2} + n_0 \nabla . \frac{dV1}{dt} = 0 \quad (9)$$

Putting equation (5) into (9)

$$\frac{\partial^2 n_1}{\partial t^2} - \frac{e n_o}{Me} \nabla \boldsymbol{E_1} = 0 \quad (10)$$

Substituting (1) into (10) gives:

$$\frac{\partial^2 n_1}{\partial t^2} - \frac{e n_o}{Me}(4\pi\rho_{Beam} - 4\pi e n_1) = 0 \quad (11)$$

Equation (11) is simplified by combining terms into a parametric known as the plasma frequency ω_p

where,

$$\omega_p^2 = \frac{4\pi e^2 n_o}{Me} \tag{12}$$

Now

$$\frac{\partial^2 n_1}{\partial t^2} + {W_p}^2 n_1 = \frac{{W_p}^2}{e}\rho_{Beam} \tag{13}$$

Substituting equation (8) into (13), we get:

$$\frac{\partial^2 n_1}{\partial t^2} + {W_p}^2 n_1 = {W_p}^2 \frac{q}{ec}\delta(r)\delta(t - \frac{z}{c}) \tag{14}$$

Equation (14) is a differential equation, which can be solved, using the Laplace transform of a system initially at rest.

$$\mathrm{L}\,(n_1) = \frac{q}{ec}\delta(r)\frac{{W_p}^2}{s^{2+W_p^2}}$$

$$n_1 = L^{-1}\left[\frac{q}{ec}\,\delta(r)\,\frac{{W_p}^2}{s^{2+W_p^2}}\right] \tag{15}$$

$$n_1 = W_p\frac{q}{ec}\delta(r)\sin\{W_p\,(t - {}^{Z}\!/_{C})\} \cup \left(t - {}^{Z}\!/_{C}\right) \tag{16}$$

Equation (16) gives the density as it is perturbed by the beam. The function $\cup\,(t)$ is heavy side function which means that the plasma is not perturbed until after the beam passes.

Now, the electric field $\boldsymbol{E_1}$ as a function of the perturbed density is solved by taking the curl of equation (3) i.e.

$$\nabla \mathrm{X}\, \nabla \times \boldsymbol{E_1} = -\frac{1}{C}\frac{\partial}{\partial t}\nabla \times \boldsymbol{B_1} \tag{17}$$

Substituting equation (4), for the curl of magnetic field into equation (17) and applying the vector identity for the curl of a curl of a vector gives

$$\nabla(\nabla.\boldsymbol{E_1}) - \nabla^2\boldsymbol{E_1} - \frac{4\pi}{c^2}\frac{\partial \boldsymbol{J_1}}{\partial t} - \frac{1}{C^2}\frac{\partial^2 \mathbf{E_1}}{\partial t^2} \tag{18}$$

The time derivative of the linearized current equation is given by:

$$\frac{\partial \boldsymbol{J_1}}{\partial t} = -\,en_o\frac{\partial \boldsymbol{V_1}}{\partial t} = \mathrm{e}^2\frac{n_o}{m_e}\boldsymbol{E_1} \tag{19}$$

Insert equations (19) and (1) into (18) and re-arrange terms to get:

$$\left(\frac{\partial^2}{\partial t^2} - C^2\,\nabla^2\right)\boldsymbol{E_1} = -\frac{4\pi \mathrm{e}^2\, n_o}{m_e}\boldsymbol{E_1} - \mathrm{C}^2\,\nabla\,(4\pi\rho - 4\pi e n_1) \tag{20}$$

Put equations (8), (12) and (16) into (20) to get:

$$\left(\frac{\partial^2}{\partial t^2} - C^2\,\nabla^2 + \mathrm{W_p^2}\right)\boldsymbol{E_1} = 4\pi q W_p^2\,\delta(\mathrm{r}) \cup \left(\mathrm{t} - \frac{z}{c}\right) Cos\left\{W_p\left(\mathrm{t} - \frac{z}{c}\right)\right\} \tag{21}$$

Here we are interested in the longitudinal accelerating and decelerating field $\boldsymbol{E_z}$. Hence we brake the gradient term in its longitudinal and transverse components

$$\nabla^2 = \nabla_\perp^2 + \frac{\partial^2}{\partial z^2} \tag{22}$$

Our interest is relativistic drive particle beams; the velocity is approximately the speed of light. Therefore, the time derivative can be replaced with a special derivative given by

$$\frac{\partial^2}{\partial t^2} = \mathrm{C}^2\frac{\partial^2}{\partial z^2} \tag{23}$$

