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The Effect of Viscosity and Thermal Conductivity on Magnetohydrodynamic Two-Phase Flow under Optically Thick Limit Radiation.

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Abstract

This paper investigates the effect of viscosity and thermal conductivity on magnetohydrodynamic two-phase flow under optically thick limit radiation in which an open-ended vertical channel is taken as the domain and the boundary is given as $-1 \leq \xi \leq 1$. The solutions for temperature, velocity and the induced magnetic field for both gas and liquid were obtained, in the optically thick limit radiation using the method of successive approximation. The Continuity, Momentum and Energy equations were formulated, non-dimensionalized and solved. We observed that increase in radiation parameter for gas and liquid increase the rate of heat transfer to the fluid (gas and liquid) and this leads to an increase in temperature. Also increase in velocity for gas and liquid decrease as the radiation parameter increases. It was also observed that an increase in radiation parameter causes an increase in the flow rate of both gas and liquid.

***Keywords*:** MHD, Two-Phase Flow, Thermal Radiation, Conductivity, Viscosity.

1.0 Introduction

The subject of two-phase gas-liquid (or, in general, multiphase) flow is increasingly important in engineering design and technology, particularly for the processing industries such as oil and gas pipelines and nuclear technology. Further, applications of two-phase flow are relevant not only to engineering and modern scientific problems, but also to natural phenomena, and hence, requires additional investigation [1 – 29].

Research on the flow of an electrically conducting fluid by electromagnetic field in a vertical channel is recently of much interest, due to its importance in the design of magneto hydrodynamic generators, shock tubes or cross-field accelerators and pumps.

Various studies have been carried out in this area, notable among them are analysis of forced convection heat transfer to an electrically conductivity liquid flowing through a vertical channel with transverse magnetic field [1, 5, 8]. The limitation of the above studies is that they do not take into account heat transfer by radiation which will be significant when we are concerned with space application and higher operating temperatures.

Gupta and Gupta considered the radiation effect on hydro magnetic convection in a vertical channel in the optically thin limit case. They obtained analytic solution for temperature, velocity and induced magnetic field [2].

The analysis in [10] provided an improvement by finding the effects of variable parameters on magnetohydrodynamic two-phase flow under an optically thin limit radiation. In spite of that, in this paper we investigate the effects of viscosity and thermal conductivity on magnetohydrodynamic two-phase flow under optically thick limit radiation.

Suneetha et al [3] worked on the magnetohydrodynamic two-phase fluid flow which has engaged the attention of a number of researchers. They worked on the thermal radiation effect on hydromagnetic free convection flow past an impulsively started vertical plate with variable surface temperature and concentration, taking into account the heat due to viscous dissipation. A parametric study was performed to liberate the influence of radiation parameter, magnetic parameter, Grashof number, Prandtl number, 'Eckert number on the velocity, temperature and concentration profiles. The numerical result reveals that an increase in thermal radiation reduces both the rise in viscous dissipation and acceleration of the flow.

Dulal Pal and Mondal worked on Hydromagnetic non-Darcy flow and heat transfer over a stretching sheet in the presence of thermal radiation and ohmic dissipation[4].

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Stamenkovic et al considered MHD flow and heat transfer of two immiscible fluids between moving plates [7]. The partial differential equations governing the flow and heat transfer are solved analytically with appropriate boundary conditions for each fluid. They observed that decrease in magnetic field inclination angle flattens out the velocity and temperature profiles. Increase in Hartmann number, velocity gradients, temperature in the middle of the channel decreases and near the plate's increases. Induced magnetic field is evidently suppressed with an increase of the Hartmann number.

Also, Chauhan and Rastogn worked on radiation effect on natural convection MHD flow in a rotating vertical porous channel partially filled with a porous medium [6]. The two infinite vertical porous plates of the channel are subjected to a constant injection velocity at the one plate and the same constant suction velocity at the other plate.

Rajesh worked on the effect of a uniform transverse magnetic field in the free convection flow of an electrically conducting fluid past an uniformly accelerated infinite, vertical porous plate through a porous medium. Expressions for the velocity field and skin friction are obtained by the Laplace transform technique [12].

Israel Cookey worked on the combined effects of thermal radiation and transverse magnetic field on steady flow of electrically conducting optically thin fluid through a horizontal channel filled with saturated porous medium and non-uniform wall temperature [15].

Ansari and Ghiasi worked on the hydrodynamical instability initial criterion in two phase stratified flow in a horizontal duct [9]. The non linear two mass and two momentum, conservation equations are used for numerical simulation using the two-phase, two-fluid model. The model was solved using finite volume and spectral methods respectively.

Also, Zeidan worked on the numerical resolution for a compressible two-phase flow model based on the theory of thermodynamically, compatible systems. The equations constitute a non homogeneous system of non linear hyperbolic conservation laws [11].

Jyothi Bala and Varma studied the unsteady MHD heat and mass transfer flow past a semi-infinite vertical porous moving plate with variable suction in the presence of heat generation and homogenous chemical reaction [13]. They analyzed the effect of magnetic field and heat and mass transfer on unsteady two dimensional laminar flow of a viscous incompressible electrically conducting fluid past a semi infinite moving vertical porous plate under the influence of a uniform transverse magnetic field with temperature dependent heat generation and homogenous first order chemical reaction. The analytical expression for the velocity, temperature and mass concentration are obtained. The effects of material parameters like Grashof number for heat transfer, Grashof number for mass transfer, Prandtl number, Magnetic parameter, permeability parameter, Schmidt number and chemical reaction parameters on velocity, temperature and mass concentration are discussed through graphs.

Anuar Ishak worked on the effect of radiation on magnetohydrodynamic (MHD) boundary layer flow of a viscous fluid over an exponentially stretching sheet [14]. The governing system of partial differential equations was transformed into ordinary differential equation before being solved numerically by an implicit finite- difference method.

Usman et al worked on the effect of variable parameters on magnetohydrodynamic two-phase flow under optically thin limit radiation in which an open ended vertical channel was taken as the domain in the interval $-1 \leq \xi \leq 1$. The solution for temperature of the liquid and gas, velocity of both liquid and gas and the induced magnetic field for liquid and gas were obtained in the optically thin limit radiation using the method of successive approximation [10].

In [10], the Continuity equation, the Momentum equation and the Energy equation were developed to study the radiation heat flux. Increase in radiation parameter for liquid and gas was found to increase the rate of heat transfer to the fluid (liquid and gas) and it led to a decrease in temperature. It was also verified that the velocity for gas and liquid decreases as the radiation parameter increases. Further, it was also found that increase in the' Hartmann number for both liquid and gas, led to decrease in radiation parameters. Likewise they also observed that an increase in radiation parameter caused an increase in the flow rate for both liquid and gas.

In this paper, the work in [10] is first reviewed by taking optically thick limit radiation to replace the thin limit radiation of the previous study. The steady flow was, taken into consideration and all the parameters were varied, using the same boundary condition.

2.0 Formulation of the Problem

This project investigates the effect of viscosity and thermal conductivity on magnetohydrodynamic two-phase flow under optically thick limit radiation. In the two-phase flow it is expected that there will he laminar flow or turbulent flow according to the Reynolds number of the flow since we have two different kinds of fluid [gas and liquid]. The Reynolds number of the gas will be different from that of the liquid.

Two-phase flow can be classified into the following

(a) Liquid-Gas flow
(b) Liquid-solid flow
(c) Gas-plasma flow
(d) Plasma-solid flow
(e) Gas-plasma flow which is the mixture of different gases.

The study further focuses attention on the situation when the liquid and the gas are of the same substance and are being mixed homogeneously under the same temperature.

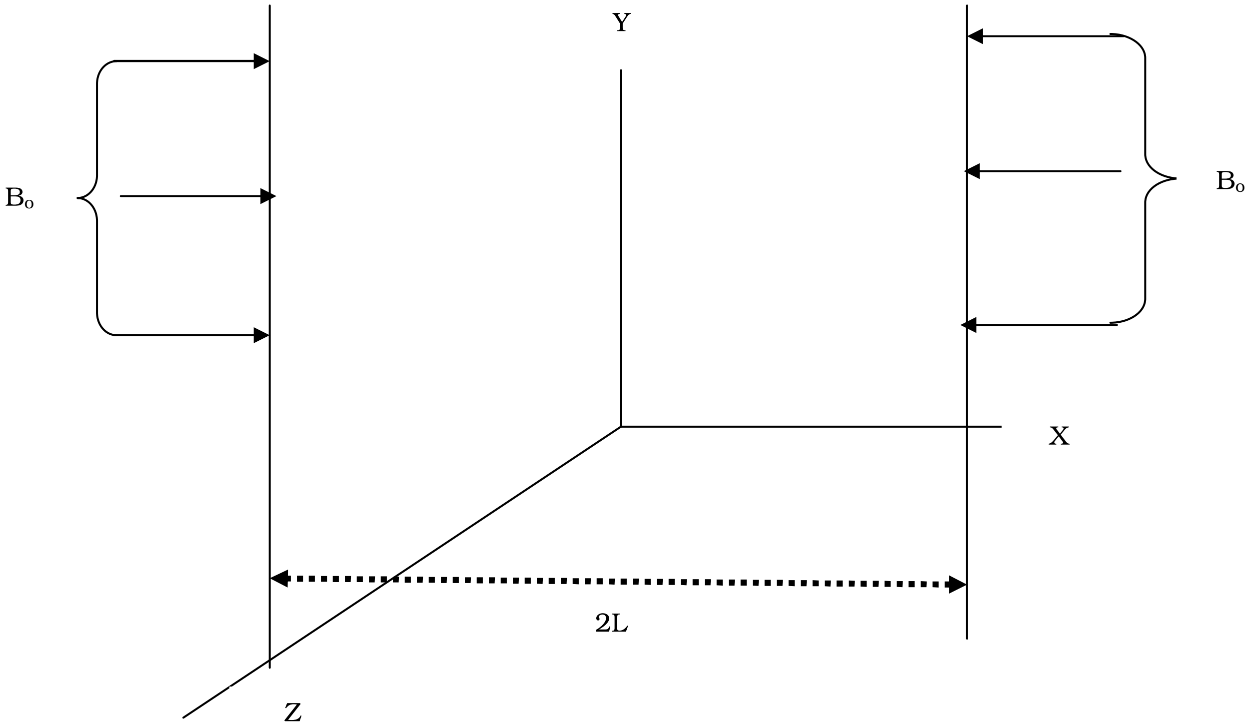

Figure 1.0: 3 – Dimensional Fluid Flow diagram

We consider two-phase magnetohydrodyanamic flow under optical thick limit radiation of an electrically conducting fluid (Liquid and gas) flowing inside an infinite vertical channel., permeated by uniform transverse magnetic field, formed by two parallel plates of distance 2L apart. (See Fig 1.0).

The diagram above consists of a vertical channel formed by two parallel plates of distance 2L. The origin is taken as the centre of the channel and we also assumed that uniform magnetic field B_0 acts transversely to the plates. Since the channel is long and for the fact that the fully developed laminar flow in a uniform magnetic field is considered and we also take into consideration the asymptotic flow valid far away from the end of the channel, thus all the physical variables except temperature and pressure are functions of Y, Y being the horizontal coordinate normal to the plate.

