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Investigating the Limit of Random Walk of Fractal Image Constituent Parts

[1]Sulaiman M. A., and [2]Usman M. A.

[1] Department of Mechanical Engineering,
Olabisi Onabanjo University, Ago-Iwoye. Ogun-State, Nigeria
[2] Department of Mechanical Engineering,
University of Ibadan, Ibadan, Nigeria

Abstract

Despite the wide applications of fractal science that has been considered in various literatures, this piece of work is a new area of fractal science which has not been exploited. Fractal images in this work are generated based on Iterated Function Systems codes (IFS), with the aid of a computer program written in Matlab 7.0. The limit image of random walks of fractal images constituent parts was investigated and a total of forty two fractal images were considered (seven (7) of these are original fractal images and thirty six (36) are fractal images reported at the end of specified random time steps (random transformation) and at the rate of six (6) per the original fractal images). Results obtained showed that in all the fractal images considered, the image changes completely from the original image with resemblance in all the moves, the space occupied is almost the same all through the moves and the image gravitates densely towards the centre. Empty spaces present in the original fractal images disappeared for all cases considered. In all cases considered the original fractal image trait can faintly be observed in each of the seven specified random steps reports. Fractal image (tree) produces mini forest form for all the seven specified random steps reported. In all, 10% of the cases studied affirmed the possibility of generating distinct new fractal images using this newly introduced approach.

Keywords: Iterated function systems codes, Fractals, random walks transformations,.

1.0 Introduction

The emerging science of complex (chaotic) systems has brought remarkable insights into the nature of the universe and life. It cuts across disciplinary boundaries as complex systems abound in physical, biological, mathematical, ecological, economic, meteorological and many other fields [1-16]. Geometrically, one can display the working of such complex systems as fantastic fractal images. A fractal image can be often viewed as a chaotic set of a dynamical system. This link between fractal geometry and complex systems is truly remarkable.

Geometry as being describe as being the cold and dry why is it so? One reason lies in its inability to describe the shape of a cloud, a mountain, a coaster line or a tree. In short it does not describe natural occurrences perfectly [2]. Euclidean geometry deals with lines and planes, circles, spheres, triangles and cones. As examples; mountains are not exactly cones, clouds are not spheres, coastlines are not circles, bark is not smooth nor do lighting travel in a straight line. All these shapes have irregular shapes that will be difficult to depict in Euclidean Geometry.

The existence of these natural patterns challenges the scientist to study those forms that Euclid leaves aside as being formless, to investigate the morphology of the amorphous. Thus Benoit B. Mandelbrot [2] conceived and developed a new geometry of nature and implemented its use in diverse fields. This theory was able to describe many of the irregular and fragmented patterns around us this he called Fractals. The striking principle discovered was that many of the irregular shapes that make up the natural world, although seemingly random and chaotic in form, have a simple originating principle [3].

The nature of fractals is reflected in the word itself, coined by B.B. Mandelbrot from the latin verb frangere “to breake” and the related adjective fractus, irregular and fragmented shape that can be subdivided in parts, each of which is a reduced copy of the whole [4]. Application of fractal theory allows description of various states of fragmenting and branching in

[1]Corresponding author: ***Sulaiman M. A.,*** E-mail: doyin_sulaiman1@yahoo.com, Tel.: +2348033454676 (Usman)

biological, ecological and other systems [5] and [6]. Fractal concepts have provided a new approach for quantifying the geometry of complex or noisy shapes and objects. Fractal geometry has been proven capable of quantifying irregular patterns, such as tortuous lines, crumpled surfaces and intricate shapes, and estimating the ruggedness of systems [2].

In-depth study into fractals geometry have proven to be a means by which we can store our information in a compact means, this can be achieved through Iterated Function System (IFS). Fractals IFS turn an image into a set of data and an algorithm for expanding it back into the original. Extended works in fractal image compression and fractal image enhancement have proven that fractal technique is a unique powerful technique in digital image processing that can achieve very good results and at same time compressing the image.

The role that random walk and Brownian motion play in parts of Mathematics, Physics and Chemistry is very important. Random walk is sometimes called the Drunkard's walk because each step is made without thought and no direction is favoured. Brownian motion is a mathematical idealization of a random walk which makes minute, independent steps. It is believed that Brownian motion is a continuous time version random walk [13]. Several researchers have worked on fractals science such as [12] worked on fractal dimensioning of Euclidean n-space, [7], also worked on random fractal aggregates so also the application of fractals in box counting in image analysis for estimating fractal dimension [8]. Despite several researches on fractals science, the writer has not been able to find research in the area of random walk of fractal images. This paper attempts to investigate the limit of random walks of fractal image constituent parts.

