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Modified Variational Iteration Method for the Analytical Solution of Nonlinear Advection Equations

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Abstract

In this paper, a Modified Variational Iteration Method (MVIM) for the solution of nonlinear advection equations is presented. The method is an elegant combination of the Taylor's approximation and the variational iteration method. The method is seen to be a very reliable alternative to some existing techniques for the nonlinear advection equations.

Keywords: Advection equation, Taylor's approximation, Lagrange Multiplier.

1.0 Introduction

This paper outlines a reliable method among many others; the method gives rapidly convergent series with specific significant features for the problem.
The nonlinear advection problem is of the form

$$U_t + UU_x = f(x,t)$$

Where

$$U = U(x,0) = g(x) \qquad (1)$$

The equation is homogeneous when $f(x,t) = 0$

This problem plays a crucial role in applied mathematics and physics. A substantial amount of research work has been directed for the study of the nonlinear problems and on advection problems in particular.
In this paper, our work stems mainly on Modified Variational Iteration Method (MVIM) which accurately compute the solutions of an advection equations. The main advantage of this method is that it can be applied directly for all types of advection equations either in form of homogeneous or nonhomogeneous.

2.0 The Method

The Variational Iteration Method was proposed by He [1-3]. In this paper a Modified Variational Iteration Method (MVIM) proposed by Olayiwola [4,5,6,11] is presented for the solution of nonlinear advection equations.
Modified Variational Iteration Method (MVIM) is the combination of Variational Iteration Method (VIM) and Taylor's approximation. We consider the following general nonlinear partial differential equation:

$$LU(x,t) + RU(x,t) + NU(x,t) = g(x,t) \qquad (2)$$

Where: L is a linear time derivative operator.
R is a linear operator which has partial derivative with respect to x
N is a nonlinear operator and
g is an inhomogeneous term.
According to Variational Iteration Method (VIM), we can construct a correction functional as follows:

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$$U_{n+1}(x,t) = U_n(x,t) + \int_0^t \lambda[LU_n + RU_n + NU_n - g]d\tau \qquad (3)$$

Where $\lambda = -1$ is a Lagrange multiplier which can be identified optimally via variational iteration method. The subscript "n" denoted n[th] approximation, U_n is considered as a restricted variation i:e $\partial U_n = 0.$ The successive approximation $U_{n+1,} n \geq 0$ of the solution U will be readily obtained upon using the determined Lagrange multiplier and any selective function $U_{0,}$ consequently, the solution is given by:

$$U = \lim_{n\to\infty} U_n \qquad (4)$$

In Modified Variational Iteration Method, $U_0(x,t)$ in equation (3) becomes:

$$U_0(x,t) = \sum_{i=1}^{3} g_i(x)t^i \qquad (5)$$

Where $g_i(x)$ can be found by substituting for $U_0(x,t)$ in (2) when t=0.

3.0 The Homogeneous Advection Problem

We first consider the following problem:

$$U_t + UU_x = 0$$
$$U(x,0) = -x \qquad (6)$$

Let

$$U^+(x,t) = U(x,0) + KU(x,0)t$$
$$U^+(x,t) = -x + kt \qquad (7)$$

Putting (7) into (6)

$$k + (-x + kt)(-1) = 0 \qquad (8)$$
$$k + x + kt = 0 \qquad (9)$$

When t=0, $k + x = 0$

$$k = -x \qquad (10)$$

Therefore (7) becomes

$$U^+(x,t) = -x - xt \qquad (11)$$

Similarly,

$$U^{++}(x,t) = -x - xt + kt^2 \qquad (12)$$

Then,

$$U^{++}(x,t) = -x - xt - xt^2 \qquad (13)$$

Therefore, $$U^{+++}(x,t) = -x - xt - xt^2 - xt^3 \qquad (14)$$

This method has eliminated the noise term encounter while using Variational Iteration Method as observed in $U_2(x,t)$ in [7]