In optically thick limit radiation according to [14] the radiation flux vector is given as

$$q_R = \frac{-4\sigma}{3k}\frac{\partial T^4}{\partial y} \tag{1}$$

$$T = T^*(Y) + NZ \tag{2}$$

For the problem under consideration the following conditions are satisfied.

$$\underline{V} = (V_x, V_y, V_z)$$

$$\underline{V} = (O, V(y), O)$$

$$\underline{B} = (B_x, B_y, B_z,$$

$$\underline{B} = (O, B(y), O)$$

$$q_R = (O, q_{r,} O)$$

$$\frac{\partial P_g}{\partial t} + \nabla . \underline{v} = 0$$

$$\frac{\partial P_g}{\partial t} + \nabla\left(\rho_g V_g\right) = 0 \qquad (3)$$

where $\underline{V_g} = (U_g, V_g, W_g)$

Then equation (3) can be written as:

$$\frac{\partial P_g}{\partial t} + \frac{\partial}{\partial x}(\rho_g U_g) + \frac{\partial}{\partial y}(\rho_g V_g) + \frac{\partial}{\partial Z}(\rho_g W_g) = 0 \qquad (4)$$

The continuity equation for the liquid is given as

$$\frac{\partial P_L}{\partial t} + \nabla(\rho_L V_L) = 0 \qquad (5)$$

where $V_L = \left(U_L, V_L, W_L\right)$

The equation (5) can be written as

$$\frac{\partial P_L}{\partial t} + \frac{\partial}{\partial x}(P_L U_L) + \frac{\partial}{\partial y}(P_L V_L) + \frac{\partial}{\partial Z}(\rho_L W_L) = 0 \qquad (6)$$

Since the flow is steady then equation (4) and equation (6) becomes

(i.e $\frac{\partial P_q}{\partial t} = \frac{\partial P_L}{\partial t} = 0$

$$\left\{\rho_g \frac{\partial U_g}{\partial x} + \frac{\partial V_g}{\partial t} + \frac{\partial W_g}{\partial Z}\right\} + V_g(y)\nabla\rho_g = 0 \qquad (7)$$

$$\left\{\rho_L \frac{\partial U_L}{\partial x} + \frac{\partial V_L}{\partial t} + \frac{\partial W_L}{\partial Z}\right\} + V_L(y)\nabla\rho_L = 0 \qquad (8)$$

respectively, but since

$V_g = \left(O, V_g(y), O\right)$, $V_l = (O, V_L(y), O)$, hence

$$\frac{\partial U_g}{\partial x} = \frac{\partial V_g}{\partial y} = \frac{\partial W_g}{\partial Z} = 0 = \frac{\partial U_L}{\partial x} = \frac{\partial U_L}{\partial y} = \frac{\partial W_L}{\partial Z}$$

Now the equation (7) and equation (8) reduce to

$$V_g(y)\nabla\rho_g = 0$$
$$and\, V_L(y)\nabla P_L = 0 \qquad (9)$$

3.0 Momentum Equation

The momentum equation for gas and liquid respectively are given in terms of components as:

In X – component, we have

$$\rho_g \frac{DU_g}{Dt} = B_{x(g)} + \frac{\partial P_{(x,x)(g)}}{\partial x} + \frac{\partial P_{(y,x)(g)}}{\partial y} + \frac{\partial P_{(Z,x)(g)}}{\partial z} + k_{gL}(U_L - U_g) \qquad (10a)$$

$$\rho_L \frac{DU_L}{Dt} = B_{x(L)} + \frac{\partial P_{(x,x)L}}{\partial x} + \frac{\partial P_{(y,x)L}}{\partial y} + \frac{\partial P_{(Z,x)L}}{\partial z} + k_{Lg}(U_g - U_L) \qquad (10b)$$

In Y – component, we have

$$\rho_g \frac{DV_g}{Dt} = B_{x(g)} + \frac{\partial P_{(x,x)g}}{\partial x} + \frac{\partial P_{(y,y)g}}{\partial y} + \frac{\partial P_{(Z,y)g}}{\partial z} + k_{gL}(U_L - U_g) \qquad (11a)$$

$$\rho_L \frac{DV_L}{Dt} = B_{x(L)} + \frac{\partial P_{(x,y)L}}{\partial x} + \frac{\partial P_{(y,y)L}}{\partial y} + \frac{\partial P_{(Z,x)L}}{\partial z} + k_{Lg}(U_g - U_L) \qquad (11b)$$

In Z – component, we have

$$\rho g \frac{DW_g}{Dt} = B_{Z(g)} + \frac{\partial P_{(x,Z)g}}{\partial x} + \frac{\partial P_{(y,Z)g}}{\partial y} + \frac{\partial P_{(Z,Z)g}}{\partial z} + k_{gL}(U_L - U_g) \qquad (12a)$$

$$\rho g \frac{DW_L}{Dt} = B_{Z(L)} + \frac{\partial P_{(x,Z)L}}{\partial x} + \frac{\partial P_{(y,Z)L}}{\partial y} + \frac{\partial P_{(Z,Z)L}}{\partial z} + k_{Lg}(U_g - U_L) \qquad (12b)$$

where B_x, B_y, and B_z are the body forces in $X, Y,$ and Z directions respectively U, V, W are the velocity components, while P and ρ are pressure and density respectively.

Since the channel width is constant, then all derivatives along x goes to zero, therefore (11a) and equation (11b) can be reduced to

$$\rho_g \frac{DV_g}{Dt} = B_{y(g)} \frac{\partial p_g}{\partial y} + K_{gL}(U_L - U_g) \qquad (13a)$$

$$\rho_l \frac{DV_L}{Dt} \quad B_{y(l)} \frac{\partial P_L}{\partial y} + K_{Lg}(U_g - U_L) \qquad (13b)$$

where $K_{gL} = K_{Lg}$ is the friction coefficient between the gas and the liquid which gives the interaction force between them. For steady flow and neglecting the channel porosity such that injection from above and suction below is neglected then we have

$$0 = \frac{1}{\rho_g}\left[B_{y(g)} - \frac{\partial P_g}{\partial y}\right] + E_T$$

Likewise the liquid equation becomes

$$0 = \frac{1}{\rho_L}\left[B_{g(L)} - \frac{\partial P_L}{\partial y}\right] + E_T$$

where

$$E_T = K_{gL}(U_L - U_g)$$

It then follows that the momentum equations (12a and 12b) reduced to

$$\frac{1}{\rho_g}\left\{B_z(g) - \partial \frac{P_g}{\partial z} + \frac{\partial}{\partial y}\left[\mu_g(\alpha\theta^1 + 1)\frac{\partial W_g}{\partial y}\right]\right\} + E_T = O \qquad (14a)$$

$$\frac{1}{\rho_L}\left\{B_z(L) - \partial \frac{P_L}{\partial z} + \frac{\partial}{\partial y}\left[\mu_L(\alpha\theta^1 + 1)\frac{\partial W_g}{\partial y}\right]\right\} + E_T = O \qquad (14b)$$

where

$\mu = \mu_g(\alpha\theta^1 + 1)$ and $\mu_L(\alpha\theta^1 + 1)$

Since B ≠ B_o (a constant) then we assume that the magnetic intensity is of the form.

$$B_{z(g)} = \frac{B_O}{\mu_g}\frac{\partial B_g}{\partial y} + \rho_g g\beta(\theta^1 + NZ) \qquad (15a)$$

$$B_Z(L) = \frac{B_0}{\mu_g}\frac{\partial B_L}{\partial y} + \rho_L\ g\beta\left[\theta^1 + NZ\right] \qquad (15b)$$

where θ^1 – is the temperature difference
β – is the volume of coefficient expansion
B_L – for liquid
B_g – for gas

Substitute equation (15a) and equation (15b) into equation (14a) and equation (14b) respectively, we have.

$$\frac{1}{\rho_g}\left\{B_{Z(g)} - \frac{\partial P_g}{\partial z} + \frac{\partial}{\partial y}\left\{\left[(\mu_g(\alpha\theta^1 + 1)\frac{\partial W_g}{\partial y}\right]\right\}\right\} + E_T = 0$$

$$\frac{1}{\rho_g}\left\{\frac{B_0}{\mu_g}\frac{\partial B_g}{\partial y} + \rho_g\ g\beta\ (\theta^1 + NZ) - \frac{\partial}{\partial Z}P_g + \frac{\partial}{\partial y}\left[\mu_g\ (\alpha\theta^1\ + 1)\frac{\partial W_g}{\partial y}\right]\right\} + E_T = 0$$

$$\frac{B_0}{\rho_g\mu_g}\ \frac{\partial B_g}{\partial y} + g\beta(\theta^1 + NZ) - \frac{1}{\rho_g}\ \frac{\partial P_g}{\partial z} + \frac{\mu_g}{\rho_g}\ \frac{\partial}{\partial y}(\alpha\theta^1 + 1)\frac{\partial W_g}{\partial y} + \frac{E_T}{P_g} = 0$$

$$\frac{B_o}{\rho_g\mu_g}\frac{\partial B_g}{\partial y} + g\beta\,\theta^1 + g\beta NZ\,\frac{-I}{\rho_g}\ \frac{\partial P_g}{\partial z} + \frac{\mu_g}{\rho_g}\ a\frac{\partial\theta^1}{\partial y}\ \frac{\partial W_g}{\partial y} + \frac{\mu g}{\rho_g}(\alpha\theta^1\ + 1)\ \frac{\partial W_g}{\partial y^2} + \frac{E_T}{\rho_g} = 0$$

$$\frac{B_0\partial B_g}{\rho_g\mu_g\partial y} + g\beta\theta^1 + \frac{\mu_g}{\rho_g}\alpha\frac{\partial\theta^1}{\partial y}\frac{\partial W_g}{\partial y} + \frac{\mu_g}{\mu_g}(\alpha\theta^1 + 1)\frac{\partial^2 W_g}{\partial y^2} + \frac{E_T}{P_g} = -g\beta NZ + \frac{1}{\rho_g}\frac{\partial\rho_g}{\partial Z} \qquad (16a)$$

likewise for the liquid i.e substituting equation (15b) into equation (14b)
We have;

$$\frac{B_0}{\rho_L\mu_L}\frac{\partial B_L}{\partial y} + g\beta\,\theta^1 + \frac{\mu_L}{\rho_L}\alpha\frac{\partial\theta^1}{\partial y}\frac{\partial W_L}{\partial y} + \frac{\mu_L}{\rho_L}(\alpha\theta^1 + 1)\frac{\partial W_L}{\partial y^2} + \frac{E_T}{P_L} = -\ g\beta NZ + \frac{1}{\rho_L}\frac{\partial\rho_L}{\partial Z} \qquad (16b)$$

4.0 Energy Equation

The conservation of energy equation is given by

$$\rho_g C_p\frac{DT_g}{Dt} = \frac{DP_g}{Dt} + \frac{\partial}{\partial x}(K\frac{\partial T_g}{\partial x}) + \frac{\partial}{\partial y}\ (K\frac{\partial T_g}{\partial y}) + \frac{\partial}{\partial Z}(K\frac{\partial T_g}{dZ}) - \nabla q_R \qquad (17a)$$

$$\rho_L C_p\frac{DT_g}{Dt} = \frac{DP_g}{Dt} + \frac{\partial}{\partial x}(K\frac{\partial T_g}{\partial x}) + \frac{\partial}{\partial y}\ (K\frac{\partial T_g}{\partial y}) + \frac{\partial}{\partial Z}(K\frac{\partial T_g}{dZ}) - \nabla q_R \qquad (17b)$$

where $T_L = T_g = T$
T - Is the temperature inside the fluid
K - Is the variable thermal conductivity parameter
ρ_g - is the density of gas
ρ_L - is the density for the liquid
C_p - is the specific heat at constant pressure and
q_R - is the radiative heat flux

Since there is no variation along x – direction, then we have

$$\frac{\partial}{\partial x}\frac{(K\partial T)}{\partial x} = \frac{\partial}{\partial Z}\frac{(K\partial T)}{\partial Z} = 0$$

By virtue of the definition of T in equation (2) and for the fact that the flow is steady flow,

$$\frac{\partial T}{\partial x} = 0 \quad and$$

$$U_g = V_g = 0$$

Then equation (15a) and equation (15b) become

$$C_p \rho_g W_g N = \frac{\partial}{\partial y} k \frac{dT}{\partial y} - \frac{\partial q_R}{\partial y} \tag{18a}$$

$$C_p \rho_L W_L N = \frac{\partial}{\partial y} k \frac{\partial T}{\partial y} - \frac{\partial q_R}{\partial y} \tag{18b}$$

Where

$N = \frac{\partial T}{\partial Z}$ - is the vertical temperature gradient

Also, in view of the fact that $K = K_0(\alpha\theta^1 + 1)$ equation (16a) becomes,

$$C_p \rho_g W_g N = \frac{\partial}{\partial y}(K_0(\alpha\theta^1 + 1)\frac{\partial \theta^1)}{\partial y} - \frac{\partial q_R}{\partial y} \tag{19}$$

Simplifying equation (17) we have;

$$C_P \rho_g W_g N = \frac{\partial}{\partial y}(K_o\, a \frac{(\partial\theta^1)^2}{\partial y} + K_o\,(a\theta^1 + 1)\frac{\partial^2\theta^1}{\partial y^2} - \frac{\partial q_R}{\partial y}$$

divide through by $C_p \rho_g$. we have

$$W_g N = \frac{1}{C_p \rho_g}\left\{K_O\, \alpha \frac{(\partial\theta^1)^2}{\partial y} + K_o(\alpha\theta^1 + 1)\frac{\partial^2\theta^1}{\partial y^2} - \frac{\partial q_R}{\partial y}\right\} \tag{20}$$

In the optically thick limit, the flow absorbs its own emitted radiation. This implies that there is self absorption.

The thermal radiation heat flux relation is given as:

$$q_R = -\frac{4\sigma}{3k}\frac{\partial T^4}{\partial y} \tag{21}$$

Where σ- is the Stefan boltzman constant

K- is the Roseland mean absorption coefficient.