A further quantity is the plasma wave number given by

$$\mathrm{K}^2_{\mathrm{p}} \equiv \frac{W_p^2}{C^2} \tag{24}$$

Putting equation (22), (23) and (24) into (21) yields:

$$(\nabla_\perp^2 - \mathrm{K}^2_{\mathrm{p}})\,\boldsymbol{E_1} = 4\pi q \mathrm{K}^2_{\mathrm{p}}\delta(r) \cup \left(\mathrm{t} - \frac{z}{c}\right) Cos\left\{W_p\left(\mathrm{t} - \frac{z}{c}\right)\right\} \tag{25}$$

Equation (25) is modified Helmholtz equation for the electric field driven by an impulse, which contain a delta function therefore, the response of 2-D Greens' function of scalar Helmholtz equation in cylindrical geometry is given by:

$$G(\rho) = -\frac{1}{2\pi}K_0(\mathrm{K_P}\rho) \tag{26}$$

Substitute equation (26) into (25) to get:

$$\boldsymbol{E_Z} = -\,2\mathrm{q}\,{\mathrm{K_p}}^2 K_0(K_P\mathrm{r}) \cup \left(\mathrm{t} - \frac{z}{c}\right) Cos\left\{W_p\left(\mathrm{t} - \frac{z}{c}\right)\right\} \tag{27}$$

Equation (27) is longitudinal electric fields $\boldsymbol{E_Z}$ which result from a beam approximated by a delta function.

Integrating equation (27) over the charge distribution gives peak accelerating field resulting from a Gaussian distribution, given as:

$$\mathbf{e}\boldsymbol{E_Z} = \sqrt{n_p}\,\frac{n_b}{n_p}\,\frac{\sqrt{2\pi k_p\,\sigma_z} e^{\frac{-K_p^2 \sigma_z^2}{2}}}{1+\frac{1}{K_p^2\sigma_r^2}} \text{Sin } K_p(\, z- ct) \text{ (ev/cm)} \qquad (28)$$

Equation (28) indicates that the Wakefield excited by the bunch oscillates sinusoidally with frequency determined by the plasma density and its phase velocity travels at the speed of light. The accelerating gradient increases linearly with increasing charge. And the field excited by the electron and positron beams are equal in magnitude but opposite in phase.

The dynamics of plasma electrons that are being perturbed by the space charge field of the electron or positron beam are extremely complicated and PIC simulations (OOPIC) are used in order to gain insight beyond what was obtained from linear theory [6].

4.0 The OOPIC code

The Simulation code used in this study is an object-oriented two-dimensional relativistic electromagnetic particle-in-cell code written in C++. OOPIC can simulate many physical system including plasmas, beams of charged particles with self consistent and externally generated electric and magnetic fields, low –to- moderate density neutral gases and wide variety of boundary conditions. It has electrostatic and electromagnetic field solvers of 2d geometries in both x-y (slab) and r-z (cylindrical) coordinates [7].

OOPIC provide a convenient and intuitive GUI for use on Microsoft windows, Mac osx, and Linux systems, as well as a batch mode to run jobs from the command line.

The OOPIC physics kernel has been used by researchers around the world since 1995 to simulate a wide range of challenging problems. These include plasma display panels, ion implantation, high power microwave devices, and next generation particle accelerator concepts. The code is somewhat rare among PIC simulations in its ability to handle ionization of background neutral gases via electron impact or field- induced tunneling effects [8].

Particle-in-cell codes have proven to be an extremely powerful and accurate tool for simulating PWFA dynamics. OOPIC code is used to simulate the evolution of the positron driver and the wake produced by the beam.

5.0 Simulation

This simulation models a beam-plasma wake field acceleration using beam plasma, such that the background plasma is pre-ionized (ions assumed stationery), the beam density exceeds electron plasma density ($n_b > n_p$), so that the beam blows out plasma electrons near the symmetry axis. Also positron beam is Gaussian in z and r. Once the beam enters the grid and is close to the far edge of the simulation region, a moving window algorithm is invoked so that the beam can be modeled for long times. Simulation region is bounded by conductors in order to captured lost particles and avoid any charge build up such that: electric fields parallel to the boundaries are forced to zero. The beam and plasma parameters used in this simulation are listed in the Table 1.

