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An Application of the Extended Generalized Riccati Equation Mapping Method to the Konopelchenko-Dubrovsky Equation.

***Kolebaje O. T., Adenodi R. A., Ojo O. L. and Akinyemi P.**

Department of Physics,
Adeyemi College of Education, Ondo, Nigeria.

Abstract

Exact hyperbolic, trigonometric and rational travelling wave solutions to the Konopelchenko-Dubrovsky (KD) equation via the extended generalized Riccati equation mapping method are presented in this paper. The twenty one travelling wave solutions obtained were verified by putting them back into the KD equation with the aid of Mathematica. This shows that the extended generalized Riccati equation mapping method is a powerful tool for finding exact solution to nonlinear partial differential equations in physics, mathematics and other applications.

Keywords: extended generalized Riccati equation mapping method, Konopelchenko-Dubrovsky equation, travelling wave solutions, solitons, nonlinear partial differential equations.

1.0 Introduction

Nonlinear partial differential equations (NLPDE) with solitons solutions appear in many applications such as Quantum field theory, Optics, Plasma physics [1], Biology, Fluid mechanics [2], Solid-state physics [3], Chemical Kinetics [4], etc.

Various powerful methods have been employed to construct exact solutions to nonlinear partial differential equations. These methods include the inverse scattering transform [5], the Backlund transform [6-7], the Darboux transform [8], the Hirota bilinear method [9], the tanh-function method [10-11], the sine-cosine method [12], the exp-function method [13], the generalized Riccati equation method [14], the Homogenous balance method [15], (G'/G) expansion method [16] etc.

The extended generalized Riccati equation mapping method introduced by [17] uses the generalized Riccati equation which has 27 different classes of solution as its auxiliary solution. The aim of this paper is to apply the extended generalized Riccati equation mapping method to construct travelling wave solutions of the Konopelchenko-Dubrovsky (KD) equation. The Konopelchenko-Dubrovsky equation is a (2+1)-dimensional coupled system of equation in u and v.

2.0 Description Of The Extended Generalized Riccati Equation Mapping Method.

Here we introduce the extended generalized Riccati equation mapping method for finding travelling wave solutions of nonlinear partial differential equations. Consider a nonlinear equation of two independent variables x and t of the form:

$$P(u, u_t, u_x, u_{tt}, u_{xx}, u_{xt}, \dots) = 0 \qquad (1)$$

P is a polynomial in $u(x,t)$ and its derivatives with respect to x and t. To begin, we transform equation (1) into a nonlinear ordinary differential equation by introducing the variable ξ given by:

$$u(x,t) = u(\xi) \qquad \xi = \mu x + ct \qquad (2)$$

Where μ and c are arbitrary constant and equation (2) reduces to a nonlinear ordinary differential equation of the form

$$Q(u, u', u'', u''' \dots) = 0 \qquad (3)$$

The method assumes that the solution to equation (1) can be expressed as a polynomial in (G'/G)

$$u(\xi) = \sum_{i=0}^{m} \alpha_i \left(\frac{G'}{G}\right)^i \qquad \alpha_m \neq 0 \qquad (4)$$

Where $\alpha_0, \alpha_1, \dots, \alpha_m$ are constants to be determined and $G(\xi)$ satisfies the generalized Riccati equation of the form

$$G'(\xi) = p + rG(\xi) + sG^2(\xi) \qquad s \neq 0 \qquad (5)$$

*Corresponding author: ***Kolebaje O. T.,*** E-mail: olusolakolebaje2008@gmail.com, Tel+2347038680315

Where p, r and s are arbitrary constants. Equation (3) is integrated as long as all the terms contain derivatives, where integration constants are considered to be zero. To determine m, we consider the homogenous balance between the highest order derivative and the highest order nonlinear term(s).