Then, equation (21) can be linearized by expanding T into the Taylor series about T_∞ and neglecting higher order terms to give.

$$T^4 = 4T_\infty^3 T - 3T_\infty^4 \tag{22}$$

Then, substituting equation (22) into equation (21)

$$q_R = -\frac{4\sigma}{3K}\frac{\partial}{\partial y}\left(4T_\infty^3 T - 3T_\infty^4\right) \tag{23}$$

$$q_R = -\frac{4\sigma}{3K}.\; 4T_\infty^3 \frac{\partial T_\infty^4}{\partial y}$$

$$q_R = -\frac{16\sigma T_\infty^3}{3K}\frac{\partial T}{\partial y}$$

Substitute equation (23) into equation (20) we have

$$W_g N = \frac{1}{C_p \rho_g} K_o \alpha \left\{ K_0 \alpha \frac{(\partial\theta^1)^2}{\partial y} + K_o(\alpha\theta^1 + 1)\frac{\partial^2\theta^1}{\partial y^2} + \frac{16\sigma}{3k} T_\infty^3 \frac{\partial^2 T}{\partial y^2} \right\} \qquad (24a)$$

It becomes

$$W_g N = \frac{1}{C_p \rho_g} K_o \alpha \left\{ K_0 \alpha \frac{(\partial\theta^1)^2}{\partial y} + K_o(\alpha\theta^1 + 1)\frac{\partial^2\theta^1}{\partial y^2} + C\theta^1 \right\} \qquad (24b)$$

where,

$$\theta^1 = T_\infty^3 \frac{\partial^2 T}{\partial y} and$$

$$C = \frac{16\sigma}{3K}$$

5.0 Magnetic Induction Equation

By using Maxwell's equations and ohm's law which connect the electric field with current density, we can obtain the magnetic equation for both gas and liquid respectively as.

$$\frac{d^2 B_g}{dy^2} + B_o \mu_g \sigma \frac{dW_g}{dy} = O \qquad (25a)$$

$$\frac{d^2 B_L}{dy^2} + B_o \mu_L \sigma \frac{dW_L}{dy} = O \qquad (25b)$$

Where,

B_g — is the induced magnetic field for gas
B_L — is the induced magnetic field for liquid
B_0 - is the applied magnetic field.
σ _ is the electrical conductivity.
μ_g __ is the magnetic permeability for gas
μ_L - is the magnetic permeability for liquid.

Henceforth, thermal conductivity and viscosity are treated as variable parameters that is

$$\mu = \mu_o \left[\alpha\theta^1 1\right] \quad \text{and}$$

$$K = K_o \left[\alpha\theta^1 1\right]$$

We incorporate into the momentum equation for gas and momentum equation for liquid, the magnetic body force in addition to the body force due to gravity in the Y and Z directions yielding

$$\frac{B_o}{\mu_g}\frac{dB_g}{dy} + \frac{dP_g}{d_y} + E_T = \mathrm{O} \qquad (26a)$$

$$\frac{B_o}{\mu_L}\frac{dB_L}{dy} + \frac{dP_L}{d_y} + E_T = \mathrm{O} \qquad (26b)$$

Eliminating the pressure terms in equation (16a and 16b) we integrate equation (16a) with respect to y, we have;

$$P_g = -B^2{}_g + P(Z) + E_T \qquad (27)$$

Differentiating the above equation (3.27) with respect to Z we have,

$$\frac{\partial P_g}{\partial Z} = \frac{\partial F(Z)}{\partial Z} \qquad (28)$$

Which is the pressure force in Z direction substituting equation (28) into the momentum equation (16a) we have

$$\frac{B_0}{\rho_g \mu_g}\frac{\partial B_g}{\partial y} + g\beta\theta^1 + \frac{\mu_g}{\rho_g}\alpha\frac{\partial \theta^1}{\partial y}.\frac{\partial W_g}{\partial y} + \frac{(\alpha\theta^1 + 1}{\rho_g}\mu_g \frac{\partial^2 W_g}{\partial y^2} + \frac{E_T}{P_g} = -g\beta NZ + \frac{1}{\rho_g}\frac{\partial F}{\partial y}(Z) \qquad (29)$$

We can see clearly in equation (29) that the right side is a function of Z only while the left hand side is a function of y only. As a result the right hand side can be represented by a constant C_1 as shown in the next equation.

$$\frac{B_o}{\mu_g \rho_g}\frac{dB_g}{dy} + g\beta\theta^1 + \frac{\mu_g}{\rho g}\alpha\frac{d\theta^1}{dy}.\frac{dWg}{dy} + \frac{(\alpha\theta^1 + 1)}{\rho_g}\mu g\frac{d^2 Wg}{dy^2} + \frac{E_T}{\rho_g} = C_1 \qquad (30)$$

The following non-dimensional quantities are introduced.

$$\delta = \frac{K_o}{C_\rho \rho_g}, \xi = \frac{y}{L}, V_g, V_g = \frac{LW_g}{\delta}$$

$$t = -\frac{\theta^1}{NL}, b_g = \frac{B_g}{B_o}$$

$$M_g = B_o L\left[\frac{\delta}{\rho_g V_g}\right]^{1/2}$$

$P_{mg} = \delta\delta\mu_g$ = the magnetic prandt Number for gas

$R_{mg} = \dfrac{g\beta\mu L4}{\delta\mu_g}$ = Raleigh Number for gas

$$\xi = \frac{y}{L} \quad = y \ = \xi L$$

$$\frac{d}{dy} = \frac{d}{d\xi}.\frac{d\xi}{dy} = \frac{1}{L}\frac{d}{d\xi}$$

$$\frac{d^2}{dy^2} = \frac{1}{L^2}\frac{d^2}{d\xi^2}$$

$$\frac{B_o}{\mu_g \rho_g}\frac{dB_g}{dy} + g\beta\theta^1 + \frac{\mu g}{\rho_g}\alpha\frac{d\theta^1}{dy}.\frac{dW_g}{dy} + \frac{\left(\alpha\theta^1 + 1\right)}{\rho_g}\mu_g\frac{d^2 W_g}{dy^2} + \frac{E_T}{\rho_g} = C_1$$

$$\frac{B_o}{\mu_g \rho_g}.\frac{1}{L}\frac{db_g}{d\xi} + g\beta(-NLt) + \frac{\mu_g}{\rho_g}\alpha.\frac{1d}{Ld\xi}(-NLt)\frac{1d}{Ld\xi}\left(\delta V_g\right) + \frac{\left(\alpha(-NLt)+1\right)}{\rho_g}\frac{\mu_g \delta d^2}{L^3}\frac{V_g}{d\xi^2} + \frac{E_T}{\rho_g} = C_1$$

$$\frac{Bo^2}{L\rho_g \mu_g}\frac{db_g}{d\xi} - g\beta NLt - \frac{\mu_g}{\rho_g}.\frac{\alpha NL\delta}{L^2}\frac{dt}{d\xi}.\frac{dV_g}{d\xi} + \frac{\left(-\alpha.NLt+1\right)}{\rho_g}\frac{\mu_g \delta}{L^3}\frac{d^2 V_g}{d\xi^2} + \frac{E_T}{\rho_g} = C_1$$

$$\frac{Bo^2}{L\rho_g \mu_g}\frac{db_g}{d\xi} - g\beta NLt - \frac{\delta\mu_g \alpha N}{\rho_g L^2} + \frac{\delta\mu_g}{\rho_g L^3}\frac{d^2 V_g}{d\xi^2} + \frac{\alpha\mu Lt}{\rho_g}\frac{\delta\mu_g}{L^3}\frac{d^2 V_g}{d\xi^2} + \frac{E_T}{\rho_g} = C_1$$

Multiply through by $\dfrac{\rho_g L^3}{\delta\mu_g}$, we have

$$\frac{Bo^2}{\delta\mu_g}\frac{L^2}{\delta\mu_g}.\frac{db_g}{d\xi} - \frac{\rho_g gBL^2 t}{\delta\mu_g} - L\alpha N\frac{dt}{d\xi}.\frac{dV_g}{d\xi} + \frac{d^2 V_g}{d\xi^2} - \alpha NLt\frac{d^2 V_g}{d\xi^2} + \frac{L^2 V_g E_T}{\delta\mu_g} = \frac{L^2 \rho_g}{\delta\mu_g} = C_1$$

But $B_0{}^2 = \frac{M^2{}_g}{\delta L^2}\rho_g V_g$ and $gBNL^4 = R_{ag}\delta V_g$

$$\frac{M^2{}_g}{L^2}\frac{\rho_g}{\delta\mu_g}\frac{V_g L^2}{\delta\mu_g}.\frac{db_h}{d\xi} - \rho_g t\frac{R_{ag}}{\delta\mu_g}\delta V_g - L\alpha N\frac{dt}{d\xi}.\frac{dV_g}{d\xi} + \frac{d^2V_g}{d\xi^2} - \alpha NLt\frac{d^2V_g}{d\xi^2} + \frac{L^3E_T}{\delta\mu_g} = \frac{L^3\rho_g}{\delta\mu_g}C_1$$

But $\alpha LN = F_g$, and $V_g = \frac{\mu_g}{\delta V_g\rho_g}$

$$\frac{M_g{}^2}{Pm_g}.\frac{db_g}{d\xi} - C_3R_{ag}t - F_g\frac{dt}{d\xi}.\frac{dV_g}{d\xi} + \frac{d^2V_g}{d\xi^2} - F_g t\frac{d^2V_g}{d\xi^2} + \frac{L^3E_T}{\delta\mu_g} = \frac{L^3\rho_g}{\delta\mu_g}C_1$$

$$\frac{M_g{}^2}{Pm_g}.\frac{db_g}{d\xi} - C_3R_{ag}t - F_g\frac{dt}{d\xi}.\frac{dV_g}{d\xi} + (1 - F_g t)\frac{d^2V_g}{d\xi^2} + \frac{L^3E_T}{\delta\mu_g} = \frac{L^3\rho_g}{\delta\mu_g}C_1$$

Let $C_2 = L^3\rho_g C_1$

We have

$$\frac{M_g{}^2}{Pm_g}.\frac{db_g}{d\xi} - C_3R_{ag}t - F_g\frac{dt}{d\xi}.\frac{dV_g}{d\xi} + (1 - F_g t)\frac{d^2V_g}{d\xi^2} + \frac{L^3E_T}{\delta\mu_g} = \frac{L^3E_T}{\delta\mu_g} = C_2$$

$$\frac{d^2B_g}{dy^2} + B_0\sigma\mu_g\frac{dW_g}{dy} = 0$$

$$\frac{1}{L^2}\frac{d^2b_g}{d\xi^2}B_0 + B_0\sigma\mu_g\frac{\delta dV_g}{L^2d\xi} = 0$$

$$\frac{B_0}{L^2}\frac{d^2b_g}{d\xi^2} + B_0\frac{\sigma\mu_g\delta}{L^2}\frac{dV_g}{d\xi} = 0$$

Multiply through by $\frac{L^2}{B_o}$

We have

$$\frac{d^2b_g}{d\xi^2} + \delta\mu_g\sigma\frac{dV_g}{d\xi} = 0$$

Let $P_{mg} = \delta\mu_g$

$$\frac{d^2b_g}{d\xi^2} + \sigma P_{mg}\frac{dV_g}{d\xi} = 0$$

Divide through by P_{mg}

$$\frac{1}{P_{mg}}\frac{d^2b_g}{d\xi^2} + \sigma\frac{dV_g}{d\xi} = 0$$

Let $\sigma = 1$
We have

$$\frac{1}{P_{mg}}\frac{d^2 b_g}{d\xi^2} + \frac{dV_g}{d\xi} = 0 \qquad (31)$$

$$W_g N = \frac{1}{C_\rho \rho_g}\left\{K_0 \alpha\left(\frac{d\theta^1}{dy}\right)^2 + K_0(\alpha\theta^1 + 1)\frac{d^2\theta^1}{dy^2} + C\theta^1\right\}$$

$$\frac{\delta V_g}{L}\left(\frac{-\theta}{Lt}\right) = \alpha\frac{K_0}{C_p p_g}\left(\frac{d\theta^1}{dy}\right)^2 + \frac{K_o}{C_p p_g}(\alpha\theta^1 + 1)\frac{d^2\theta^1}{dy^2} + \frac{C\theta^1}{C_p p_g}$$

$$-\frac{\delta V_g \theta^1}{L^2 t} = \frac{\alpha\delta C_p p_g}{C_p p_g}\left(\frac{1}{L}\frac{d}{d\xi}(-tNL)\right)^2 + \frac{\delta C_p p_g}{C_p p_g}(\alpha(-tNL) + \frac{1}{L^2}\frac{d^2(-tNL)}{d\xi^2} + \frac{C(-tNL)}{C_p p_g}$$

$$-\frac{\delta V_g(-tNL}{L^2 t} = -\frac{\alpha\delta N^2 L^2}{L^2}\left(\frac{dt}{d\xi}\right)^2 + \frac{\delta\alpha N^2 L^2 t}{L^2}\frac{d^2 t}{d\xi^2} - \frac{\delta NL}{L^2}\frac{d^2 t}{d\xi^2} - \frac{CNLt}{C_p p_g}$$

$$\frac{\delta N V_g}{L} = -\alpha N^2 \delta\left(\frac{dt}{d\xi}\right)^2 + \delta\alpha N^2 t\frac{d^2 t}{d\xi^2} - \frac{\delta N}{L}\frac{d^2 t}{d\xi^2} - \frac{CNLt}{C_p p_g}$$

Multiply through by $\frac{L}{\delta N}$, we have

$$V_g = \alpha LN\left(\frac{dt}{d\xi}\right)^2 + \alpha NLt\frac{d^2 t}{d\xi^2} - \frac{d^2 t}{d\xi^2} - \frac{CL^2 t}{\delta C_p p_g}$$

$$V_g = F_g\left(\frac{dt}{d\xi}\right)^2 + F_g t\frac{d^2 t}{d\xi^2} - \frac{d^2 t}{d\xi^2} - \frac{CL^2 t}{\delta C_p p_g}$$

$$V_g = F_g\left(\frac{dt}{d\xi}\right)^2 + (F_g t - 1) - \frac{d^2 t}{d\xi^2} - E_n t \qquad (32)$$