The simulation region is a 2-d cylindrical geometry, simulation parameters were radial and longitudinal grid sizes with r $=9x10^{-4}$m, z = $5.4x10^{-4}$m, grid points n_r = 32, n_z = 192, for a total of num cells= 32 x 192 =6144. The 28.5Gev positron beam is injected into plasma with density $n_p = 2.1x10^2 e^- cm^{-3}$, the corresponding grid size $d_r = d_z = 20 \times 10^{-6}$m, time step (dt) chosen = $0.41 \times d_z/c = 2.74 \times 10^{-14}$.The electron plasma frequency $\omega_p = 8.6 \times 10^6$ rad/s. The wakefield was measured after the beam fully propagated into the plasma, at this point in the plasma, the wakefields are more or less fully excited and do not change shape or magnitude as the drive beam propagates further into the plasma although parts of the beam itself can dramatically focus and defocus in the transverse directions in response to the wakefield.

Table 1: Beam and plasma parameters used in the simulation

PARAMETERS	**VALUE**
Pi	3.14159
Speed of light	2.99979×10^8m/s
Mass of Positron	9.1095×10^{-31}kg
Charge	+1c
Positron energy	28.5×10^9ev
Length of lithium	1m
Plasma density	$2.1x10^{20}\, e^- cm^{-3}$

6.0 Results

The wakefield produced was measured after the beam fully propagated into the plasma. Fig. 2 shows the initial 28.5 GeV beam in cylindrical coordinates, at an initial energy the beam is stiff and the beam particles do not physically move in the longitudinal direction, the dimension are in meters with r on vertical axis and z on horizontal axis. Fig. 3 shows the corresponding wake field or plasma wake. The crossing of particle trajectories in the wake is an indication of highly non laminar flow, which cannot be modeled with a fluid code. The structure of the wake is independent of the beam radius.

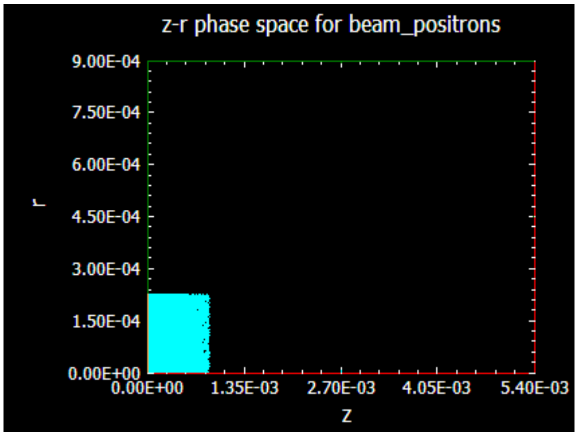

Fig. 2 Initial beam energy measured at t = 7.69282×10^{-13}

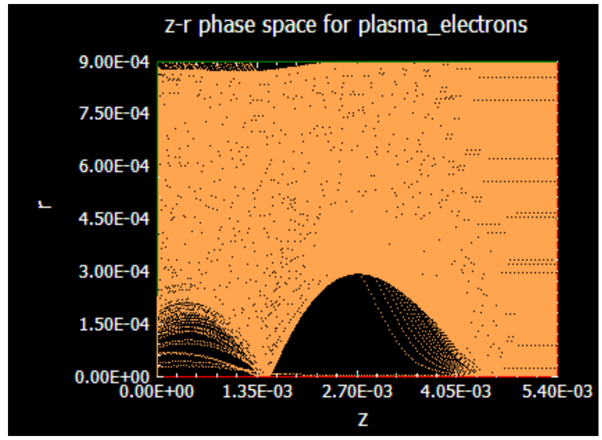

Fig. 3 Plasma wake after full propagation of the beam.

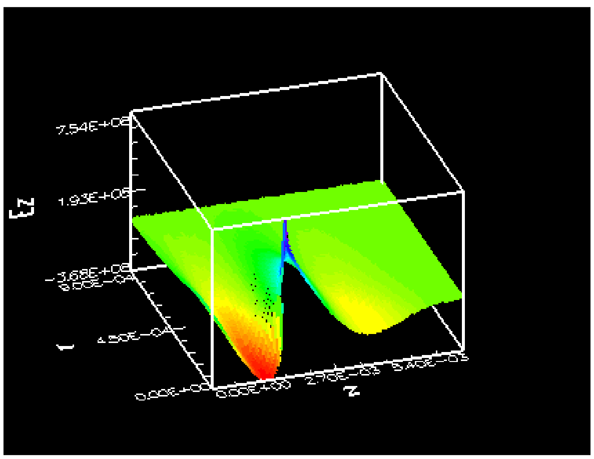

Fig. 4 Longitudinal electric field E_Z.