2.0 Methodology

Most fractals are generated by taking a set of inputs and applying them as inputs to some sort of equation. The output of this equation is then fed into itself again. This feedback is repeated over and over again until the desired number of iterations passes, or until the behavior of the values is determined. The larger the iterations the better the fractal image produced. In this work, one thousand five hundred iterations are used so that the fractal images can form appropriately.

There are various systems for generating different types of fractals. One of the classic examples of a fractal is the Mandelbrot. The Mandelbrot family of fractals is generated by using some variation of the quadratic function $Q(z)= z^2 + c$ where both z and c are complex numbers. Iterated Function Systems (IFS) form part of a range of complexity techniques known as Production Systems used in production of fractals image. Other forms of production systems include Linden Mayer Systems (L-Systems), Classifiers, Diffusion Limited Aggregation (DLA) among others. [9]

Creating an IFS fractal

- Creating an IFS fractal consists of the following steps:
- Defining a set of plane transformations,
- Drawing an initial pattern on the plane (any pattern),
- Transforming the initial pattern using the transformations defined in first step,
- Transforming the new picture (combination of initial and transformed patterns) using the same set of transformations,
- Repeating the fourth step as many times as possible (in theory, this procedure can be repeated an infinite number of times). [10].

An IFS is given by N contractive transformations $w_1, w_2. . . w_N$ on a complete metric space (X, d). Its notation is: $\{X: w_1, w_2. . . w_N\}$.
Typically, X is the k-dimensional Euclidean real space and the transformations are affine. We will restrict X to be $[0, 1]^k$
A 2-dimensional *affine transformation* $w: R^2 \rightarrow R^2$ is defined by

$$W\begin{pmatrix} x \\ y \end{pmatrix} = \begin{matrix} a_{11x} + a_{12y} + b_1 \\ a_{21x} + a_{22y} + b_2 \end{matrix}$$

$$W\begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} a_{11} + a_{12} \\ a_{21} + a_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} b_1 \\ b_1 \end{bmatrix} \qquad (3.1)$$

Where a_{ij} 's and b_i's are rational numbers.

An IFS defines a unique set A which is the fixed point of the mapping.

The result of this research is validated by comparing the result of this work with previous works. The program is written in Matlab 7.0. ,developed by Mathwoks in Boston USA. The flow diagram that generates the data points for plotting individual fractal images from their Iterated Function System codes and is depicted in Fig.1.