Substitute equation (4) with the determined value of m into equation (3), and collect all terms with the same order of $G^{-m}(\xi)$ and $G^m(\xi)$ for $m = 0, 1, 2, 3, \ldots$ together. If the coefficients of G^m vanish separately, we have a set of algebraic equations in $\alpha_0, \alpha_1, \ldots, \alpha_m, \mu, c, p, r$ and s that is solved with the aid of Mathematica.

Finally, substituting $\alpha_0, \alpha_1, \ldots, \alpha_m,\ c$ and the general solution to equation (5) into equation (4) yield the travelling wave solution of equation (1). Twenty seven different solutions of the generalized Riccati equation under four different families are presented below [18]:

Family 1: When $\phi = \sqrt{r^2 - 4sp}$, $\phi^2 > 0$ and $rs \neq 0$ or $sp \neq 0$, the hyperbolic solutions to equation (5) are:

$$G_1 = -\frac{1}{2s}\left[r + \phi \tanh\left(\frac{\phi}{2}\xi\right)\right]$$

$$G_2 = -\frac{1}{2s}\left[r + \phi \coth\left(\frac{\phi}{2}\xi\right)\right]$$

$$G_3 = -\frac{1}{2s}\left[r + \phi[\tanh(\phi\xi) \pm i\,\mathrm{sech}(\phi\xi)]\right]$$

$$G_4 = -\frac{1}{2s}\left[r + \phi[\coth(\phi\xi) \pm \mathrm{cosech}(\phi\xi)]\right]$$

$$G_5 = -\frac{1}{4s}\left[2r + \phi\left[\tanh\left(\frac{\phi}{4}\xi\right) + \coth\left(\frac{\phi}{4}\xi\right)\right]\right]$$

$$G_6 = \frac{1}{2s}\left[-r + \frac{\phi\sqrt{Q^2 + R^2} - Q\phi\cosh(\phi\xi)}{Q\sinh(\phi\xi) + R}\right]$$

$$G_7 = \frac{1}{2s}\left[-r - \frac{\phi\sqrt{R^2 - Q^2} + Q\phi\sinh(\phi\xi)}{Q\cosh(\phi\xi) + R}\right]$$

Q and R are two non-zero real constants that satisfies $R^2 - Q^2 > 0$.

$$G_8 = \frac{2p\cosh\left(\frac{\phi}{2}\xi\right)}{\phi\sinh\left(\frac{\phi}{2}\xi\right) - r\cosh\left(\frac{\phi}{2}\xi\right)}$$

$$G_9 = \frac{-2p\sinh\left(\frac{\phi}{2}\xi\right)}{-\phi\cosh\left(\frac{\phi}{2}\xi\right) + r\sinh\left(\frac{\phi}{2}\xi\right)}$$

$$G_{10} = \frac{2p\cosh(\phi\xi)}{\phi\sinh(\phi\xi) - r\cosh(\phi\xi) \pm i\phi}$$

$$G_{11} = \frac{2p\sinh(\phi\xi)}{\phi\cosh(\phi\xi) - r\sinh(\phi\xi) \pm \phi}$$

$$G_{12} = \frac{4p\sinh\left(\frac{\phi}{4}\xi\right)\cosh\left(\frac{\phi}{4}\xi\right)}{2\phi\cosh^2\left(\frac{\phi}{4}\xi\right) - 2r\sinh\left(\frac{\phi}{4}\xi\right)\cosh\left(\frac{\phi}{4}\xi\right) - \phi}$$

Family 2: When $\eta = \sqrt{4sp - r^2}$, $\eta^2 > 0$ and $rs \neq 0$ or $sp \neq 0$, the trigonometric solutions to equation (5) are:

$$G_{13} = \frac{1}{2s}\left[-r + \eta\tan\left(\frac{\eta}{2}\xi\right)\right]$$

$$G_{14} = -\frac{1}{2s}\left[r + \eta\cot\left(\frac{\eta}{2}\xi\right)\right]$$