Multiply through by

$$V_g = F_g\left(\frac{dt}{d\xi}\right)^2 + (F_g t - 1) - \frac{d^2 t}{d\xi^2} - E_n t$$

$$\frac{L}{\delta N}$$

Where

$$F_g = \alpha LN, E_n = \frac{CL2}{\delta C_\rho \rho_g}$$

Thus we have the following set of equations to be solved

$$\frac{{M_g}^2}{P_{mg}}\frac{db_g}{d\xi} - C_3 R_{ag} t - F_g dt.\frac{dV_g}{d\xi} + \frac{dV_g}{d\xi} + (1 - F_g t)d^2 V_g + \frac{L^3 E_T}{d\xi^2} = \frac{C_2}{\delta\mu_g} \qquad (33)$$

$$\frac{1}{P_{mg}}\frac{d^2 b_g}{d\xi^2}+\frac{dV_g}{d\xi}=0 \tag{34}$$

$$V_g = F_g\left(\frac{dt}{d_g}\right)^2 + \left(F_g t_g - 1\right)\frac{d^2 t}{dg^2} - E_n t \tag{35}$$

Where

$$F_g = \alpha LN$$

$$E_n = \frac{CL^2}{\delta}$$

$$C_2 = \frac{L^3 C_1}{\delta V}$$

$$C_3 = \frac{1}{\delta V}$$

$$C = \frac{\mu g}{\delta V_g \rho_g}$$

The boundary conditions for velocity and temperature are:

$V_g(\xi) = T(\xi) = 0$ at $\xi \pm 1$

$b_g(\xi) = 0$ at $\xi = \pm 1$

6.0 Analysis:-

By integrating equation (34), we have

$$\frac{1}{P_{mg}}\frac{d^2 b_g}{d\xi^2}+\frac{dV_g}{d\xi}=0 \quad \text{(Integration)}$$

$\frac{1}{P_{mg}}\frac{db_g}{d\xi}+V_g = C_3$ (Where C_3 is the constant of integration

$$\frac{1}{P_{mg}}\frac{db_g}{d\xi} = C_3 - V_g$$

$$\frac{db_g}{d\xi} = P_{mg}\left(C_3 - V_g\right)$$

Which implies that

$$V_g = C_3 - \frac{1}{P_{mg}}\frac{db_g}{d\xi} \tag{36}$$

Putting equation (36) into equation (35) we have

$$V_g = F_g\left(\frac{dt}{d\xi}\right)^2 + \left(F_g t - 1\right)\frac{d^2 t}{dg^2} - E_n t$$

Making the substitution

$$C_3 - \frac{1}{P_{mg}}\frac{db_g}{d\xi} = F_g\left(\frac{dt}{d\xi}\right)^2 + \left(F_g t - 1\right)d^2 t - \frac{E_n t}{d\xi^2} \tag{37}$$

Differentiate equation (36) and put the result into equation (33) and we get.

$$\frac{dV_g}{d\xi}=-\frac{1}{P_{mg}}\frac{d^2b_g}{d\xi^2}\ \text{ and}$$

$$\frac{d^2V_g}{d\xi^2}=-\frac{1}{P_{mg}}\frac{d^3b_g}{d\xi^3}$$

Make the substitution into, equation (33)

$$\frac{M^2}{P_{mg}}\frac{db_g}{d\xi}-C_3R_{ag}t-F_g\frac{dt}{d\xi}.\frac{dV_g}{d\xi}+(1-F_gt)\frac{d^2V_g}{d\xi^2}+\frac{L^3E_T}{\delta\mu_g}=C_2$$

$$\frac{M^2}{P_{mg}}\frac{db_g}{d\xi}-C_3R_{ag}t-F_g\frac{dt}{d\xi}.\frac{1}{P_{mg}}\frac{d^2b_g}{d\xi^2}+(1-F_gt)\left[-\frac{1}{P_{mg}}\frac{d^3b_g}{d\xi^3}\right]+\frac{L^3E_T}{\delta\mu_g}=C_2$$

Make $\frac{db_g}{d\xi}$ the subject of the formula from equation (35) and put the result into equation (37) we get from equation (36)

$$C_3-\frac{1}{P_{mg}}\frac{db_g}{d\xi}=F_g\left(\frac{dt}{d\xi}\right)^2+(F_gt-1)\frac{d^2t}{d\xi}E_nt$$

$$C_3-F_g\left(\frac{dt}{d\xi}\right)^2-(F_gt-1)\frac{d^2t}{d\xi^2}E_nt=\frac{1}{P_{mg}}\frac{1}{P_{mg}}\frac{db_g}{d\xi}$$

$$P_{mg}\left\{C_3-F_g\left(\frac{dt}{d\xi}\right)^2+(F_gt-1)-\frac{d^2t}{d\xi^2}-E_nt\right\}\frac{db_g}{d\xi} \tag{38}$$

substitute equation (38) into equation (37)

$$\frac{M_g{}^2}{P_{mg}}\frac{db_g}{d\xi}-C_3R_{ag}t+\frac{F_g}{P_{mg}}\frac{dt}{d\xi}-\frac{(1-F_gt)}{P_{mg}}\frac{d^2b_g}{d\xi}+\frac{L^3E_T}{\delta\mu_g}=C_2$$

$$M^2{}_gC_3-F_g\left(\frac{dt}{d\xi}\right)^2+(F_gt-1)-\frac{d^2t}{d\xi^2}-E_nt-C_3Ragt+\frac{F_g}{P_{mg}}\frac{dt}{d\xi}.\frac{d^2b_g}{d\xi^2}-\frac{(1-F_gt)}{P_{mg}}\frac{d^2b_g}{d\xi^2}+\frac{L^3Et}{\delta\mu_g}=C_2$$

$$\frac{M^2g}{P_{mg}}C_3-M_g{}^2F_g\left(\frac{dt}{d\xi}\right)^2M_g{}^2(F_gt-1)-\frac{d^2t}{d\xi}+M_g{}^2E_nt_g-C_3R_{ag}t_h+\frac{F_g}{d\xi} \tag{39}$$

$$\frac{d^2b}{d\xi^2}-\frac{(1-F_gt)}{P_{mg}}\frac{d^3b_g}{d\xi^3}+\frac{L^3E_T}{\delta\mu_g}=C_2 \tag{40}$$

$$-M^2g(F_gt-1)\frac{d^2t}{d\xi^2}+M^2{}_gE_nt_g-C_3R_{ag}t_g-M^2{}_gF_g\left(\frac{dt}{d\xi}\right)^2=C_2$$

$$M^2C_3-\frac{F_g}{P_{mg}}\frac{dt}{d\xi}.\frac{d^2b_g}{d\xi^2}+\frac{(1-F_gt)}{P_{mg}}\frac{d^3b_g}{d\xi^3}-\frac{L^3E_T}{\delta\mu_g}$$

$$-M^2 g\left(F_g t-1\right)\frac{d^2 t}{d\xi}+\left(M^2{}_g E_n - C_3 R_{ag}\right)t_g - M^2{}_g F_g\left(\frac{dt}{d\xi}\right)^2$$

$$\frac{\left(1-F_g\right)}{P_{mg}}t_g\,\frac{d^3 b_g}{d\xi^3}-\frac{F_g}{P_{mg}}\frac{dt}{d\xi},\frac{d^2 b_g}{d\xi^2}-\frac{L^3 E_T}{\delta\mu_g}+C_2 - M^2{}_g C_3$$

Equation (40) can be written as

$$-M^2{}_g\left(F_g t-1\right)t^{11}g+\left(M^2{}_g E_n - C_3 R_{ag}\right)t_g - M^2{}_g F_g\left(t^1{}_g\right)^2=\left(\frac{1-F_g}{P_{mg}}\right)t_g b^{111}{}_g\,\frac{F_g}{P_{mg}}t^1{}_g b^{11}{}_g - \frac{L^3 E_T}{\delta\mu_g}+C_4$$

Where

$$C_4 = C_2 - M^2{}_g C_3 \qquad (41)$$

Equations (35) and (36) also become,

$$V_g = C_3 - \frac{1}{P_{mg}}b^1{}_g \ (42)$$

$$\left(F_g t-1\right)t^{11}{}_g + F_g\left(t^1{}_g\right)^2 - E_n t = C_3 - \frac{1}{P_{mg}}b^1{}_g \qquad (43)$$

Respectively.

Now by ignoring the non-linear term in the equation (41) we have.

$$-M^2 g\left(F_g t-1\right)t^{11}g+\left(M^2{}_g E_n - C_3 R_{ag}\right)t_g - C_4\,\frac{L^3 E_T}{\delta\mu_g} \qquad (44)$$

Therefore equations (42), (43) and (44) are subject to the boundary conditions.

$$t = V = b = 0 \text{ at } \xi = \pm 1$$

NOTE: The subscript g denotes gas. We shall replace it with the subscript L to denote liquid in the same pattern.

7.0 Method of Solution

We use the method of successive approximation to solve the differential equations (45); (46) and 47) for gas and for liquid.

The required equations to be solved for in case of gas are;

$$M^2 g\left(1-F_g t\right)t^{11}{}_g - \left(C_3 R_{ag} - M^2{}_g E_n\right)t_g = C_4 - \frac{L^3 E_T}{\delta\mu_g} \qquad (45)$$

$$\left(F_g t-1\right)t^{11}{}_g + F_g\left(t^1{}_g\right)^2 - E_n t_g = C_3 - \frac{b^1{}_g}{P_{mg}} - \qquad (46)$$

$$V_g = C_3 - \frac{1}{P_{mg}}b^1{}_g \ (47)$$

Solving equation (45) we have

$$M^2{}_g\left(1-F_g t\right)t^{11}{}_g - \left(C_3 R_{ag} - M^2{}_g E_n\right)t_g = C_4 - \frac{L^3 E_T}{\delta\mu_g}$$

The equation becomes

$$M^2{}_g\left(1-F_g t\right)t^{11}g - \left(C_3 R_{ag} - M^2{}_g E_n\right)t_g = C_5 \qquad (48)$$

Where,

$$C_5 = C_4 - \frac{L^3 E_T}{\delta\mu_g}$$

To solve for the complementary solution, we have,

$$M^2{}_g\left(1-F_g t\right)t^{11}{}_g-\left(C_3R_{ag}-M^2{}_gE_n\right)t_g=0 \qquad (49)$$

$$M^2{}_g\left(1-F_g t\right)t^{11}{}_g-\left(C_3R_{ag}-M^2{}_gE_n\right)t_g$$

Let $t_{1g} = A \quad em_1^{\xi}$

$t_{1g} = A \quad m \quad em^{\xi}$

$t_{1g} = A \quad m^2 \quad em\xi$

Then substitute into equation (49) we have

$$M^2{}_g\left(1-F_g t\right)A^1{}_gM^2e^{m\xi}=\left(C_3R_{ag}-M_g{}^2E_n\right)A\ e\ m\xi$$

Divide both sides by A e m^{ξ} we have

$$M^2=\frac{C_3R_{ag}M^2{}_gE_n}{M^2{}_g(1-F_gt)}$$

$$M^2=+\sqrt{\frac{C_3R_{ag}M^2{}_gE_n}{M^2{}_g(1-F_gt)}}$$

Therefore; $M_1=\sqrt{\frac{C_3R_{ag}M^2{}_gE_n}{M^2{}_g(1-F_gt)}}$ and

$$M_2=-\sqrt{\frac{C_3R_{ag}M^2{}_gE_n}{M^2{}_g(1-F_gt)}}$$

Therefore; $t_{ic}=a_1e^{m\xi}+a_2e^{m_2\xi}$

To get the particular solution. i.e t_{ip}

Let $t_{ip}=C$

$t^1 1p=0$ and $t^{11}1p=0$

Substitute into the equation above, we have

$$M^2{}_g\left(1-F_gt\right)(0)-\left(C_3R_{ag}\_M^2{}_gE_n\right)C=C_5$$

We have

$$\left(C_3R_{ag}\_M^2{}_gE_n\right)C=C_5$$

$$C=\frac{-C_5}{\left(C_3R_{ag}\_M^2{}_gE_n\right)}$$

Therefore the general solution is

$$t_{1g}=a_1e^{m\xi}+a_2e^{m_2\xi}-\frac{C_5}{\left(C_3R_{ag}\_M^2{}_gE_n\right)} \qquad (50)$$

Applying the boundary conditions, t_{1g} = 0 at ξ = ± 1 and solve the two equations simultaneously, we have

$$a_1e^{-m_1}+a_2e^{m_2}-\frac{C_5}{\left(C_3R_{ag}\_M^2{}_gE_n\right)}=0 \qquad (51)$$

$$a_1e^{-m_1}+a_2e^{-m_2}-\frac{C_5}{\left(C_3R_{ag}\_M^2{}_gE_n\right)}=0 \qquad (52)$$