FLOW CHART

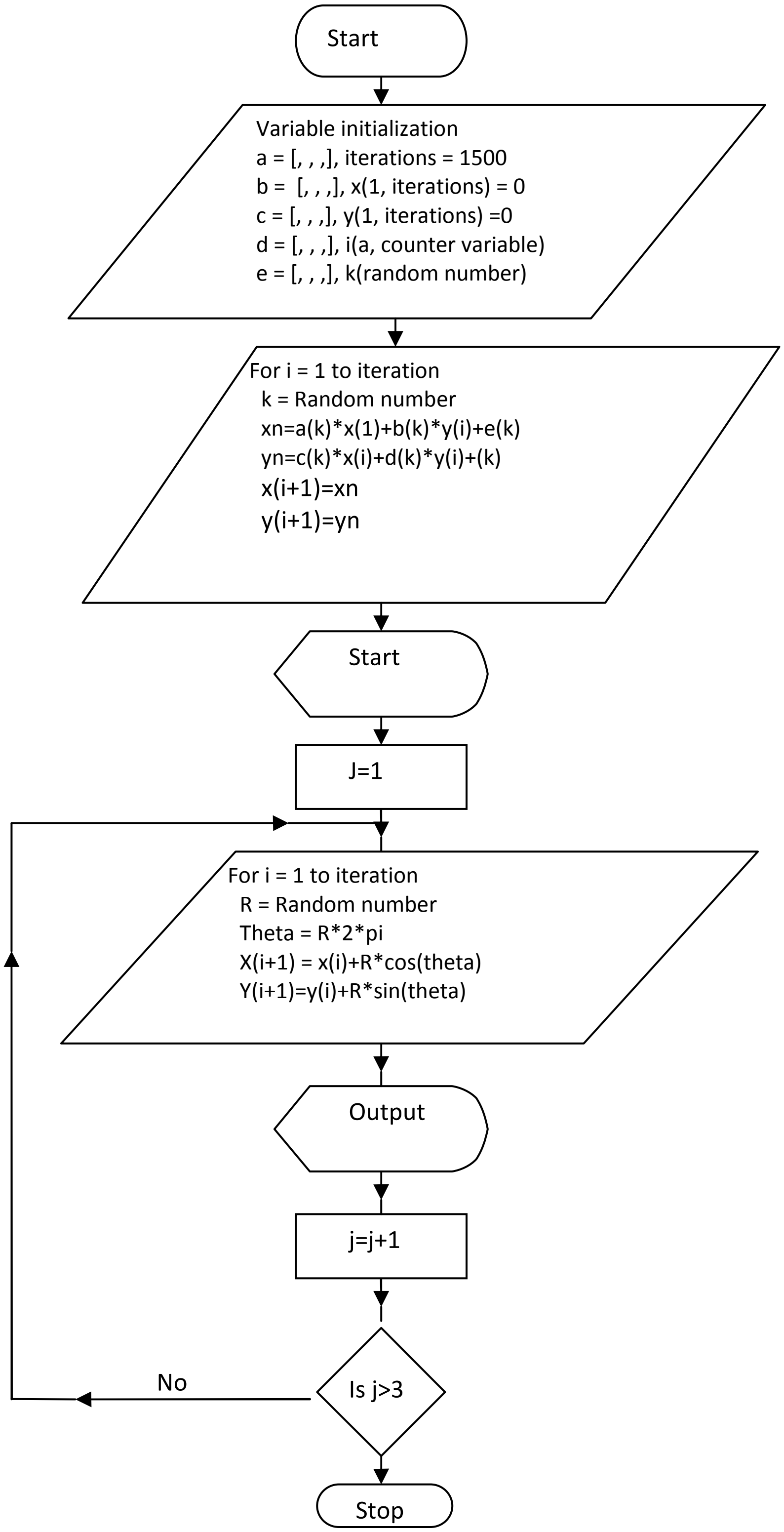


Fig 1: flow diagram that generates the data

3.0 Result and Discussion

The various existing iterated function systems codes (IFS) are fed as input into the program codes presented in the appendix. The program, as discuss earlier, is written in Math lab. 6.5. The random numbers used in this work are computer based generated and the random walk are observed at the first, third, hundredth, five hundredth and one thousandth random move. In this work, seven fractal images were considered and for each, the fractal image was allowed to move randomly for eight times so as to arrive at a good judgment and clear picture over the 'move' at different time of the random walks. So, this work presents a total of forty images altogether as follows:

TABLE 1: IFS code for fractal image No 1

w_i	a	b	C	D	e	f
1	0	0.00	0.00	0.16	0.00	0.00
2	0.85	0.04	-0.04	0.85	0.00	1.60
3	0.20	-0.26	-0.23	0.22	0.00	1.60
4	-0.15	0.28	0.26	0.24	0.00	0.44

Source: [11]

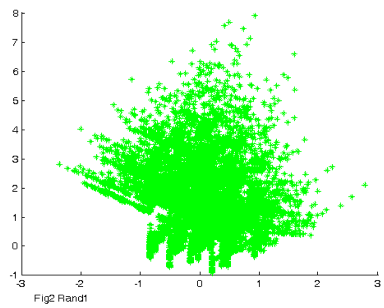


Fig 2.: Fractal Image No1

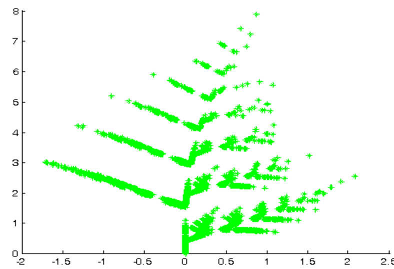


Fig 3: fractal image No1 at the end of 1st random move.