$$G_{15} = \frac{1}{2s}\left[-r + \eta[\tan(\eta\xi) \pm \sec(\eta\xi)]\right]$$

$$G_{16} = -\frac{1}{2s}\left[r + \eta[\cot(\eta\xi) \pm \mathrm{cosec}(\eta\xi)]\right]$$

$$G_{17} = \frac{1}{4s}\left[-2r + \eta\left[\tan\left(\frac{\eta}{4}\xi\right) - \cot\left(\frac{\eta}{4}\xi\right)\right]\right]$$

$$G_{18} = \frac{1}{2s}\left[-r + \frac{\pm\eta\sqrt{Q^2 - R^2} - Q\eta\cos(\eta\xi)}{Q\sin(\eta\xi) + R}\right]$$

$$G_{19} = \frac{1}{2s}\left[-r - \frac{\pm\eta\sqrt{Q^2 - R^2} + Q\eta\cos(\eta\xi)}{Q\sin(\eta\xi) + R}\right]$$

Q and R are two non-zero real constants that satisfies $Q^2 - R^2 > 0$.

$$G_{20} = \frac{-2p\cos\left(\frac{\eta}{2}\xi\right)}{\eta\sin\left(\frac{\eta}{2}\xi\right) + r\cos\left(\frac{\eta}{2}\xi\right)}$$

$$G_{21} = \frac{2p\sin\left(\frac{\eta}{2}\xi\right)}{\eta\cos\left(\frac{\eta}{2}\xi\right) - r\sin\left(\frac{\eta}{2}\xi\right)}$$

$$G_{22} = \frac{-2p\cos(\eta\xi)}{\eta\sin(\eta\xi) + r\cos(\eta\xi) \pm \eta}$$

$$G_{23} = \frac{2p\sin(\eta\xi)}{\eta\cos(\eta\xi) - r\sin(\eta\xi) \pm \eta}$$

$$G_{24} = \frac{4p\sin\left(\frac{\eta}{4}\xi\right)\cos\left(\frac{\eta}{4}\xi\right)}{2\eta\cos^2\left(\frac{\eta}{4}\xi\right) - 2r\sin\left(\frac{\eta}{4}\xi\right)\cosh\left(\frac{\eta}{4}\xi\right) - \eta}$$

Family 3: When $p = 0,\ rs \neq 0$ and g_1 is an arbitrary constant, then we have more solutions to equation (5):

$$G_{25} = \frac{-rg_1}{s[g_1 + \cosh(r\xi) - \sinh(r\xi)]}$$

$$G_{26} = \frac{-r[\cosh(r\xi) + \sinh(r\xi)]}{s[g_1 + \cosh(r\xi) + \sinh(r\xi)]}$$

Family 4: When $p = r = 0,\ s \neq 0$ and d_1 is an arbitrary constant, then the rational solutions to equation (5) are:

$$G_{27} = \frac{-1}{s\xi + d_1}$$

3.0 Application

In this section, we apply the extended generalized Riccati equation mapping method to construct travelling wave solution of the Konopelchenko-Dubrovsky (KD) equation given by [19],

$$u_t - u_{xxx} - 6buu_x + {}^3/_2\, a^2u^2u_x - 3v_y + 3au_xv = 0 \qquad (6a)$$

$$u_y = v_x \qquad (6b)$$

Where a and b are real parameters. Equation (6) is a nonlinear integrable evolution equation on two spatial dimensions and one temporal. For $u_y = 0$, equation (6a) becomes the Gardner equation. Equation (6a) is the Kadomtsev-Petviashvili equation for $a = 0$ and the modified Kadomtsev-Petviashvili for $b = 0$ [19-20].

We make the transformation $u(x,y,t) = u(\xi),\ v(x,y,t) = v(\xi)$ and $\xi = \mu x + \beta y + ct$.