Let K = $\dfrac{C_5}{\left(C_3R_{ag} - M^2{}_g E_n\right)}$

Then the equations (51) and (52) become

$$a_1e^{m}{}_1 + a_2e^{m}{}_2 - k = 0 \quad (53)$$

$$a_1e^{-m}{}_1 + a_2e^{-m}{}_2 - k = 0 \quad (54)$$

from equation (53) we have

$$a_1e^{m}{}_1 = k - a_2e^{m}{}_2 \quad (55)$$

$$a_1 = \frac{K - a_2e^{m}{}_2}{e^{m}{}_1}$$

Substitute equation (55) into equation (44), we have

$$a_1e^{-m}{}_1 + a_2e^{-m}{}_2 = k$$

$$e^{-m}{}_1\left[\frac{K - a_2e^{m}{}_2}{e^{m}{}_1}\right] + a_2e^{-m}{}_2 = k$$

$$\left(K - a_2e^{m}{}_2\right)e^{-2m}{}_1 + a_2e^{-m}{}_2 = k$$

$$Ke^{-2m}{}_1 - -a_2e^{m}{}_2 -{}^{2m}{}_1 + a_2e^{-m}{}_2 = k$$

$$a_2\left(e^{-2m}{}_1 - e^{m2-2m}{}_1\right) = k - ke^{-2m}1$$

$$a_2 = \frac{k\left(1 - e^{-2m}{}_1\right)}{e^{-2m}{}_1 - e^{m2-2m}1}$$

Substitute the value of a_2 into equation (55)

$$a_1 = \frac{k - a_2e^{m}{}_2}{e^{m}{}_1}$$

$$a_1 = \frac{k - k\left(1 - e^{2m}{}_1\right)e^{m}{}_2}{\dfrac{e^{-m}{}_2 -}{e^{m}{}_1} e^{m}{}_2{}^{-2m}{}_1}$$

$$a_1 = \frac{k\left(e^{-m}{}_2 - e^{m}{}_2{}^{-2m}{}_1\right) - k\left(1 - e^{-2m}{}_1\right)e^{m}{}_2}{e^{m}{}_1\left(e - m_2 - e^{m}{}_2{}^{-2m}{}_1\right)}$$

$$a_1 = \frac{K\left(e^{-m}{}_2 - e^{m}{}_2\right)}{e^{-m}{}_1 -{}^{m}{}_2 - e^{m}{}_2 -{}^{m}{}_1}$$

Differentiating equation (50) twice we have:

$$t_{1g} = a_1e^{m\xi} + a_2e^{m}{}_2\xi - \frac{C_5}{C_3R_{ag} - M_g{}^2E_n}$$

$$t^1{}_{1g} = a_1m^2{}_1e^{m}{}_1{}^{\xi} + a_2m_2{}^2e^{m}{}_2{}^{\xi}$$

Also $\left(t^1{}_{1g}\right)^2 = \left(t^1{}_{1g}\right)\left(t^1{}_{1g}\right)$

$$= \left(a_1m_1e^{m}{}_1{}^{\xi} + a_2m_2e^{m}{}_2{}^{\xi}\right)\left(a_1m_1e^{m}{}_1{}^{\xi} + a_2m_2e^{m}{}_2{}^{\xi}\right)$$

$$= a_1m^2{}_1e^{m}{}_1{}^{\xi} + 2a_1a_2m_1m_2e^{({}^{m}{}_1 + {}^{m}{}_2)\xi} + a_2{}^2m_2{}^2e^2m_2{}^{\xi} \quad (55b)$$

Substitute equation (55b) into equation (46) we have i.e $\left(F_g - 1\right)t^{11}{}_g + F_g\left(t^1{}_g\right)^2 - E_n t_g = C_3 - \frac{1}{P_{mg}} b^1{}_g$

$$F_g t^{11}{}_g - t^{11}{}_g + F_g\left(t^1{}_g\right)^2 - E_n t_g + \frac{1}{P_{mg}} b^1{}_g = C_3$$

$$F_g\left(a_1 m_1{}^2 e^{m_1\xi} + a_2 m_2 e^{m_2\xi}\right) - a_1 m_1{}^2 e^{m_1\xi} + a_2 m_2 e^{m_2\xi} + F_g\left(a^2{}_1 m_1{}^2 e^{2m\xi} + 2a_1 a_2 m_1 m_2 e^{(m_1+m_2)\xi} + a_2{}^2 m_2{}^2 e^2 m^{2\xi}\right) -$$

$$E_n\left(a_1 e^{m_1\xi} + a_2 e^{m_2\xi} - \frac{C_5}{C_3 R_{ag} - M_g{}^2 E_n} + \frac{1}{p_{mg}} b^1{}_g = C_3\right.$$

$$F_g a_1 m_1{}^2 e^{m_1\xi} + a_2 m_2 e^{m_2\xi} - a_1 m_1{}^2 e^{m_1\xi} + a_2 m_2 e^{m_2\xi} +$$

$$F_g a^2{}_1 m^2{}_1 e^{2m_1\xi} + 2F_g a_1 a_2 m_1 m_2 e^{(m_1(m_1+m_2)\xi} + F_g a^2{}_2 m^2{}_2 e^{m_2\xi} +$$

$$E_n\left(a_2 e^{m2\xi} + \frac{C_5}{C_3 R_{ag} - M_g{}^2 E_n} + \frac{1}{P_{mg}} b^1{}_g = C_3\right. \qquad (56)$$

Integrate equation (56) and use the boundary condition b = 0 at ξ = 1 to obtain the constant of integration. The integration gives,

$$\frac{F_g a_1 m^2{}_1 e^{m_1\xi}}{m_1} + \frac{F_g a_2 m^2{}_2 e^{m_2\xi}}{m_2} - \frac{a_1 m^2{}_1 e^{m_1\xi}}{m_1} + \frac{a_2 m^2{}_2 e^{m_2\xi}}{m_2}$$

$$\frac{F_g a^2{}_1 m^2{}_1 e^{2m_1\xi}}{2m_1} + \frac{2F_g a_1 a_2 m_1 m_2 e^{(m_1+m_2)\xi}}{m_1 + m_2} + \frac{F_g a^2{}_2 m^2{}_2 e^{2m_2\xi}}{2m_2} - \frac{E_n a_2 e^{m_2\xi}}{m_2} + \frac{C_5\xi}{C_3 R_{ag} - M_g{}^2 E_n} + \frac{1}{P_{mg}} b_g = C_3\xi + C_6$$

$$F_g a_1 m_1 e^{m_1\xi} + F_g a_2 m_2 e^{m_2\xi} - a_1 m_1 e^{m_1\xi} + a_2 m_2 e^{m_2\xi} +$$

$$\frac{F_g a^2{}_1 m_1 e^{2m_1\xi}}{2} + \frac{2F_g a_1 a_2 m_1 m_2 e^{(m_1+m_2)\xi}}{(m_1 + m_2)} + \frac{F_g a^2{}_2 m^2{}_2 e^{2m_2\xi}}{2m_2} -$$

$$\frac{E_n a_2 e^{m_2\xi}}{m^2} + \frac{C_5 E_n \xi}{\left(C_3 R_{ag} - M_g{}^2 E_n\right)} + \frac{b_g}{P_{mg}} - C_3\xi = C_6 \qquad (57)$$

Therefore substitute $b = 0$ and $\xi = 1$ into equation (57) to get the constant of integration C_6. We have

$$\frac{F_g a_1{}^2 m_1 e^{2m_1}}{2} + \frac{2F_g a_1 a_2 m_1 m_2 e^{(m_1+m_2)}}{(m_1 + m_2)} + \frac{F_g a^2{}_2 m_2{}^2 e^{2m_2}}{2m_2} -$$

$$\frac{C_5 E_n}{\left(C_3 R_{ag} - M_g{}^2 E_n\right)} +$$

Therefore equation (57) becomes

$$a_1 m_1\left(F_g - 1 - \frac{E_n}{m^2{}_1}\right)e^{m_1\xi} + a_2 m_2\left(F_g - 1 - E_n\right)e^{m_2\xi} + F_g\,\frac{a_1{}^2 m_1 e^2 m_1{}^\xi}{2} +$$

$$\frac{2F_g a_1 a_2 m_1 m_2 e^{(m_1+m_2)\xi}}{(m_1 + m_2)} + \frac{F_g a_2{}^2 m_2{}^2 e^{2m_2\xi}}{2} - \frac{E_n C_5 \xi}{\left(C_3 R_{ag} - M_g{}^2 E_n\right)} - C_3\xi = C_6 \qquad (58)$$

Hence,

$$b_{1g} = P_{mg} \frac{(a_1 m_1 (F_g - 1 - E_n)}{m_1^2} e^{m_1 \xi} + a_2 m_2 \frac{(F_g - 1 - E_n)}{m_2^2} e^{m_2 \xi} + F_g a_1^2 m_1^{\xi} +$$

$$\frac{2F_g a_1 a_2 m_1 m_2 e^{(m_1 + m_2)\xi}}{(m_1 + m_2)} + \frac{F_g a_2^2 m_2^2 e^{2m_2 \xi}}{2} - \frac{E_n C_5 \xi}{(C_3 R_{ag} - M_g^2 E_n)} - C_3 = C_6 \quad (59)$$

$$b_{1g} = P_{mg} \frac{(a_1 m_1 (F_g - 1 - E_n)}{m_1^2} e^{m_1 \xi} + a_2 m_2 \frac{(F_g - 1 - E_n)}{m_2^2} e^{m_2 \xi} + F_g a_1^2 m_1 e^{2m_1 \xi} +$$

$$\frac{2F_g a_1 a_2 m_1 m_2 e^{(m_1 + m_2)\xi}}{(m_1 + m_2)} + \frac{F_g a_2^2 m_2^2 e^{2m_2 \xi}}{2} - \frac{E_n C_5 \xi}{(C_3 R_{ag} - M_g^2 E_n)} - C_3 \xi) -$$

$$a_1 m_1 \frac{(F_g - 1 - E_n)}{m_1^2} e^{m_1} + a_2 m_2 \frac{(F_g - 1E_n)}{m_2^2} e m_2 - F_g \frac{a_1^2 m_1 e^{2_1 \xi}}{2}$$

$$\frac{2F_g a_1 a_2 m_1 m_2 e^{(m_1 + m_2)\xi}}{(m_1 + m_2)} + \frac{F_g a_2^2 m_2 e^{2m_2}}{2} - \frac{E_n C_5}{(C_3 R_{ag} - M_g^2 E_n)} - C_3 = C_6 \quad (60)$$

To get V_{1g}, we differentiate equation (60) above and substitute it into equation (47).
The differentiation gives.

$$b_{1g} = P_{mg} [D_1 m_1 e^{m_1 \xi} + D_2 m_2 e^{m_2 \xi} + F_g a_1^2 m_1^2 e^{2m_1 \xi} +$$
$$2F_g a_1 a_2 m_1 m_2 e^{(m_1 + m_2)\xi} + F_g a_2^2 m_2 e^{2m_2} +$$
$$\frac{E_n C_5}{(C_3 R_{ag} - M_g^2 E_n)} - C_3]$$

$$V_{1g} = [D_1 m_1 e^{m_1 \xi} + D_2 m_2 e^{m_2 \xi} + F_g a_1^2 m_1^2 e^{2m_1 \xi} +$$
$$2F_g a_1 a_2 m_1 m_2 e^{(m_1 + m_2)\xi} + F_g a_2^2 m_2 e^{2m_2 \xi} + \frac{E_n C_5}{(C_3 R_{ag} - M_g^2 E_n)} \quad (61)$$

Where,

$$D_1 = a_1 m_1 \frac{(F_h - 1 - E_n)}{m_1^2}$$

$$D_2 = a_2 m_2 \frac{(F_h - 1 - E_n)}{m_2^2}$$

Now to solve for equation (41) we differentiate equation (60) thrice and equation (50) once, then substitute into equation (41)

$$b^1{}_{1g} = P_{mg} [D_1 m_1 e^{m_1 \xi} + D_2 m_2 e^{m_2 \xi} + F_g a_1^2 m_1^2 e^{2m_1 \xi} +$$
$$2F_g a_1 a_2 m_1 m_2 e^{(m_1 + m_2)\xi} + F_g a_2^2 m_2^2 e^{2m_2} +$$
$$\frac{E_n C_5}{(C_3 R_{ag} - M_g^2 E_n)} - C_3]$$

$$b^{11}{}_{1g} = -P_{mg} [D_1 m_1 e^{m_1 \xi} + D_2 m_2 e^{m_2 \xi} + F_g a_1^2 m_1^2 e^{2m_1 \xi} +$$

$$2F_g a_1 a_2 m_1 m_2 (m_1 + m_2) e(^{m_1 + m_2})^{\xi} + 2F_g a_2{}^2 m_2{}^3 e^{2m_2 \xi} \Big]$$

$$b^{11}{}_{1g} = -P_{mg} \Big[D_1 m_1{}^3 e^{m_1 \xi} + D_2 m_2{}^3 e^{m_2 \xi} + 4F_g a_1{}^2 m_1{}^4 e^{2m_1 \xi} +$$

$$4F_g a_2{}^2 m_2{}^4 e^{2m_2 \xi} + 2F_g a_1 a_2 m_1 m_2 (m_1 + m_2)^2 e(^{m_1 + m_2})^{\xi} \Big]$$