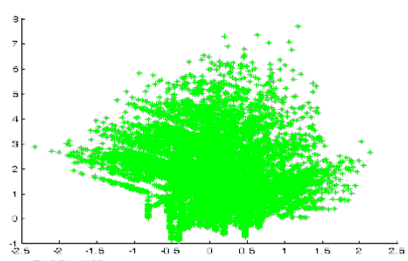


Fig 4: fractal image No 3 at the end of 3rd random move

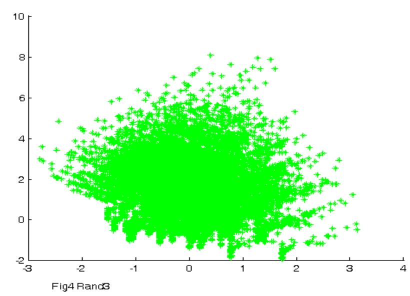


Fig 5 : fractal image No 4 at the end of 100th random move

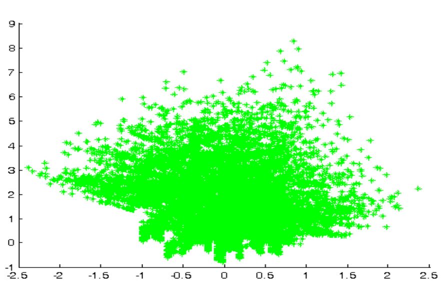


Fig 6: fractal image No 6 at the end of 500th random move.

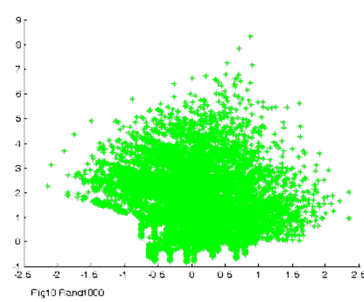


Fig 7: fractal image No 9 at the end of 1000th random move.

Table.2: IFS code for fractal image No2

w_i	A	b	C	D	e	f
1	0.5000	0.0000	0.0000	0.5000	0.0000	0.0000
2	0.5000	0.0000	0.0000	0.5000	0.5000	0.0000
3	0.5000	0.0000	0.0000	0.5000	0.2500	0.5000

Source: Source: [11]

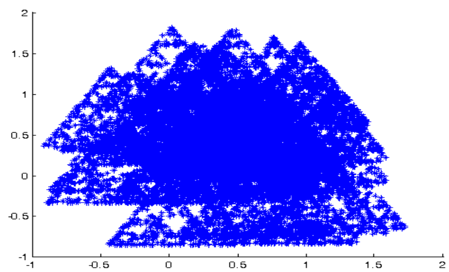


Fig 8: fractal image No 2

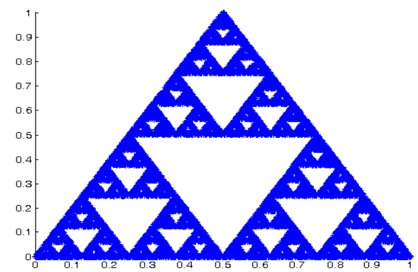


Fig 9: fractal image No1 at the end of 1st random move.

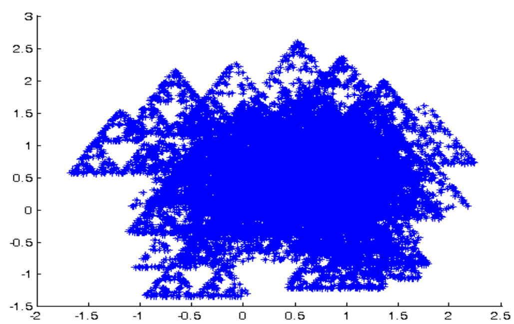


Fig 10: fractal image No2 at the end of 3rd random move.

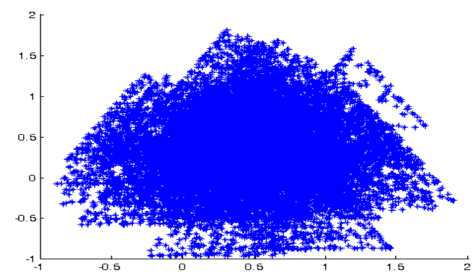


Fig 11: fractal image No 4 at the end of 100th random move.

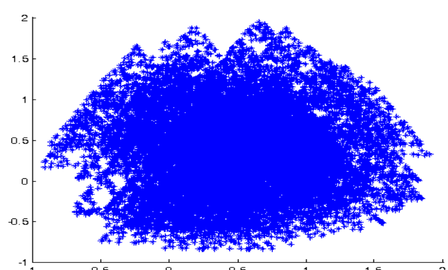


Fig 12: fractal image No 6 at the end of 500th random move.