Equation (6) becomes

$$cu' - \mu^3u''' - 6b\mu uu' + {}^3/_2\, a^2\mu u^2u' - 3\beta v' + 3a\mu u'v = 0 \qquad (7a)$$

$$\beta u' = \mu v' \qquad (7b)$$

Integrating equation $(7b)$ with respect to ξ yields

$$v = \frac{\beta}{\mu}u \qquad (8)$$

Substituting equation (8) into equation (7a) gives an ordinary differential equation in only u

$$cu' - \mu^3u''' - 6b\mu uu' + {}^3/_2\, a^2\mu u^2u' - \frac{3\beta^2}{\mu}u' + 3a\beta u'u = 0 \qquad (9)$$

Integrating equation (9) with respect to ξ gives

$$\left(c - \frac{3\beta^2}{\mu}\right)u + {}^3/_2\,(a\beta - 2b\mu)u^2 + {}^1/_2\, a^2\mu u^3 - \mu^3u'' = 0 \qquad (10)$$

To get m, we consider the homogenous balance between the highest order derivative u'' and the highest order nonlinear term u^3.

$$3m = m + 2 \implies m = 1$$

Then equation (4) becomes

$$u(\xi) = \alpha_1\left(\frac{G'}{G}\right) + \alpha_0 \qquad \text{where } \alpha_1 \neq 0 \qquad (11)$$

Where α_0 and α_1 are constants to be determined later. Substituting equation (5) in equation (9)

$$u(\xi) = \alpha_1(pG^{-1} + r + sG) + \alpha_0 \qquad (12)$$

Substituting equation (10) and its derivatives into equation (8) and collecting the coefficients of $G^{\pm m}(\xi)$ together yields a simultaneous set of nonlinear algebraic equations in $\alpha_0, \alpha_1, r, s, p$ and c. These equations are:

$$-2p^3\mu^3a_1 + \frac{1}{2}p^3\alpha^2\mu a_1{}^3 = 0$$

$$-3p^2r\mu^3a_1 + \frac{3}{2}p^2(\alpha\beta - 2b\mu)a_1{}^2 + \frac{3}{2}p^2\alpha^2\mu a_0a_1{}^2 + \frac{3}{2}p^2r\alpha^2\mu a_1{}^3 = 0$$

$$cpa_1 - \frac{3p\beta^2a_1}{\mu} - pr^2\mu^3a_1 - 2p^2s\mu^3a_1 + 3p(\alpha\beta - 2b\mu)a_0a_1 + \frac{3}{2}p\alpha^2\mu a_0{}^2a_1 + 3pr(\alpha\beta - 2b\mu)a_1{}^2 + 3pr\alpha^2\mu a_0a_1{}^2 + \frac{3}{2}pr^2\alpha^2\mu a_1{}^3 + \frac{3}{2}p^2s\alpha^2\mu a_1{}^3 = 0$$

$$ca_0 - \frac{3\beta^2a_0}{\mu} + \frac{3}{2}(\alpha\beta - 2b\mu)a_0{}^2 + \frac{1}{2}\alpha^2\mu a_0{}^3 + cra_1 - \frac{3r\beta^2a_1}{\mu} - 2prs\mu^3a_1 + 3r(\alpha\beta - 2b\mu)a_0a_1 + \frac{3}{2}r\alpha^2\mu a_0{}^2a_1 + \frac{3}{2}r^2(\alpha\beta - 2b\mu)a_1{}^2 + 3ps(\alpha\beta - 2b\mu)a_1{}^2 + \frac{3}{2}r^2\alpha^2\mu a_0a_1{}^2 + 3ps\alpha^2\mu a_0a_1{}^2 + \frac{1}{2}r^3\alpha^2\mu a_1{}^3 + 3prs\alpha^2\mu a_1{}^3 = 0$$

$$csa_1 - \frac{3s\beta^2a_1}{\mu} - sr^2\mu^3a_1 - 2ps^2\mu^3a_1 + 3s(\alpha\beta - 2b\mu)a_0a_1 + \frac{3}{2}s\alpha^2\mu a_0{}^2a_1 + 3sr(\alpha\beta - 2b\mu)a_1{}^2 + 3sr\alpha^2\mu a_0a_1{}^2 + \frac{3}{2}sr^2\alpha^2\mu a_1{}^3 + \frac{3}{2}ps^2\alpha^2\mu a_1{}^3 = 0$$