Differentiation of equation (50) gives,

$$t^1{}_{1g} = a_1 m_1 e^{m_1 \xi} + a_2 m_2 e^{m_2 \xi}$$

Therefore substitute the above differentiations into the RHS of equation (41), we have,

$$\frac{(1 - F_g)}{P_{mg}} t_g b^{111}{}_g - \frac{F_g}{P_{mg}} t^1{}_g b^{111}{}_g - \frac{L^3 E_T}{\delta \mu_g} + C_4$$

$$\frac{(1 - F_g)}{P_{mg}} \left(a_1 e^{2m_1 \xi} + a_2 e^{m_2 \xi} \right) - \frac{C_5}{\left(C_3 R_{ag} - M_g{}^2 E_n \right)} \Big(- P_{mg} (D_1 m_1{}^3 e_1{}^{\xi} +$$

$$D_2 = m_2{}^3 e^{m_2 \xi} + 4F_g a_1{}^2 m_1{}^4 e^{2m_1 \xi}\, 4F_g a_2{}^2 m_2{}^4 e^{2m_1 \xi} +$$

$$2F_g a_1 a_2 m_1 m_2 (m_1 + m_2)^2 e(^{m_1 + m_2})^{\xi} \Big)$$

$$+ F_g D_1 a_1 m_1{}^2 m_2 e(^{m_1 + m_2})^{\xi} \Big) + F_g D_2 a_1 m_1 m_2{}^2 e(^{m_1 + m_2})^{\xi} + F_g D_2 a_2 m_2{}^3$$

$$e^{2m_2 \xi} + 2F_g{}^2 a_1{}^3 m_1{}^4 s e^{3m_1 \xi} + 2F_g{}^2 a_1{}^2 a_2 m_1{}^3 m_2 e(^{2m_1 + m_2})^{\xi} +$$

$$2F_g{}^2 a^2{}_1 a_2 m_1{}^2 m_2 (m_1 + m_2) e(^{2m_1 + m_2})^{\xi} + 2F_g{}^2 a_2{}^3 m_2{}^4 e^{3m_2 \xi} + C_5$$

$$\left(2F_g D_1 a_1 m_1{}^3 - 4F_g{}^2 a_1{}^2 m_1{}^4 C_7 - D_1 a_1 m_1{}^3 + 4F_g C_7 a_1{}^2 m_1{}^4 \right) e^{2m_1 \xi} +$$

$$\left(2F_g D_2 a_2 m_2{}^3 - 4F_g{}^2 a_2{}^2 m_2{}^4 C_7 - D_2 a_2 m_2{}^3 + 4F_g C_7 a_2{}^2 m_2{}^4 \right) e^{2m_2 \xi} +$$

$$\left(C_7 D_1 m_1{}^3 - F_g D_1 m_1{}^3 C_7 \right) e^{m_2 \xi} + \left(C_7 D_2 m_2{}^3 - F_g D_2 m_2{}^3 C_7 \right) e^{m_2 \xi}$$

$$F_g D_2 a_2 m_2{}^3 a_1 + F_g D_1 a_2 m_1{}^3 + 2F_g{}^2 a_1 a_2{}^2 m_1 m_2 (m_1 + m_2)^2 - 2F_g{}^2 a_1$$

$$a_2 m_1 m_2 C_7 (m_1 + m_2)^2 - D_2 a_1 m_2{}^3 - D_1 a_2 m_1{}^3 +$$

$$2F_g a_1 a_2 m_1 m_2 C_7 (m_1 + m_2)^2 + F_g D_1 a_2 m_1{}^2 m_2 + F_g D_2 a_1 m_1 m_2{}^2$$

$$e(^{m_1 + m_2})^{\xi} +$$

$$\left(2F_g D_2 a_2 m_2{}^3 - 4F_g{}^2 a_2{}^2 m_2{}^4 C_7 - D_2 a_2 m_2{}^3 + 4F_g C_7 a_2{}^2 m_2{}^4 \right) e^{2m_2 \xi} +$$

$$\left(C_7 D_1 m_1{}^3 - F_g D_1 m_1{}^3 C_7 \right) e^{m_2 \xi} + \left(C_7 D_2 m_2{}^3 - F_g D_2 m_2{}^3 C_7 \right) e^{m_2 \xi}$$

$$F_g D_2 a_2 m_2{}^3 a_1 + F_g D_1 a_2 m_1{}^3 + 2F_g{}^2 a_1 a_2{}^2 m_1 m_2 (m_1 + m_2)^2 - 2F_g{}^2 a_1$$

$$a_2 m_1 m_2 C_7 (m_1 + m_2)^2 - D_2 a_1 m_2{}^3 - D_1 a_2 m_1{}^3 +$$

$$2F_g a_1 a_2 m_1 m_2 C_7 (m_1 + m_2)^2 + F_g D_1 a_2 m_1{}^2 m_2 + F_g D_2 a_1 m_1 m_2{}^2$$

$$e(^{m_1 + m_2})^{\xi} +$$

$$(6F_g{}^2 a_1{}^3 m_1{}^4 - 4F_g a_1{}^3 a_1{}^4) e^{3m_1 \xi} + (6F_g{}^2 a_2{}^3 m_2{}^4 - 4F_g a_2{}^3 m_2{}^4) e^{3m_2 \xi} +$$

$$2F_g{}^2 a^2{}_1 a_2 m_1 m_2 (m_1 + m_2)^2 - 2F_g a^2{}_1 a_2 m_1 m_2 (m_1 + m_2)^2 +$$

$$2F_g{}^2 a^2{}_1 a_2 m_1 m_2 (m_1 + m_2)^2 - 2F_g a^2{}_1 a_2 m_1 m_2 (m_1 + m_2)^2 +$$

$$(6F_g{}^2a_1{}^3m_1{}^4-4F_ga_1{}^3a_1{}^4)e^{3m_1\xi}+(6F_g{}^2a_2{}^3m_2{}^4-4F_ga_2{}^3m_2{}^4)e^{3m_2\xi}+$$
$$2F_g{}^2a^2{}_1a_2m_1m_2(m_1+m_2)^2-2F_ga^2{}_1a_2m_1m_2(m_1+m_2)^2+$$
$$2F_g{}^2a^2{}_1a_2m_1m_2(m_1+m_2)^2e(^{2m_1+m_2})^{\xi}+(4F_g{}^2a_1a_2{}^2m_2a_2{}^2m_2{}^4-4F_ga_1a_2{}^2$$
$$M_2{}^2-2F_ga_1a_2{}^2m_1m_2(m_1+m_2)^2e(^{m_1+2m_2})^{\xi}+C_5$$

Where

$$C_7=-\frac{C_5}{\left(C_3R_{ag}-M_g{}^2E_n\right)}$$
$$C_5=C_4-\frac{L^3E_T}{\left(\delta\mu_g\right)} \qquad (62)$$

The complementary equation becomes

$$t_{2C}=a_5e^{m_1\xi}+a_4e^{m_2\xi} \qquad (63)$$

The particular equation is

$$t_{2p}=a_5e^{2m_1\xi}+a_6e^{2m_2\xi}+a_7e^{m_1\xi}+a_8e^{m_2\xi}a_ge(^{m_1+m_2})^{\xi}+$$
$$a_{10}e^{3m_1\xi}+a_{11}e^{3m_2\xi}+a_{12}e^{(2m_1+m_2)^{\xi}}+a_{13}e^{(m_1+2m_2)^{\xi}}a_{14} \qquad (64)$$

we differentiate equation (64) twice and substitute into the LHS of equation (41) we have

$$t^1{}_{2p}=2a_5m_1e^{2m_1\xi}+2a_6m_2e^{2m2g}+a_7m_1e^{m1g}+a_8m_2e^{m2g}+$$
$$a_9(m_1+m_2)e(^{m_1+m_2})^{\xi}+3a_{10}m_1e^{3m_1\xi}+3a_{\mu}m_2e^{3m_2\xi}+a_{12}(2m_1+m_2)$$
$$e(^{2m_1+m_2\xi})+a_{13}(m_1+2m_2)e(^{m_1+2m_2})^{\xi}$$
$$t^{11}{}_{2p}=4a_5m_1{}^2e^{2m_1\xi}+4a_6m_2{}^2e^{2m_2\xi}+a_7m_1{}^2e^{m_1\xi}+a_8m_2{}^2e^{m_2\xi}+$$
$$a_9(m_1+m_2)^2e^{(m_1+m_2)\xi}+9a_{10}m_1{}^2e^{3m_1\xi}+9a_{11}m_2{}^2e^{3m_2\xi}+$$
$$a_{12}(2m_1+m_2)^2e^{(2m_1+m_2)}+a_{13}(2m_1+m_2)^2e^{(m_1+2m_2)\xi}$$

Now make the substitution into equation (41)

$$-M_g{}^2(F_g-1)t^{11}{}_g+(M_g{}^2E_n-C_3R_{ag}t_g-M_g{}^2F_g(t^1{}_g)^2$$
$$-M_g{}^2(F_g-1)(4a_5m_1{}^2e^{2m_1\xi}+4a_6m_2{}^2e^{2m_2\xi}+a_7m_1{}^2e^{m_1\xi}+a_8m_2{}^2e^{m_2\xi}$$
$$a_9(m_1+m_2)^2e^{(m_1+m_2)\xi}+9a_{10}m_1{}^2e^{3m_1\xi}+9a_{11}m_2{}^2e^{3m_2\xi}+$$
$$a_{12}(2m_1+m_2)^2e^{(2m_1+m_2)}+a_{13}(2m_1+m_2)^2e^{(m_1+2m_2)\xi}$$
$$M_g{}^2E_n-C_3R_{ag})(a_5e^{2m_1\xi}+a_6e^{2m_2\xi}+a_7e^{m_1\xi}+a_8e^{m_2\xi}+$$
$$a_9e(m_1+m_2)_g\,a_{10}e^{3m1g}+a_{11}e^{3m2}{}_g+a_{12}e^{(2m_1+m_2)}{}_g+$$
$$a_{13}e^{(m_1+2m2)}g+a_{14}) \qquad (65)$$

Now comparing the coefficient of the exponential terms in equations (62) and (65); we have

$$a_5=\frac{2F_gD_1a_1m_1{}^3-4F_g{}^2a^1{}_2m_1{}^4C_7-D_1a_1m_1{}^3+4F_gC_7a^2{}_1m^1{}_4}{-4M^2{}_g\left(F_g-1\right)m_1{}^2+\left(M_g{}^2E_n-C_3R_{ag}\right)}$$
$$a_6=\frac{2F_gD_2a_2m_2{}^3-4F_g{}^2a^2{}_2m_2{}^4C_7-D_2a_2m_2{}^3+4F_gC_7a^2{}_2m^4{}_2}{-4M_g{}^2\left(F_g-1\right)+\left(M_g{}^2E_n-C_3R_{ag}\right)}$$

$$a_7 = \frac{C_7 D_1 m_1^{\ 3} - F_g D_1 m_1^{\ 3} C_7}{-M^2{}_g\left(F_g - 1\right) + M_1^{\ 2} + \left(M_g^{\ 2} E_n - C_3 R_{ag}\right)}$$

$$a_8 = \frac{C_7 D_2 m_2^{\ 3} - F_g D_2 m_2^{\ 3} C_7}{-M^2{}_g\left(F_g - 1\right) + m_{21}^{\ 2} + \left(M_g^{\ 2} E_n - C_3 R_{ag}\right)}$$

$$a_9 = \frac{F_g D_2 a_2 m_2^{\ 3} a_1 + F_g D_1 a_2 m_1^{\ 3} + 2F^2{}_g a_1 a^2{}_2 m_1 m_2 (m_1 + m_2)^2}{-2F_g^{\ 2} a_1 a_2 m_1 m_2 C_7 (m_1 + m_2)^2 - D_1 a_1 m_1^{\ 3} - D_1 a_2 m_1^{\ 3}}$$

$$+ \frac{2F_g a_1 a_2 m_1 C_7 (m_1 + m_2)^2 + F_g D_1 a_2 m_1^{\ 2} m_2 + F_g D_2 a_1 m_1 m^2{}_2}{-m^2{}_g (F_g - 1)(m_1 + m_2)^2 + (M_g^{\ 2} E_n - C_3 R_{ag})}$$

$$a_{10} = \frac{6F_g^{\ 2} a_1^{\ 3} m_1^{\ 4} - 4Fga_1^{\ 3} m_1^{\ 4}}{-9M_g^{\ 2}(F_g - 1)m_1^{\ 2} + (M^2{}_g E_n - C_3 R_{ag})}$$

$$a_{11} = \frac{6F_g^{\ 2} a_2^{\ 3} m_1^{\ 4} - 4Fga_1^{\ 3} m_1^{\ 4}}{-9M_g^{\ 2}(F_g - 1)m_1^{\ 2} + (M^2{}_g E_n - C_3 R_{ag})}$$

$$a_{12} = \frac{2F_g^{\ 2} a_1^{\ 2} a_2 m_1 a_2 (m_1 + m_2)^2 - 2F_g a_1^{\ 2} a_2 m_1 m_2^{\ (m_1+m_2)^2} +}{\dfrac{2F_g^{\ 2} a_1^{\ 2} a_2 m_1^{\ 2} m^2 (m_1 + m_2}{M_g^{\ 2}(F_g - 1)(2m_1 + m_2)^2 (M_g^{\ 2} E_n - C_3 R_{ag})}}$$

$$a_{13} = \frac{4F_g^{\ 2} a_1 a_2^{\ 2} m_2^{\ 4} - 4F_g a_1 a_2^{\ 3} m_2^{\ 4} - 2F_g a_1 a_2^{\ 2} m_1 m_2 (m_1 + m_2)^2}{-M_g^{\ 2}(F_g - 1)(m_1 + 2m_2)^2 + (M_g^{\ 2} E_n - C_3 R_{ag})}$$

$$a_{14} = C_8$$

Hence,

$$t_{2g} = t_{2c} + t_{2p}$$

$$t_{2g} = a_3 e^{m_1 \xi} + a_4 e^{m_2 \xi} + a_5 e^{2m_1 \xi} + a_6 e^{2m_2 \xi} + a_7 e^{m_1 \xi} + a_8 e^{m_2 \xi} + \tag{66}$$

$$a_a e^{(m_1 + m_2)\xi} + a_{10} e^{3m_1 \xi} + a_{11} e^{3m_2 \xi} + a_{12} e^{2m_1 + m_2)\xi} + a_{13} e^{(m_1 + 2m_2)\xi} + a_{14}$$