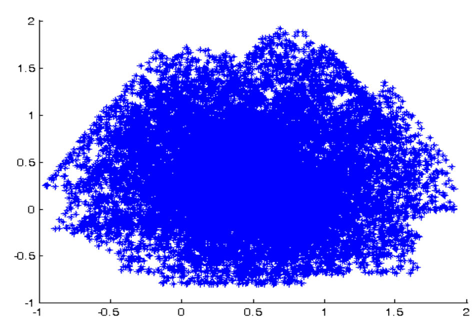
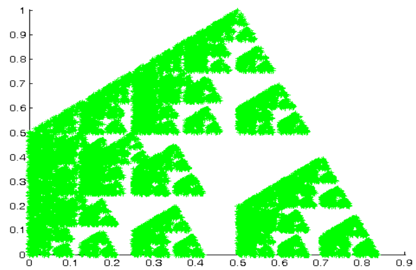
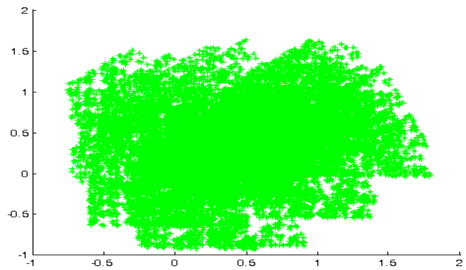


Fig 13: fractal image No 9 at the end of 1000th random move.

Table.3: IFS code for fractal image No 3

W	a	b	C	D	e	f
1	0.5	0	0	0.5	0	0
2	0.5	0	0	0.5	0	0.25
3	0.5	0	0	0.5	0.5	0
4	0.5	0	0	0.5	0.25	0.25

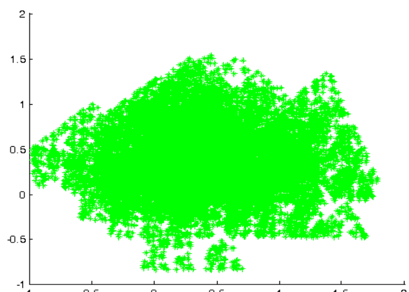




Source: [11]

Fig 14: fractal image No 3

Fig 15: fractal image No1 at the end of 1st random move



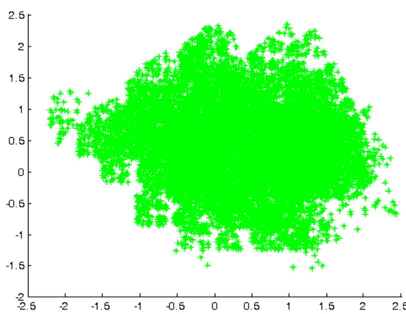
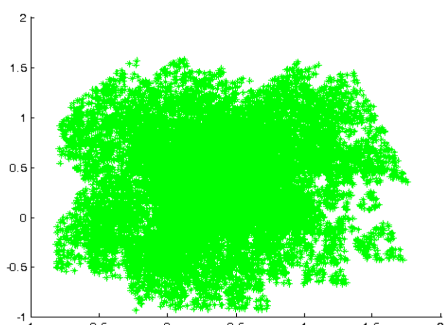


Fig 16: fractal image No3 at the end of 3rd random mov

Fig 17: fractal image No 4 at the end of 100th random move



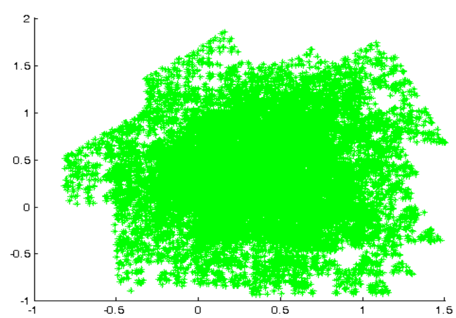


Fig 18: fractal image No5 at the end of 500th random move

Fig 19: fractal image No 6 at the end of 1000th random move

Table 4: The IFS codes of the next fractal image to be consider is

Wi	a	b	C	D	e	f
1	0	0	0	0.16	0	1
2	0.85	0.04	-0.04	0.85	0	1.6
3	0.2	-0.26	0.23	0.22	0	1.6
4	-0.15	0.28	0.26	0.24	0	0.44

Source: Source: [11]