$$-3s^2r\mu^3a_1 + \frac{3}{2}s^2(\alpha\beta - 2b\mu)a_1{}^2 + \frac{3}{2}s^2\alpha^2\mu a_0a_1{}^2 + \frac{3}{2}rs^2\alpha^2\mu a_1{}^3 = 0$$

$$-2s^3\mu^3a_1 + \frac{1}{2}s^3\alpha^2\mu a_1{}^3 = 0$$

Solving this algebraic system of equation with the aid of Mathematica yields the solution

$$\alpha_0 = \frac{-a\beta + 2b\mu \mp ra\mu^2}{a^2\mu} \qquad \alpha_1 = \pm\frac{2\mu}{a}$$

$$c = \frac{-12ab\mu\beta + 12b^2\mu^2 + 9a^2\beta^2 - (r^2 + 8sp)a^2\mu^4}{2\mu a^2}$$

Substituting the solution to the nonlinear algebraic equation and the general solution to the generalized Riccati equation (5) into equation (11), we obtain different travelling wave solutions of the KD equation.

Family 1: The hyperbolic solutions to equation (6), when $\phi = \sqrt{r^2 - 4sp}$, $\phi^2 > 0$ and $rs \neq 0$ or $sp \neq 0$ are:

$$\alpha_0 = \frac{-a\beta + 2b\mu \mp ra\mu^2}{a^2\mu} \qquad \alpha_1 = \pm\frac{2\mu}{a}$$

$$\xi = \mu x + \beta y + \left[\frac{-12ab\mu\beta + 12b^2\mu^2 + 9a^2\beta^2 - (r^2 + 8sp)a^2\mu^4}{2\mu a^2}\right]t$$

$$u_1(\xi) = \alpha_1\left(\frac{\phi^2 \operatorname{sech}^2\left(\frac{\phi}{2}\xi\right)}{2\left[r + \phi\tanh\left(\frac{\phi}{2}\xi\right)\right]}\right) + \alpha_0$$

$$v_1(\xi) = \frac{\alpha_1\beta}{\mu}\left(\frac{\phi^2 \operatorname{sech}^2\left(\frac{\phi}{2}\xi\right)}{2\left[r + \phi\tanh\left(\frac{\phi}{2}\xi\right)\right]}\right) + \frac{\alpha_0\beta}{\mu}$$

$$u_2(\xi) = \alpha_1\left(\frac{-\phi^2 \operatorname{cosech}^2\left(\frac{\phi}{2}\xi\right)}{2\left[r + \phi\coth\left(\frac{\phi}{2}\xi\right)\right]}\right) + \alpha_0$$

$$v_2(\xi) = \frac{\alpha_1\beta}{\mu}\left(\frac{-\phi^2 \operatorname{cosech}^2\left(\frac{\phi}{2}\xi\right)}{2\left[r + \phi \coth\left(\frac{\phi}{2}\xi\right)\right]}\right) + \frac{\alpha_0\beta}{\mu}$$

$$u_3(\xi) = \alpha_1\left(\frac{\pm i\phi^2}{\phi\cosh(\phi\xi) + r[\pm i + \sinh(\phi\xi)]}\right) + \alpha_0$$

$$v_3(\xi) = \frac{\alpha_1\beta}{\mu}\left(\frac{\pm i\phi^2}{\phi\cosh(\phi\xi) + r[\pm i + \sinh(\phi\xi)]}\right) + \frac{\alpha_0\beta}{\mu}$$

$$u_4(\xi) = \alpha_1\left(\frac{\phi^2}{r \mp r\cosh(\phi\mu) \mp \phi\sinh(\phi\mu)}\right) + \alpha_0$$