Modified Variational Iteration Method (MVIM) Formula:

$$U_{n+1}(x,t) = U_n^{+++}(x,t) - \int_0^t [\frac{\partial U_n^{+++}(x,\xi)}{\partial \xi} + U_n^{+++}(x,\xi)\frac{\partial U_n^{+++}(x,\xi)}{\partial x}]d\xi, n \geq 0. \qquad (15)$$

Where

$$U(x,t) = -x - xt - xt^2 - xt^3 \qquad (16)$$

Substitute for $U(x,t), U(x,\xi), \frac{\partial U}{\partial \xi}, \frac{\partial U}{\partial x}$ in (15.0)

We have,

$$U_{n+1}(x,t) = U_n^{+++}(x,t) - \int_0^t [\frac{\partial U_n^{+++}(x,\xi)}{\partial \xi} + U_n^{+++}(x,\xi)\frac{\partial U_n^{+++}(x,\xi)}{\partial x}]d\xi, n \geq 0.$$

$$U_{n+1}(x,t) = -x - xt - xt^2 - xt^3 - \int_0^t (-x - 2x\xi - 3x\xi^2) + (-x - x\xi - x\xi^2 - x\xi^3)(-1 - \xi - \xi^2 - \xi^3)$$

$$U_1 = -x - xt - xt^2 - xt^3 - \int_0^t [-x - 2x\xi - 3x\xi^2 + x + x\xi + x\xi^2 + x\xi^3 + x\xi + x\xi^2 + x\xi^3 + x\xi^4 + x\xi^2 + x\xi^3 + x\xi^4 + x\xi^5 + x\xi^3 + x\xi^4 + x\xi^5 + x\xi^6]d\xi$$

$$U_1 = -x - xt - xt^2 - xt^3 - xt^4 - small\ term \tag{17}$$

To know U_2

$$U_2 = -x - xt - xt^2 - xt^3 - xt^4 - xt^5 - small\ term \tag{18}$$

$$\lim_{n\to\infty} U_n = \frac{x}{t-1} \tag{19}$$

Which is the exact solution.

4.0 The Non-Homogeneous Advection Problem

Consider the problem:

$$U_t + UU_x - U = e^t$$
$$U(x,0) = 1 + x \tag{20}$$

Let

$$U_1^+ = 1 + x + kt$$

$$\text{Put } U_{1,}^+ U_{t,}^+ U_x^+ \text{ into (20)}$$

$$k + (1 + x + kt)(1) - (1 + x + kt) - e^t = 0 \tag{21}$$

$$k = 1 \tag{22}$$

Substitute for k in U_1

$$U^+ = 1 + x + t \tag{23}$$

Similarly

$$U^{++} = 1 + x + t + \frac{1}{2}t^2 \tag{24}$$

$$U^{+++} = 1 + x + t + \frac{1}{2}t^2 + \frac{1}{6}t^3 \tag{25}$$

By using Modified Variational Iteration Method (MVIM)

$$U_{n+1}(x,t) = U_n^{+++}(x,t) - \int_0^t [\frac{\partial U_n^{+++}(x,\xi)}{\partial \xi} + U_n^{+++}(x,\xi)\frac{\partial U_n^{+++}(x,\xi)}{\partial x} - U_n^{+++}(x,\xi) - e^{\xi}]d\xi \tag{26}$$

Substitute for $U_n(x,t)$, $U_n(x,\xi)$, $\frac{\partial U_n(x,\xi)}{\partial \xi}$ and $\frac{\partial U_n(x,\xi)}{\partial x}$ in MVIM Formula above.

$$U_1 = x + e^t \tag{27}$$

Which is the exact solution.

5.0 Discussion and Conclusion

In this paper, we have presented a Modified Variational Iteration Method proposed in [4,5,6, 11] to both the homogeneous and non-homogeneous nonlinear advection equation. The method proved to be elegant and reliable. It is observed that the solution converges to the analytical solution after one or two iteration. This method has eliminated the noise term and the problem of trial functions. Also, the linearization of the original problem is totally eliminated.

It is also observed that this method requires very less computation and computational terms. This method is recommended for the solution of strongly nonlinear partial differential equations where the analytical solution does not exist.

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