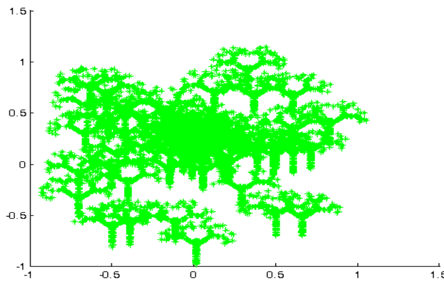


Fig 20: fractal image No 4

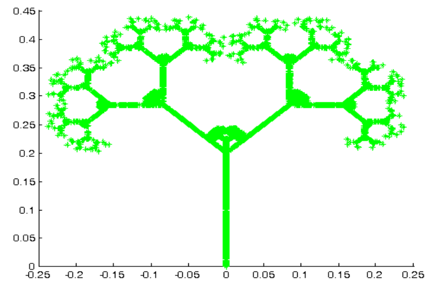


Fig 21: fractal image No1 at the end of 1^{st} random move

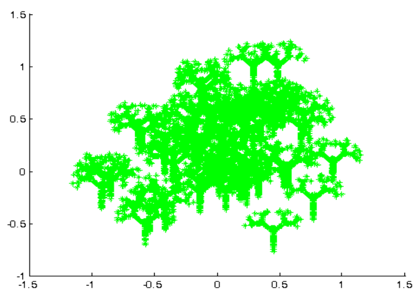


Fig 22: fractal image No3 at the end of 3^{rd} random move

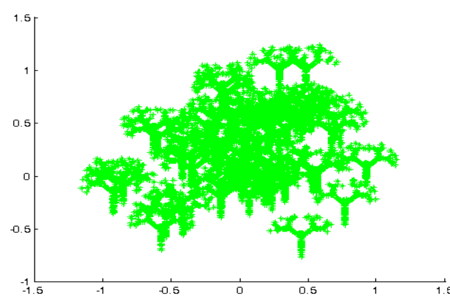


Fig 23: fractal image No 4 at the end of 100^{th} random move

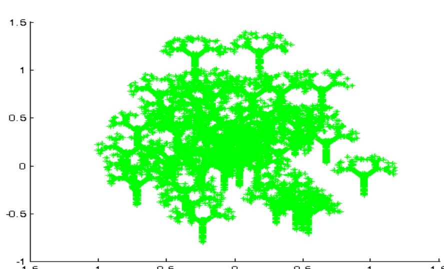


Fig 24: fractal image No5 at the end of 500^{th} random move

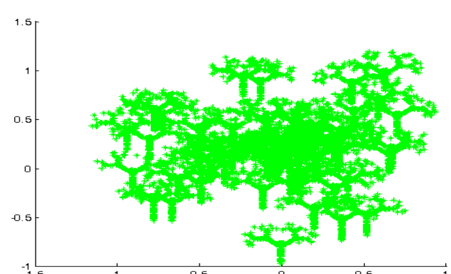


Fig 25: fractal image No 6 at the end of 500^{th} random move

Table 5: The next to be considered fractal image IFS codes is

W	a	b	C	D	e	G
1	0.14	0.01	0	0.51	-0.08	-1.31
2	0.43	0.52	-0.45	0.5	1.49	-0.75
3	0.45	-0.49	0.47	0.47	-1.62	-0.74
4	0.49	0	0	0.51	0.02	1.62

Source: Source: [11]

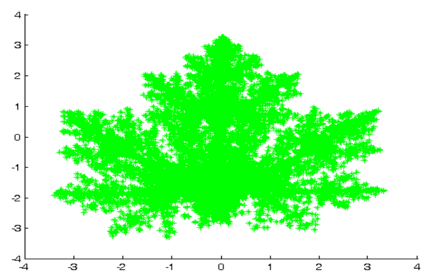


Fig 26: fractal image No 5

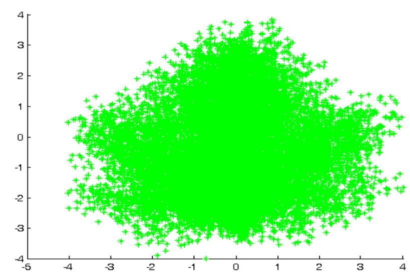


Fig 27: fractal image No1 at the end of 1st random move

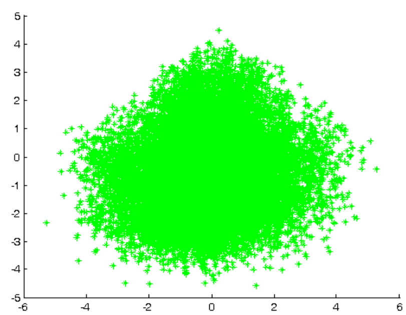


Fig 28: fractal image No3 at the end of 3rd random move

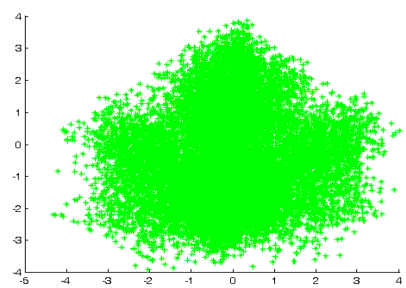


Fig 29: fractal image No 4 at the end of 100th random move.