$$v_4(\xi) = \frac{\alpha_1\beta}{\mu}\left(\frac{\phi^2}{r \mp r\cosh(\phi\mu) \mp \phi\sinh(\phi\mu)}\right) + \frac{\alpha_0\beta}{\mu}$$

$$u_5(\xi) = \alpha_1\left(\frac{-\phi^2 \operatorname{cosech}^2\left(\frac{\phi}{2}\xi\right)}{2r + \phi\left[\coth\left(\frac{\phi}{4}\xi\right) + \tanh\left(\frac{\phi}{4}\xi\right)\right]}\right) + \alpha_0$$

$$v_5(\xi) = \frac{\alpha_1\beta}{\mu}\left(\frac{-\phi^2 \operatorname{cosech}^2\left(\frac{\phi}{2}\xi\right)}{2r + \phi\left[\coth\left(\frac{\phi}{4}\xi\right) + \tanh\left(\frac{\phi}{4}\xi\right)\right]}\right) + \frac{\alpha_0\beta}{\mu}$$

$$u_6(\xi) = \alpha_1\left(\frac{Q\left[-Q\phi^2 + \phi^2\sqrt{Q^2+R^2}\cosh(\phi\xi) + R\phi^2\sinh(\phi\xi)\right]}{[R + Q\sinh(\phi\xi)]\left[rR - \phi\sqrt{Q^2+R^2} + Q\phi\cosh(\phi\xi) + Qr\sinh(\phi\xi)\right]}\right) + \alpha_0$$

$$v_6(\xi) = \frac{\alpha_1\beta}{\mu}\left(\frac{Q\left[-Q\phi^2 + \phi^2\sqrt{Q^2+R^2}\cosh(\phi\xi) + R\phi^2\sinh(\phi\xi)\right]}{[R + Q\sinh(\phi\xi)]\left[rR - \phi\sqrt{Q^2+R^2} + Q\phi\cosh(\phi\xi) + Qr\sinh(\phi\xi)\right]}\right) + \frac{\alpha_0\beta}{\mu}$$

$$u_7(\xi) = \alpha_1\left(\frac{Q\left[Q\phi^2 - \phi^2\sqrt{R^2-Q^2}\sinh(\phi\xi) + R\phi^2\cosh(\phi\xi)\right]}{[R + Q\cosh(\phi\xi)]\left[rR + \phi\sqrt{R^2-Q^2} + Q\phi\sinh(\phi\xi) + Qr\cosh(\phi\xi)\right]}\right) + \alpha_0$$

$$v_7(\xi) = \frac{\alpha_1\beta}{\mu}\left(\frac{Q\left[Q\phi^2 - \phi^2\sqrt{R^2-Q^2}\sinh(\phi\xi) + R\phi^2\cosh(\phi\xi)\right]}{[R + Q\cosh(\phi\xi)]\left[rR + \phi\sqrt{R^2-Q^2} + Q\phi\sinh(\phi\xi) + Qr\cosh(\phi\xi)\right]}\right) + \frac{\alpha_0\beta}{\mu}$$

$$u_8(\xi) = \alpha_1\left(\frac{\phi^2}{r + r\cosh(\phi\xi) - \phi\sinh(\phi\xi)}\right) + \alpha_0$$

$$v_8(\xi) = \frac{\alpha_1\beta}{\mu}\left(\frac{\phi^2}{r + r\cosh(\phi\xi) - \phi\sinh(\phi\xi)}\right) + \frac{\alpha_0\beta}{\mu}$$

$$u_9(\xi) = \alpha_1\left(\frac{\phi^2}{r - r\cosh(\phi\xi) + \phi\sinh(\phi\xi)}\right) + \alpha_0$$

$$v_9(\xi) = \frac{\alpha_1\beta}{\mu}\left(\frac{\phi^2}{r - r\cosh(\phi\xi) + \phi\sinh(\phi\xi)}\right) + \frac{\alpha_0\beta}{\mu}$$

