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The Effects of Charged Dust Inhomogeneities on the Propagation of Low Frequency Waves in a Magnetized Plasma

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Abstract

A linear dispersion relation for magnetized, collisionless dusty plasma containing Boltzmann distributed ions and electrons, and highly negatively charged micron sized dust grains has been derived. The effects of charged dust inhomogeneities on the propagation of low-frequency waves have been investigated. It is found that the presence of the charged dust grains modifies the existing plasma wave spectra and that there is damping of the normal modes even in the absence of dust charge dynamics.

***Keywords*:** Dispersion, inhomogeneity, low frequency, normal modes, damping.

1.0 Introduction

Plasmas and dust are both ubiquitous ingredients of the universe. The interplay between the two has opened up a new and fascinating research domain: that of dusty plasmas, containing charged dust grains besides the usual plasma constituents.

The study of dusty plasma is important to at least three different scientific communities of plasma research namely astrophysics and space science, semiconductor manufacturing and basic plasma physics research. Astrophysicist and space theorists were the first who took interest in the study of dusty plasmas because the places like planetary rings, comet tails and nebulae in the universe are full of dusty plasmas.

2.0 Basic Equations

We consider a three-component, warm, non-uniform, magnetized dusty plasma system consisting of negatively charged, extremely massive ($10^6 – 10^{12}$ the mass of a typical ion) dust grains, positively charged ions and electrons. This dusty plasma system is assumed to be immersed in an external static magnetic field. In the low-frequency regime of the electrostatic waves we are considering, the phase speed of the waves is small compared to the thermal speed of the electrons and ions. The inertia of both of these species may then be neglected. Hence, the electron and ion number densities are governed by Boltzmann distributions, each defined by their respective temperatures which we assume to be constant:

$$n_e = n_{e0} \exp\left(\frac{e\phi}{T_e}\right) \quad (1)$$

and

$$n_i = n_{i0} \exp\left(-\frac{e\phi}{T_i}\right) \quad (2)$$

where e is the electronic charge (without sign), $n_{e0}(n_{i0})$ the equilibrium number density of electrons (ions), ϕ the self-consistent electric field potential, and $T_e(T_i)$ the temperature of the electrons (ions)

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The dust grains, on the other hand, being the heaviest component, provide the inertia and therefore the wave dynamics are governed by the full set of dust fluid equations including the Poisson equation. Thus, we have the continuity equation

$$\frac{\partial n_d}{\partial t} + \nabla.(n_d \mathbf{u_d}) = 0 \quad (3)$$

the momentum equation

$$\frac{\partial \mathbf{u_d}}{\partial t} + (\mathbf{u_d}.\nabla)\mathbf{u_d} = -\frac{q_d}{m_d}\nabla\phi + \omega_{cd}\hat{\mathbf{e}}_\mathbf{z}\ x\ \mathbf{u_d} - \frac{V_{td}^2}{n_d}\nabla n_d \quad (4)$$

the Poisson equation

$$\nabla^2\phi = -4\pi(q_d n_d + en_i + en_e) \quad (5)$$

In eqns (3) to (5) the subscript d refers to the dust component and n_d, u_d, m_d and q_d represent the number density, fluid velocity, mass and charge, respectively (q_d includes the sign of the dust charges). Also $V_{td}\left(=\sqrt{T_d/m_d}\right)$ is the dust thermal speed and $\omega_{cd}\left(=q_d B_0/m_d\right)$ is the cyclotron frequency of the dust particles, B_0 being the external magnetic field.