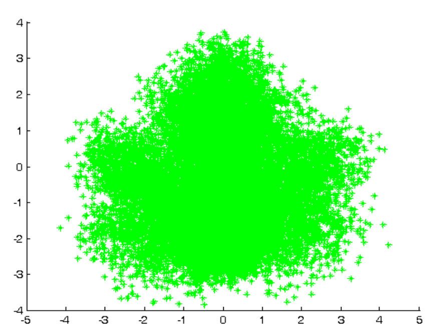


Fig 30: fractal image No5 at the end of 500th random move

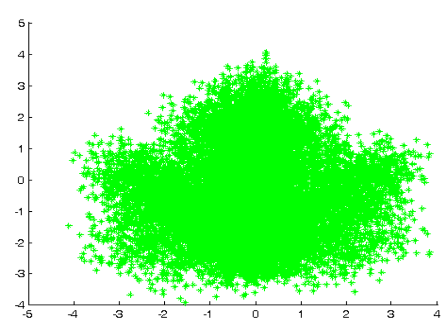


Fig 31: fractal image No 6 at the end of 1000th random move

Table 6: The IFScodes for the next to be considered is

w_i	a	b	C	d	e	F
1	0.3333	0.0000	0.0000	0.3333	0.0000	0.0000
2	0.3333	0.0000	0.0000	0.3333	0.6667	0.0000
3	0.3333	0.0000	0.0000	0.3333	0.0000	0.6667
4	0.3333	0.0000	0.0000	0.3333	0.6667	0.6667
5	0.3333	0.0000	0.0000	0.3333	0.3333	0.3333

Source: Source: [11]

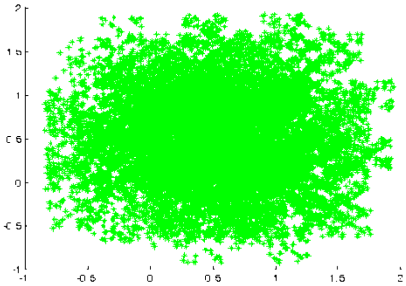


Fig 32: fractal image No 6

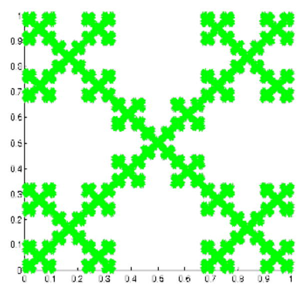


Fig 33: fractal image No1 at the end of 1st random move

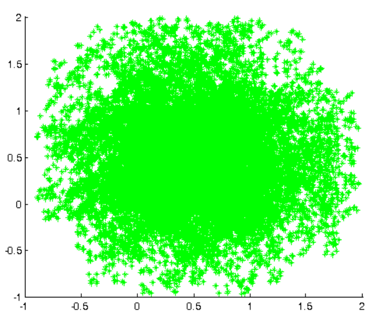


Fig 34: fractal image No3 at the end of 3rd random move

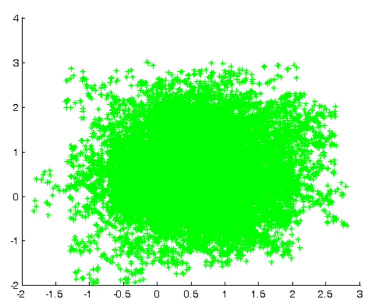


Fig 35: fractal image No 4 at the end of 100th random move.

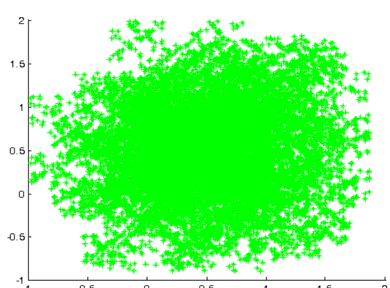


Fig 36: fractal image No5 at the end of 500th random move

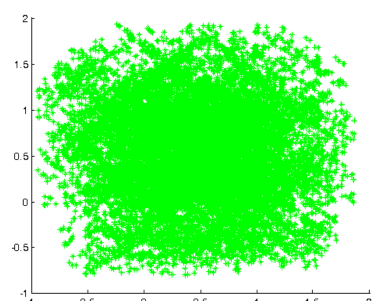


Fig 37: fractal image No 6 at the end of 1000th random move

Table 7: The next to be considered fractal image IFS code is IFS code for Fractal Image 9

w_i	a	b	C	d	e	F
1	0.0000	0.2439	0.0000	0.3053	0.0000	0.0000
2	0.7248	0.0337	-0.0253	0.7426	0.2060	0.2538
3	0.1583	-0.1297	0.3550	0.3676	0.1383	0.1750
4	0.4900	0.0000	0.0000	0.5100	0.0200	1.6200