$$u_{10}(\xi) = \alpha_1\left(\frac{\phi^2[\operatorname{sech}(\phi\xi) \mp i\tanh(\phi\xi)]}{r\cosh(\phi\xi) - \phi[\pm i + \sinh(\phi\xi)]}\right) + \alpha_0$$

$$v_{10}(\xi) = \frac{\alpha_1\beta}{\mu}\left(\frac{\phi^2[\operatorname{sech}(\phi\xi) \mp i\tanh(\phi\xi)]}{r\cosh(\phi\xi) - \phi[\pm i + \sinh(\phi\xi)]}\right) + \frac{\alpha_0\beta}{\mu}$$

$$u_{11}(\xi) = \alpha_1\left(\frac{\phi^2}{r \mp r\cosh(\phi\xi) \pm \phi\sinh(\phi\xi)}\right) + \alpha_0$$

$$v_{11}(\xi) = \frac{\alpha_1\beta}{\mu}\left(\frac{\phi^2}{r \mp r\cosh(\phi\xi) \pm \phi\sinh(\phi\xi)}\right) + \frac{\alpha_0\beta}{\mu}$$

$$u_{12}(\xi) = \alpha_1\left(\frac{\phi^2}{r - r\cosh(\phi\xi) + \phi\sinh(\phi\xi)}\right) + \alpha_0$$

$$v_{12}(\xi) = \frac{\alpha_1\beta}{\mu}\left(\frac{\phi^2}{r - r\cosh(\phi\xi) + \phi\sinh(\phi\xi)}\right) + \frac{\alpha_0\beta}{\mu}$$

Family 2: The trigonometric solutions to equation (6), when $\eta = \sqrt{4sp - r^2}$, $\eta^2 > 0$ and $rs \neq 0$ or $sp \neq 0$ are:

$$\alpha_0 = \frac{-a\beta + 2b\mu \mp ra\mu^2}{a^2\mu} \qquad \alpha_1 = \pm\frac{2\mu}{a}$$

$$\xi = \mu x + \beta y + \left[\frac{-12ab\mu\beta + 12b^2\mu^2 + 9a^2\beta^2 - (r^2 + 8sp)a^2\mu^4}{2\mu a^2}\right]t$$

$$u_{13}(\xi) = \alpha_1\left(\frac{-\eta^2\sec^2\left(\frac{\eta}{2}\xi\right)}{2\left[r - \eta\tan\left(\frac{\eta}{2}\xi\right)\right]}\right) + \alpha_0$$

$$v_{13}(\xi) = \frac{\alpha_1\beta}{\mu}\left(\frac{-\eta^2\sec^2\left(\frac{\eta}{2}\xi\right)}{2\left[r - \eta\tan\left(\frac{\eta}{2}\xi\right)\right]}\right) + \frac{\alpha_0\beta}{\mu}$$

$$u_{14}(\xi) = \alpha_1\left(\frac{-\eta^2\operatorname{cosec}^2\left(\frac{\eta}{2}\xi\right)}{2\left[r + \eta\cot\left(\frac{\eta}{2}\xi\right)\right]}\right) + \alpha_0$$

$$v_{14}(\xi) = \frac{\alpha_1\beta}{\mu}\left(\frac{-\eta^2\operatorname{cosec}^2\left(\frac{\eta}{2}\xi\right)}{2\left[r + \eta\cot\left(\frac{\eta}{2}\xi\right)\right]}\right) + \frac{\alpha_0\beta}{\mu}$$

$$u_{15}(\xi) = \alpha_1\left(\frac{-\eta^2}{r \mp \eta\cos(\eta\xi) \mp r\sin(\eta\xi)}\right) + \alpha_0$$

$$v_{15}(\xi) = \frac{\alpha_1\beta}{\mu}\left(\frac{-\eta^2}{r \mp \eta\cos(\eta\xi) \mp r\sin(\eta\xi)}\right) + \frac{\alpha_0\beta}{\mu}$$