The complete approximate solution for t_g is obtained by adding t_{1g} and t_{2g} together and obtain.

$$t_{2g} = a_1 e^{m_1 \xi} + a_2 e^{m_2 \xi} - C_7 + a_3 e^{m_1 \xi} + a_4 e^{m_2 \xi} + a_5 e^{2m_1 \xi} + a_6 e^{2m_2 \xi} +$$
$$a_7 e^{m_2 \xi} + a_8 e^{m_2 \xi} + a_9 e^{(m_1 + m_2)\xi} + a_{10} e^{3m_1 \xi} + a_{11} e^{3m_2 \xi} + a_{12} e^{(2m_1 + m_2)\xi} +$$
$$a_{13} e^{(m_1 + 2m2)}{}_g + a_{14} \tag{67}$$

Which implies that

$$t_g = (a_1 + a_3 + a_7) e^{m_1 \xi} + (a_2 + a_4 + e^{m_2 \xi} + a_5 e^{2m_1 \xi} + a_6 e^{2m_2 \xi} + a_9$$
$$e^{(m_1 + m_2)\xi} + a_{10} e^{3m_1 \xi} + a_{11} e^{3m_2 \xi} + a_{12} e^{(2m_2 + 2m_2)\xi} + a_{13} e^{(m_1 + 2m_2)\xi} + a^{14} - C_7 \tag{68}$$

Likewise with simplification, we obtain b_{2g} as

$$b_{2g} = P_{mg}(e^{m_1\xi} - e^{m_1\xi})D_1 + P_{mg}(e^{m_2\xi} - e^{m_2})D_2 + \frac{F_g a_1^2 m_1}{2}(e^{2m_1\xi} - e^{2m_1})$$
$$+\frac{F_g a_2^2 m_2}{2}(e^{2m_2\xi} - e^{2m_2}) + \frac{2F_g a_1 a_2 m_1 m_2}{(m_1+m_2)}(e^{(m_1+m_2)\xi} - e^{(m_1+m_2)} +$$
$$\frac{E_n C_5 \xi}{(C_3 R_{ag} - M_g{}^2 E_n)} - C_3(\xi+1) \qquad (69)$$

The complete approximate solution of the magnetic field for gas will be

$$b_g = b_{1g} + b_{2g}$$
$$b_g = -P_{mg} D_1 e^{m_1\xi} - P_{mg} D_2 e^{m_2\xi} + \frac{P_{mg} a_1^2 m_1}{2} - (1-F_g)e^{2m_1\xi} -$$
$$F_g a_1^2 m_1 e^{2m_1} + \frac{F_g a_2^2 m_2}{2}(1-P_{mg})e^{2m_2\xi} - F_g a_2^2 m_2 e^{2m_2} +$$
$$\frac{4F_g a_1 a_2 m_1 m_2 e^{(m_1+m_2)\xi}}{(m_1+m_2)}(1-P_{mg}) - \frac{4F_L a_1 a_2 m_1 m_2 e^{(m_1+m_2)\xi}}{(m_1+m_2)} -$$
$$\frac{E_n C_5}{(C_3 R_{ag} - M_g{}^2 E_n)}(P_{mg}\xi - 1) - \frac{(D_1 + F_g a_1^2 m_1 e^{m_1})e^{m_1}}{2}$$
$$\frac{(D_2 + F_g a_2^2 e^{m_1})e^{m_2}}{2} \qquad (70)$$

To get V_{2g}, we differentiate equation (63) and substitute into equation (47)

$$b_{2g} = P_{mg}(e^{m_1\xi} - e^{m_1})D_1 + P_{mg}(e^{m_2\xi} - e^{m_2})D_2 + \frac{F_g a_1^2 m_1}{2}(e^{2m_1\xi} - e^{2m_1})$$
$$+\frac{F_g a_2^2 m_2}{2}(e^{2m_2\xi} - e^{2m_2}) + \frac{2F_g a_1 a_2 m_1 m_2}{(m_1+m_2)}(e^{(m_1+m_2)\xi} - e^{(m_1+m_2)} +$$
$$\frac{E_n C_5 \xi}{(C_3 R_{ag} - M_g{}^2 E_n)} - C_3(\xi+1)$$
$$b^1{}_{2g} = +P_{mg} D_1 m_1 e^{m_1\xi} - P_{mg} D_2 m_2 e^{m_2\xi} + \frac{2F_g a_1^2 m_1^2 e^{2m_1\xi}}{2} +$$
$$\frac{2F_g a_2 m_2^2 e^{2m_2\xi}}{2} + \frac{2F_g a_1 a_2 m_1 m_2 (m_1+m_2)}{(m_1+m_2)} e^{(m_1+m_2)\xi} + \qquad (71)$$
$$\frac{E_n C_5}{(C_3 R_{ag} - M_g{}^2 E_n)} - C_3$$
$$b^1{}_{2g} = +P_{mg} D_1 m_1 e^{m_1\xi} + P_{mg} D_2 m_2 e^{m_2\xi} + \frac{F_g a_1^2 m_1^2 e^{2m_1\xi}}{2} +$$
$$\frac{F_g a_2^2 m_2^2 e^{2m_2\xi}}{2} + 2F_g a_1 a_2 m_1 m_2 (m_1+m_2) e^{(m_1+m_2)\xi} +$$

$$\frac{E_nC_5}{(C_3R_{ag}-M_g{}^2E_n)}-C_3 \tag{72}$$

Substitute the above equation into equation (47)

$$V_{2g}=C_3=\frac{1}{P_{mg}}b^1{}_{2g}$$

$$V_{2g}=C_3=\frac{1}{P_{mg}}\Big[P_{mg}D_1m_1e^{m_1\xi}+P_{mg}D_2m_2e^{m_2\xi}+\frac{F_ga_1{}^2m_1{}^2e^{2m_1\xi}}{P_{mg}}-$$

$$\frac{F_ga_2{}^2m^2{}_2e^{2m_1\xi}}{P_{mg}}-\frac{2F_ga_1a_2m_1m_2e^{(m_1+m_2)\xi}+}{P_{mg}}$$

$$\frac{E_nC_5}{(C_3R_{ag}-M_g{}^2E_n)}-C_3\Big]$$

$$V_{2g}=(1+P_{mg})C_3-D_1m_1e^{m_2\xi}-\frac{F_ga_1{}^2m_1{}^2e^{2m_1\xi}}{P_{mg}}-$$

$$\frac{F_ga^2{}_2m^2{}_2e^{2m_2\xi}}{P_{mg}}-\frac{2F_ga_1a_2m_1m_2e^{(m_1+m_2)\xi}}{P_{mg}}-$$

$$\frac{E_nC_5}{(C_3R_{ag}-M_g{}^2E_n)}-C_3$$

$$V_{2g}=(1+P_{mg})C_3-D_1m_1e^{m_1\xi}-D_2m_2e^{m_2\xi}-\frac{F_ga_1{}^2m_1{}^2e^{2m_1\xi}}{P_{mg}}-$$

$$\frac{F_ga^2{}_2m^2{}_2e^{2m_2\xi}}{P_{mg}}-\frac{2F_ga_1a_2m_1m_2e^{(m_1+m_2)\xi}}{P_{mg}}-\frac{E_nC_5}{(C_3R_{ag}-M_g{}^2E_n)} \tag{73}$$

Thus the velocity for gas (V_g) becomes

$$V_g=V_{1g}+V_{2g}$$

$$V_g=\left(1-\frac{1}{P_{mL}}\right)C_3+F_ga_1{}^2m_1{}^2e^{2m_1\xi}\left(1-\frac{1}{P_{mL}}\right)+F_ga^2{}_2m^2{}_2e^{2m_2\xi}\left(1-\frac{1}{P_{mL}}\right)$$

$$\frac{E_nC_5}{(C_3R_{ag}-M_g{}^2E_n)}-C_3\left(1-\frac{1}{P_{mL}}\right) \tag{74}$$

Similarly, following the same method of solution of that of gas and we have that of liquid becoming.

$$t_l=(a_1+a_3+a_7)e^{m_1\xi}+(a_2+a_4+a_8)e^{m_2\xi}+a_5e^{2m_1\xi}+a_6e^{2m_2\xi}+$$

$$a_9e(m_1+m_2)^\xi+a_{10}e^{3m_1\xi}+a_{11}+e^{3m_2\xi}+a_{12}e(^{2m_1+m_2})^\xi+a_{13}ee(^{m_1+2m_2})^\xi+a^{14}-C_7 \tag{75}$$

Likewise

$$b_L=-P_{mL}D_1e^{m_1\xi}-P_{mL}D_2e^{m_2\xi}+\frac{P_{mL}a_1{}^2m_1}{2}-(1-F_L)e^{2m_1\xi}-$$

$$F_La^2{}_1m_1e^{2m_1}+\frac{F_La^2{}_1m_2}{2}(1-P_{mL})e^{2m_2\xi}-F_La_2{}^{2m_2}+$$

$$\frac{4F_La_1a_2m_1m_2e^{(m_1+m_2)\xi}}{(m_1+m_2)}(1-P_{mg})-\frac{4F_ga_1a_2m_1m_2e^{(m_1+m_2)\xi}}{(m_1+m_2)}-$$

$$\frac{E_n C_5}{(C_3 R_{ag} - M_L{}^2 E_n)} - (P_{mL} - 1) - (D_1 + F_L a_1{}^2 m_1 e^{m_1}) e^{m_1}$$

$$(D_2 + F_L a_2{}^2 m_2 e^{m_2}) e^{m_2} \tag{76}$$

and

$$V_L = \left(1 + \frac{1}{P_{mL}}\right) C_3 + F_L a_1{}^2 m_1{}^2 e^{2m_1 \xi}\left(1 - \frac{1}{P_{mL}}\right) + F_L a_2{}^2 m_2{}^2 e^{2m_2 \xi}\left(1 - \frac{1}{P_{mL}}\right)$$

$$\frac{E_n C_5}{(C_3 R_{aL} - M_L{}^2 E_n)} - C_3\left(1 - \frac{1}{P_{mL}}\right) \tag{77}$$

8.0 Determination of the Temperature Profile

The temperature profile for gas and liquid is obtained by evaluating equation (68) and equation (75).

$$t_g = (a_1 + a_3 + a_7) e^{m_1 \xi} + (a_2 + a_4 + a_8) e^{m_2 \xi} + a_5 e^{2m_1 \xi} + a_6 e^{2m_2 \xi} +$$

$$a_9 e(m_1 + m_2)^{\xi} + a_{10} e^{3m_1 \xi} + a_{11} + e^{3m_2 \xi} + a_{12} e(^{2m_1 + m_2})^{\xi} + a_{13} e(^{m_1 + 2m_2})^{\xi} + a^{14} - C_7$$

$$t_l = (a_1 + a_3 + a_7) e^{m_1 \xi} + (a_2 + a_4 + a_8) e^{m_2 \xi} + a_5 e^{2m_1 \xi} + a_6 e^{2m_2 \xi} +$$

$$a_9 e(m_1 + m_2)^{\xi} + a_{10} e^{3m_1 \xi} + a_{11} + e^{3m_2 \xi} + a_{12} e(^{2m_1 + m_2})^{\xi} + a_{13} e(^{m_1 + 2m_2})^{\xi} + a^{14} - C_7$$

9.0 Determination of the Velocity Profile

The velocity profile for gas and liquid is given by equation (74) and equation (77).