Source: Source: [11]

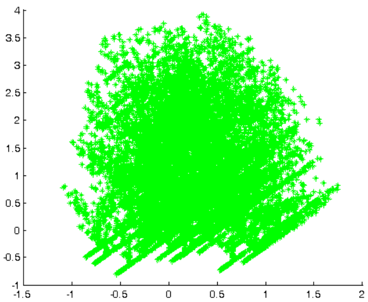


Fig 38: fractal image No 7

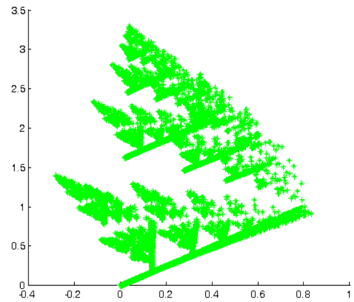


Fig 39: fractal image No1 at the end of 1st random move

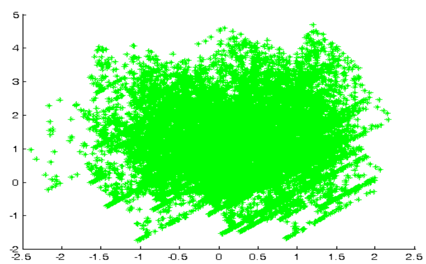


Fig 40: fractal image No3 at the end of 3rd random move

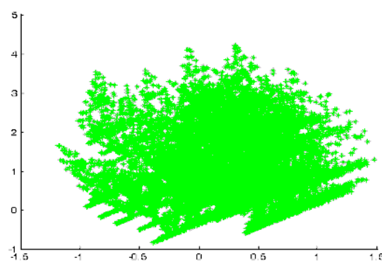


Fig 41: fractal image No 4 at the end of 100th random move.

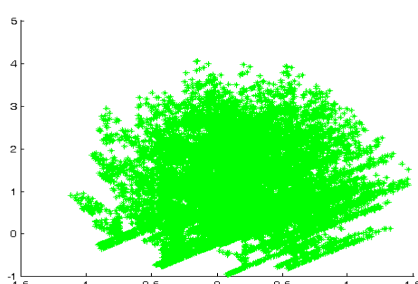


Fig 42: fractal image No5 at the end of 500th random move

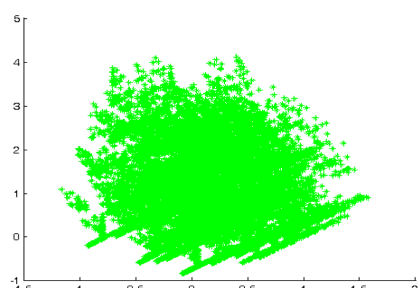


Fig 43: fractal image No 6 at the end of 1000th random move

Observations

From the fractal image presented above between Fig 2 to Fig 43 it is observed that:

- the image change completely from the original image but there is a resemblance in all the moves.
- in each case it is found that the area (space) occupied is almost the same all through the moves
- the image gravitate densely towards the centre.
- the original image trait can still be faintly observed in all the random moves.
- the spaces in the original image disappears.
- the original image collapsed completely.

4.0 Conclusion

From the investigation of the limit image of random walks constituents' parts carried out in this work, the following conclusions can be drawn:

For all the fractal images considered:

- the image change completely from the original image but there is a resemblance in all the moves.
- in each case the area (space) occupied is almost the same all through the moves.
- the image gravitate densely towards the centre.
- empty spaces present in the original fractal images disappeared for all cases considered

Specifically, a new image entirely, in the form of mini forest, was created by fractal image No.4 (Tree).In all, 10% of the cases studied affirmed the possibility of generating distinct new fractal images using this newly introduce approach. This work has considered 2 dimensional fractal images. Further work can still be carried out in the area of 3 dimensional fractal images, image compression and coding of object with the aid of Iterated function system (IFS).

5.0 References

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