When we consider only charging from ion and electron fluxes, q_d is calculated from the relation [1]

$$\frac{\partial q_d}{\partial t} + (\mathbf{u_d}.\nabla)q_d = I_e + I_i \quad (6)$$

where I_e is the electron current and I_i is the ion current. When the streaming velocities of the electrons and ions are much smaller than their respective thermal velocities, the expressions for the electron and ion currents for spherical grains of radius R are given by [1 - 2].

$$I_e = -\pi R^2 e\left(\frac{8T_e}{\pi m_e}\right)^{1/2} n_e \exp\left(\frac{e\psi}{T_e}\right) \quad (7)$$

$$I_i = \pi R^2 e\left(\frac{8T_i}{\pi m_i}\right)^{1/2} n_i\left(1 - \frac{e\psi}{T_i}\right) \quad (8)$$

where ψ is the dust grain surface potential relative to the plasma potential and is given by $\psi = \frac{q_d}{R}$. Equations (3) – (6) constitute a complete set of governing equations, and a linear dispersion relation can be obtained by Fourier analyzing the perturbed equations around an equilibrium. Accordingly, we expand $\Psi = \Psi_0 + \Psi_1$ where $\Psi = (n_d, \mathbf{u_d}, \phi_d, q_d)$ and the subscript "0" and "1" denote equilibrium and perturbed quantities, respectively. Thus

$$\begin{aligned} n_d &= n_{d0}(x) + n_{d1}(x,t) \\ \mathbf{u_d} &= 0 + \mathbf{u_{d1}}(x,t) \\ \phi &= 0 + \phi_1(x,t) \\ q_d &= q_{d0}(x) + q_{d1}(x,t) \\ \psi &= \psi_0(x) + \psi_1(x,t) \end{aligned} \quad (9)$$

We next linearize equations (3) – (6) by using equation (9) and neglecting squares and products of small fluctuations. The linearized forms of these equations are

$$\frac{\partial n_{d1}}{\partial t}+(\mathbf{u_{d1}}.\nabla)n_{d0}+n_{d0}(\nabla.\mathbf{u_d})=0 \tag{10}$$

$$\frac{\partial \mathbf{u_{d1}}}{\partial t}=-\frac{q_{d0}}{m_d}\nabla\phi_1+\omega_{cd}\hat{\mathbf{e}}_{\mathbf{z}}\ x\ \mathbf{u_{d1}}-\frac{V_{td}^2}{n_{d0}}\nabla n_{d1}+\frac{V_{td}^2}{n_{d0}}\frac{n_{d1}}{n_{d0}}\nabla n_{d0} \tag{11}$$

$$\left(\frac{\partial}{\partial t}+\omega_1\right)q_{d1}=-(\mathbf{u_{d1}}.\nabla)q_{d0}-R\omega_2\phi_1 \tag{12}$$

$$\left(1-\lambda_D^2\nabla^2\right)\phi_1=4\pi\lambda_D^2 q_{d0}n_{d1}+4\pi\lambda_D^2 n_{d0}q_{d1} \tag{13}$$

where

$$\omega_1=\frac{R}{\sqrt{2\pi}}\left[\frac{\omega_{pi}}{\lambda_{Di}}+\frac{\omega_{pe}}{\lambda_{De}}\exp\left(\frac{e\psi_0}{T_e}\right)\right] \tag{14}$$

$$\omega_2=\frac{R}{\sqrt{2\pi}}\left[\frac{\omega_{pi}}{\lambda_{Di}}\left(1+\frac{e\psi_0}{T_i}\right)+\frac{\omega_{pe}}{\lambda_{De}}\exp\left(\frac{e\psi_0}{T_e}\right)\right] \tag{15}$$

In equation (13), $\lambda_D=\left(\lambda_{De}^{-2}+\lambda_{Di}^{-2}\right)^{-1/2}$ is the global Debye length and $\lambda_{Dj}^2=T_j/4\pi e^2 n_{j0}$.