$$V_g = \left(1 - \frac{1}{P_{mL}}\right) C_3 + F_g a_1{}^2 m_1{}^2 e^{2m_1 \xi}\left(1 - \frac{1}{P_{mL}}\right) + F_g a_2{}^2 m_2{}^2 e^{2m_2 \xi}\left(1 - \frac{1}{P_{mL}}\right)$$

$$+ \frac{E_n C_5}{(C_3 R_{ag} - M_g{}^2 E_n)}\left(1 - \frac{1}{P_{mL}}\right)$$

and

$$V_L = \left(1 - \frac{1}{P_{mL}}\right) C_3 + F_L a_1{}^2 m_1{}^2 e^{2m_1 \xi}\left(1 - \frac{1}{P_{mL}}\right) + F_L a_2{}^2 m_2{}^2 e^{2m_2 \xi}\left(1 - \frac{1}{P_{mL}}\right)$$

$$+ \frac{E_n C_5}{(C_3 R_{aL} - M_L{}^2 E_n)}\left(1 - \frac{1}{P_{mL}}\right)$$

10.0 The Induced Magnetic Field for Gas and Liquid

The induced magnetic field for gas and liquid is given by equation (71) and equation (76).

$$b_g = -P_{mg} D_1 e^{m_1 \xi} - P_{mg} D_2 e^{m_2 \xi} + \frac{P_{mg} a_1{}^2 m_1}{2} - (1 - F_g) e^{2m_1 \xi} -$$

$$F_g a^2{}_1 m_1 e^{2m_1} + \frac{F_g a_1{}^2 m_2}{2}\left(1 - P_{mg}\right) e^{2m_2 \xi} - F_g a_2{}^2 m_2 e^{2m_2} +$$

$$\frac{4F_g a_1 a_2 m_1 m_2 e^{(m_1+m_2)\xi}}{(m_1+m_2)}(1-P_{mg}) - \frac{4F_L a_1 a_2 m_1 m_2 e^{(m_1+m_2)\xi}}{(m_1+m_2)} -$$

$$\frac{E_n C_5}{(C_3 R_{ag} - M_g{}^2 E_n)} - (P_{mg} - 1) - (D_1 + \frac{F_g a_1{}^2 m_1 e^{m_1}) e^{m_1}}{2}$$

$$\frac{(D_2 + F_g a_2{}^2 m_2 e^{m_2}) e^{m_2}}{2}$$

and

$$b_L = -P_{mL} D_1 e^{m_1 \xi} - P_{mL} D_2 e^{m_2 \xi} + \frac{P_{mL} a_1{}^2 m_1}{2} - (1 - F_L) e^{2m_1 \xi} -$$

$$F_L a_1{}^2 m_1 e^{2m_1} + \frac{F_L a^2{}_1 m_2}{2}(1 - P_{mL}) e^{2m_2 \xi} - F_L a_2{}^2 m_2 e^{2m_2} +$$

$$\frac{4F_L a_1 a_2 m_1 m_2 e^{(m_1+m_2)\xi}}{(m_1+m_2)}(1-P_{mg}) - \frac{4F_L a_1 a_2 m_1 m_2 e^{(m_1+m_2)\xi}}{(m_1+m_2)} -$$

$$\frac{E_n C_5}{(C_3 R_{aL} - M_L{}^2 E_n)} - (P_{mL} - 1) - \frac{(D_1 + F_L a_1{}^2 m_1 e^{m_1}) e^{m_1}}{2}$$

$$(D_2 + F_L a_2{}^2 m_2 e^{m_2}) e^{m_2}$$

The constants C_3 and C_5 are obtained by using the boundary condition

$$b = V = 0 \quad at \quad \xi = \pm 1$$

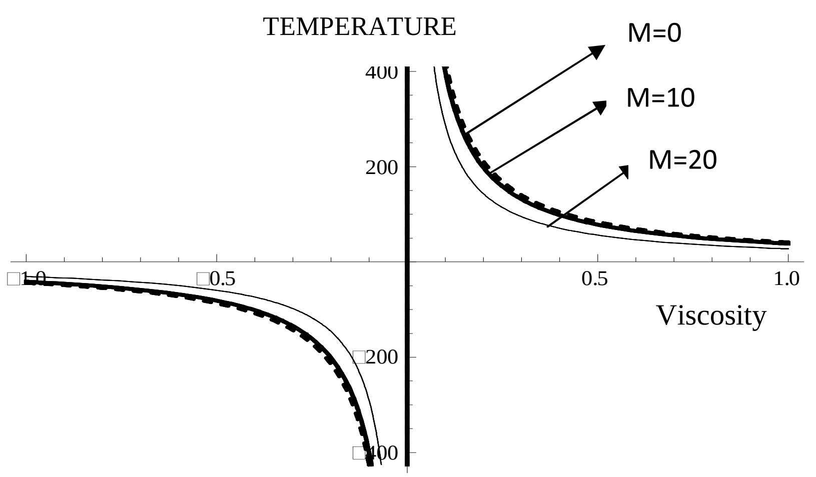

Figure 1: Effect of Viscosity on Temperature for Gas (M = Hartman number)

The Effect of Viscosity and Thermal Conductivity on… ***Usman and Onitilo J of NAMP***

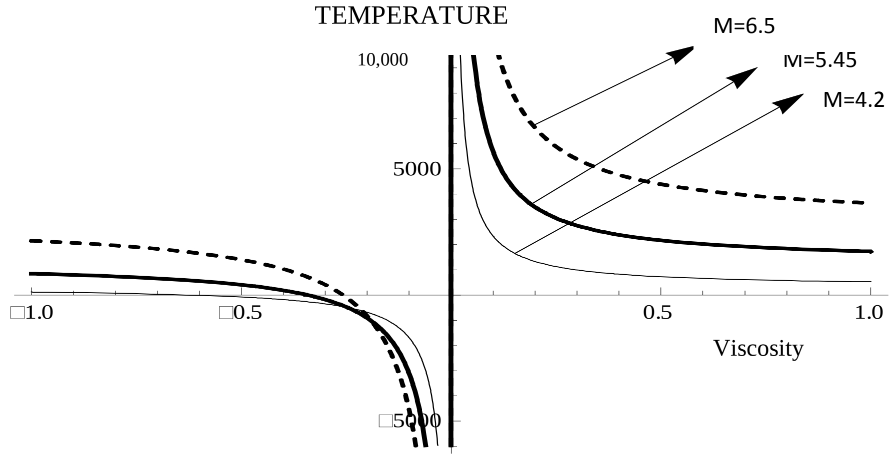

Figure 2: Effect of Viscosity on Temperature for liquid (M = Hartman number)

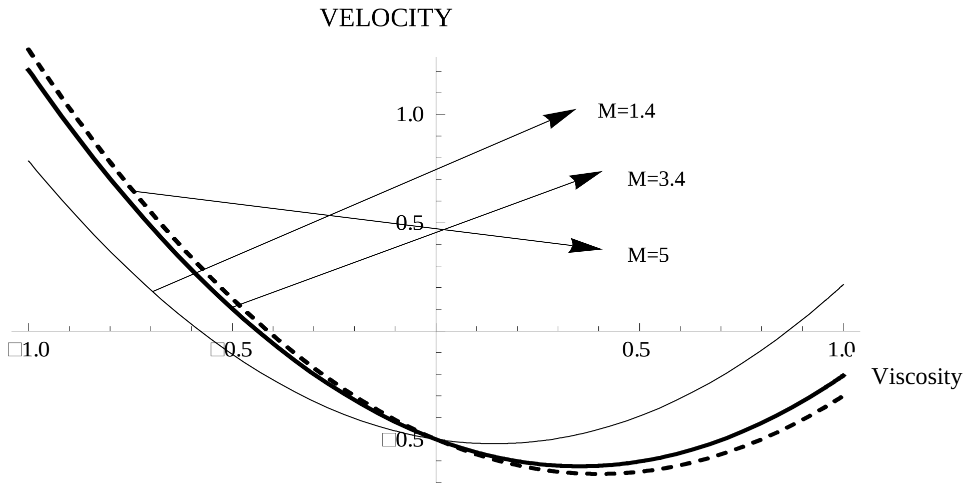

Figure 3: Effect of Viscosity on Velocity for Gas (M = Hartman number)

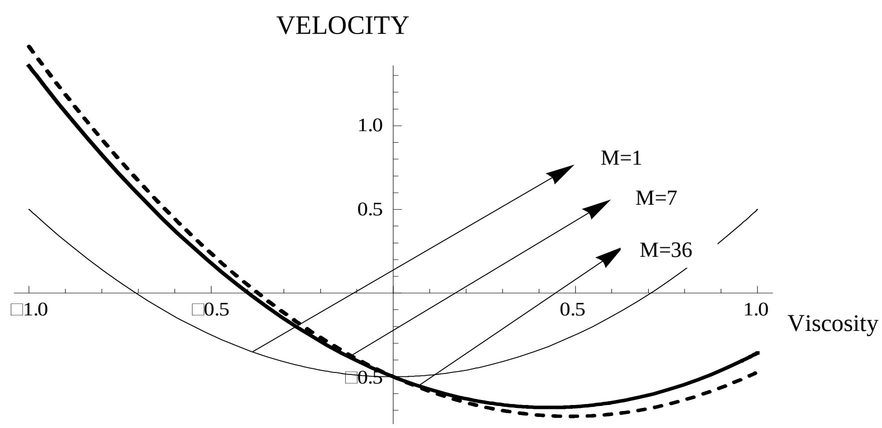

Figure 4: Effect of viscosity on Velocity for liquid (M = Hartman number)

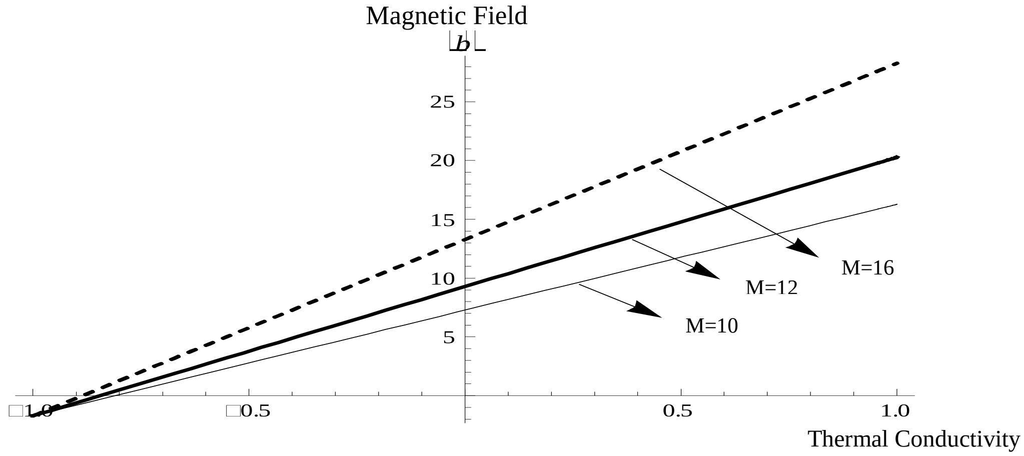

Figure 5: **Effect of thermal conductivity on Induced Magnetic Field** for Gas (M = Hartman number)

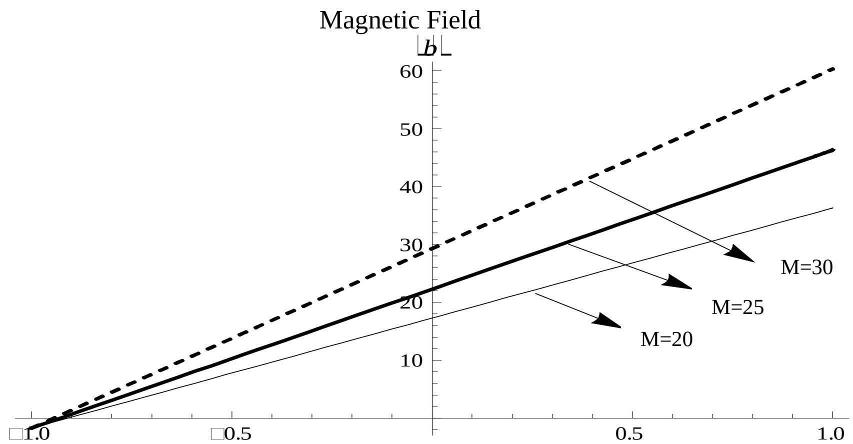

Figure 6: Effect of thermal conductivity on Induced Magnetic Field for liquid (M = Hartman number)

11.0 Discussion and Conclusion

This study examines the effect of viscosity and thermal conductivity on magnetohydrodynamic two-phase flow under optically thick limit radiation. The domain of consideration is an open-ended vertical channel, in which the flow is taken along the vertical axes and the velocity which is a function of y that is v(y) and the other axes are taken as zero, that is (V_x,V_y,V_z) = (O, V(y), O).

This study considered the two states of matter that is gas and liquid. The radiation which is one of the parameters considered in this study for both gas and liquid increase the rate of heat transfer to the gas and liquid, which leads to increase in temperature.

It can be seen clearly from Figures 1 and 2 that increase in viscosity for gas and liquid, with constant radiation parameter gives increases in temperature.

Figures 3 and 4, the velocity of both gas and liquid increase as the viscosity increases.

The induced magnetic field for gas and liquid increases when the radiation parameter is constant with increase in thermal conductivity. This can be seen from Figures 5 and 6.

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