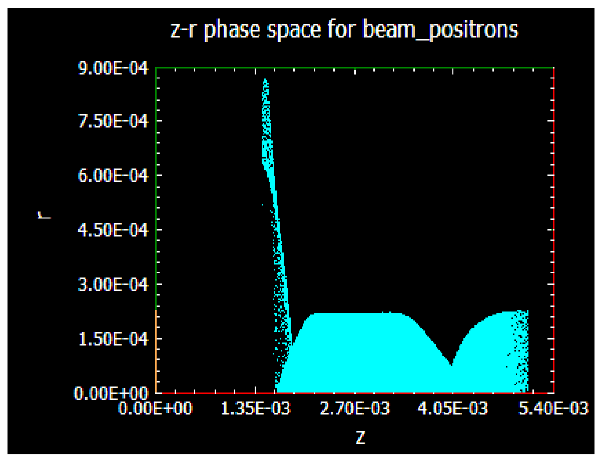

Fig. 5 Acceleration of the beam

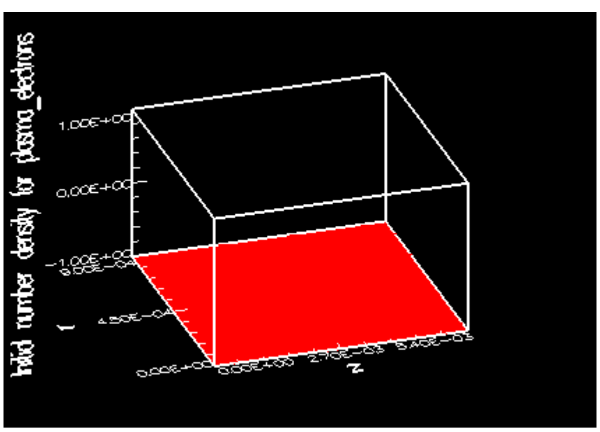

Fig. 6 Initial number density for plasma electrons.

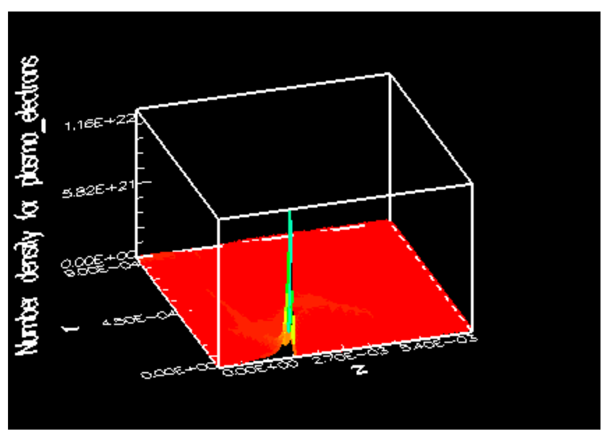

Fig. 7 Number density of plasma electrons after propagation of the beam.

The peak accelerating field (longitudinal electric field) as a function of perturbed density generated by the wake is shown in Fig. 4 with higher resolution, the peak field on axis is greater than 1GV/m. Fig. 5 shows the resulting acceleration of beam particles after 1m propagating through the lithium plasma.

The initial number density of plasma electrons and the corresponding number density of plasma electrons after the positron beam passes or after the full propagation of the beam are indicated below in Figs 6 and 7. This shows that the plasma is not perturbed until after the beam passes through the plasma.

7.0 Conclusion

We have investigated the interaction of a positron beam with plasma using the object-oriented particle-in-cell code OOPIC, which is time explicit and fully electromagnetic. It is found that the plasma electrons are blown out creating a large amplitude wake of about 3×10^{-4} m, driven by a strong axial electric field E_Z of order 1 GV/m, which is used in accelerating the electrons to high energies. This is in good agreement with the Stanford linear accelerator centre (SLAC) E-157 experiment [9].

Acknowledgments

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