Also $\omega_{pj}^2=4\pi e^2 n_{j0}/m_j$

3.0 Derivation of the Dispersion Relation

Without loss of generality, we consider only waves propagating in the *x*-axis, so that

$$\nabla=\hat{\mathbf{e}}_{\mathbf{x}}\frac{\partial}{\partial x} \tag{16}$$

On the other hand we shall take the static magnetic field to be along the *z*-direction:

$$\mathbf{B_0}=\hat{\mathbf{e}}_{\mathbf{z}}B_0 \tag{17}$$

If we assume that first order terms vary as $\sim\exp[i(kx-\omega t)]$ where ω and k are the frequency and wave number, respectively, we Fourier transform eqns (10) – (13), i.e. we set $\frac{\partial}{\partial t}=-i\omega$ and $\frac{\partial}{\partial x}=ik$. Eqn (10) then gives

$$-i\omega n_{d1}+n_{d0}\left[\frac{1}{n_{d0}}\frac{dn_{d0}}{dx}+ik\right]u_{dx}=0 \tag{18}$$

Eqn (11) gives

$$-i\omega\mathbf{u_{d1}}=-i\frac{q_{d0}}{m_d}k\phi_1\hat{\mathbf{e}}_{\mathbf{x}}+\omega_{cd}\left(\hat{\mathbf{e}}_{\mathbf{y}}u_{dx}-\hat{\mathbf{e}}_{\mathbf{x}}u_{dy}\right)$$
$$-ikV_{td}^2\frac{n_{d1}}{n_{d0}}\hat{\mathbf{e}}_{\mathbf{x}}+k_{Ln}V_{td}^2\frac{n_{d1}}{n_{d0}}\hat{\mathbf{e}}_{\mathbf{x}} \tag{19}$$

Eqn (12) gives

$$(\omega_1-i\omega)q_{d1}=-u_{dx}\frac{dq_{d0}}{dx}-R\omega_2\phi_1 \tag{20}$$

Eqn (13) gives

$$\left(1+k^2\lambda_D^2\right)\phi_1=4\pi\lambda_D^2 q_{d0}n_{d1}-4\pi\lambda_D^2 n_{d0}q_{d1} \tag{21}$$

First of all, eqn (18) yields

$$\frac{n_{d1}}{n_{d0}} = \frac{k - ik_{Ln}}{\omega} u_{dx} \tag{22}$$

where k_{Ln} is the scale length for the dust number density gradient and is given by

$$k_{Ln} = \frac{1}{n_{d0}} \frac{dn_{d0}}{dx} \tag{23}$$

Substituting this in eqn (19) we obtain the components of the fluid velocity as

$$u_{dx} = \frac{q_{d0} k\omega}{m_d \left[\omega^2 - \omega_{cd}^2 - \left(k^2 + k_{Ln}^2\right) V_{td}^2\right]} \phi_1 \tag{24}$$

$$u_{dy} = \frac{iq_{d0} k\omega_{cd}}{m_d \left[\omega^2 - \omega_{cd}^2 - \left(k^2 + k_{Ln}^2\right) V_{td}^2\right]} \phi_1 \tag{25}$$

$$u_{dz} = 0 \tag{26}$$

Eqn (26) shows that there is no drift of particles in the *z*-direction. Using eqn (24) in eqn (20) and eqn (22) we obtain

$$q_{d1} = -\left[\frac{q_{d0}^2 k k_{Lq} \omega}{m_d \left(\omega_1 - i\omega\right)\left[\omega^2 - \omega_{cd}^2 - \left(k^2 + k_{Ln}^2\right) V_{td}^2\right]} + \frac{R\omega_2}{\omega_1 - i\omega} \right] \phi_1 \tag{27}$$

and

$$n_{d1} = \frac{k\left(k - ik_{Ln}\right) q_{d0} n_{d0}}{m_d \left[\omega^2 - \omega_{cd}^2 - \left(k^2 + k_{Ln}^2\right) V_{td}^2\right]} \phi_1 \tag{28}$$

k_{Lq} in eqn (27) is the scale length for the dust charge density gradient and is given by

$$k_{Lq} = \frac{1}{q_{d0}} \frac{dq_{d0}}{dx} \tag{29}$$

Substituting eqn (27) and (28) in eqn (21) we obtain the following dispersion relation

$$\left[1 + k^2 \lambda_D^2 + f\Delta\right]\left[\omega^2 - \omega_{cd}^2 - \left(k^2 + k_{Ln}^2\right) V_{td}^2\right]$$

$$= k^2 \lambda_D^2 \omega_{pd}^2 - \frac{k k_{Lq} \lambda_D^2 \omega \omega_1 \omega_{pd}^2}{\omega_1^2 + \omega^2} - i\left[k k_{Ln} \omega_{pd}^2 \lambda_D^2 + \frac{k k_{Lq} \lambda_D^2 \omega^2 \omega_{pd}^2}{\omega_1^2 + \omega^2} \right] \tag{30}$$

where *f* is the fugacity and is given by

$$f = 4\pi n_{d0} R \lambda_D^2 \tag{31}$$

and

$$\Delta(\omega) = \frac{\omega_2}{\omega_1 - i\omega} \tag{32}$$

Eqn (30) gives the modified dispersion relation for low frequency electrostatic waves due to dust charge inhomogeneities over arbitrary fugacity range.

4.0 Discussion of Result

Eqn (30) which gives the general expression for the dispersion relation for low frequency electrostatic waves due to dust charge inhomogeneities over arbitrary fugacity range looks quite horrible. The problem will, however, become more tractable if we consider the case in which $|\omega| << \omega_1$. This means that we may neglect ω/ω_1 compared to unity. The distinct advantage of this approximation is that we now have an expression that permits comparison with the results obtained by other authors. With this approximation, eqn (30) reduces to

$$\left[1 + k^2\lambda_D^2 + f\delta\right]\left[\omega^2 - \omega_{cd}^2 - \left(k^2 + k_{Ln}^2\right)V_{td}^2\right] = k^2\lambda_D^2\omega_{pd}^2 - ikk_{Ln}\omega_{pd}^2\lambda_D^2 \qquad (33)$$

where $\delta = \omega_2/\omega_1$.

The vast majority of studies that have taken into account dust charge variation effects have come up with a major conclusion: In tenuous dusty plasma characterized by low fugacity f $(f << 1)$ grain charge fluctuations typically leads to damping of wave modes, which otherwise propagates as normal modes [1,3 - 5].

We see from the last term in eqn (33) that even in the absence of grain charge fluctuations there will be damping. This is due to the presence of the dust number density gradient (characterized by k_{Ln}). This is one of the effects we have seen in this study.

Let us make some further assumptions. If the plasma is unmagnetized, homogeneous and tenuous, eqn (30) reduces to

$$1 + \frac{1}{k^2\lambda_D^2} = \frac{\omega_{pd}^2}{\omega^2 - k^2V_{td}^2} \qquad (34)$$

This is just the dust-acoustic wave predicted in [6] and shown to exist experimentally in [7]. In the presence of a magnetic field this dust-acoustic wave is modified to

$$1 + \frac{1}{k^2\lambda_D^2} = \frac{\omega_{pd}^2}{\omega^2 - \omega_{cd}^2 - k^2V_{td}^2} \qquad (35)$$

In the long wavelength regime, where $k^2\lambda_D^2 \to 0$ eqn (35) reduces to

$$\omega^2 = \omega_{cd}^2 + k^2V_{td}^2 \qquad (36)$$

which is the electrostatic dust cyclotron wave obtained in [8].

In the absence of inhomogeneities and if there is no magnetic field eqn (30) becomes

$$\frac{\omega^2}{k^2} = \frac{\omega_{pd}^2\lambda_D^2}{1 + k^2\lambda_D^2 + f\delta} + V_{td}^2 \qquad (37)$$

in the high fugacity range $(f\delta >> 1)$. This is the Dust-Coulomb wave predicted by Rao [9].

5.0 Conclusion

In this paper, we have presented a comprehensive study of the propagation of low-frequency waves in magnetized, collisionless dusty plasmas. The effects of charged dust inhomogeneities on the propagation of these waves were investigated. It is found that the presence of the charged dust grains modifies the existing plasma wave spectra and that there is damping of the normal modes even in the absence of dust charge dynamics. The results presented here should be helpful in interpreting the low-frequency electrostatic noises in space and astrophysical plasma.

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