$$u_{16}(\xi) = \alpha_1\left(\frac{-\eta^2}{r \pm \eta\sin(\eta\xi) \mp r\cos(\eta\xi)}\right) + \alpha_0$$

$$v_{16}(\xi) = \frac{\alpha_1\beta}{\mu}\left(\frac{-\eta^2}{r \pm \eta\sin(\eta\xi) \mp r\cos(\eta\xi)}\right) + \frac{\alpha_0\beta}{\mu}$$

$$u_{17}(\xi) = \alpha_1\left(\frac{-\eta^2\operatorname{cosec}^2\left(\frac{\eta}{2}\xi\right)}{2r + \eta\left[\cot\left(\frac{\eta}{4}\xi\right) - \tan\left(\frac{\eta}{4}\xi\right)\right]}\right) + \alpha_0$$

$$v_{17}(\xi) = \frac{\alpha_1\beta}{\mu}\left(\frac{-\eta^2\operatorname{cosec}^2\left(\frac{\eta}{2}\xi\right)}{2r + \eta\left[\cot\left(\frac{\eta}{4}\xi\right) - \tan\left(\frac{\eta}{4}\xi\right)\right]}\right) + \frac{\alpha_0\beta}{\mu}$$

$$u_{18}(\xi) = \alpha_1\left(\frac{Q\left[-Q\eta^2 \pm \eta^2\sqrt{Q^2 - R^2}\cos(\eta\xi) - R\eta^2\sin(\eta\xi)\right]}{[R + Q\sin(\eta\xi)]\left[rR \mp \eta\sqrt{Q^2 - R^2} + Q\eta\cos(\eta\xi) + Qr\sin(\eta\xi)\right]}\right) + \alpha_0$$

$$v_{18}(\xi) = \frac{\alpha_1\beta}{\mu}\left(\frac{Q\left[-Q\eta^2 \pm \eta^2\sqrt{Q^2 - R^2}\cos(\eta\xi) - R\eta^2\sin(\eta\xi)\right]}{[R + Q\sin(\eta\xi)]\left[rR \mp \eta\sqrt{Q^2 - R^2} + Q\eta\cos(\eta\xi) + Qr\sin(\eta\xi)\right]}\right) + \frac{\alpha_0\beta}{\mu}$$

$$u_{19}(\xi) = \alpha_1\left(\frac{Q\left[-Q\eta^2 \mp \eta^2\sqrt{Q^2 - R^2}\cos(\eta\xi) - R\eta^2\sin(\eta\xi)\right]}{[R + Q\sin(\eta\xi)]\left[rR \pm \eta\sqrt{Q^2 - R^2} + Q\eta\cos(\eta\xi) + Qr\sin(\eta\xi)\right]}\right) + \alpha_0$$

$$v_{19}(\xi) = \frac{\alpha_1\beta}{\mu}\left(\frac{Q\left[-Q\eta^2 \mp \eta^2\sqrt{Q^2 - R^2}\cos(\eta\xi) - R\eta^2\sin(\eta\xi)\right]}{[R + Q\sin(\eta\xi)]\left[rR \pm \eta\sqrt{Q^2 - R^2} + Q\eta\cos(\eta\xi) + Qr\sin(\eta\xi)\right]}\right) + \frac{\alpha_0\beta}{\mu}$$

$$u_{20}(\xi) = \alpha_1\left(\frac{-\eta^2}{r + r\cos(\eta\xi) + \eta\sin(\eta\xi)}\right) + \alpha_0$$

$$v_{20}(\xi) = \frac{\alpha_1\beta}{\mu}\left(\frac{-\eta^2}{r + r\cos(\eta\xi) + \eta\sin(\eta\xi)}\right) + \frac{\alpha_0\beta}{\mu}$$

$$u_{21}(\xi) = \alpha_1\left(\frac{-\eta^2}{r - r\cos(\eta\xi) - \eta\sin(\eta\xi)}\right) + \alpha_0$$

$$v_{21}(\xi) = \frac{\alpha_1\beta}{\mu}\left(\frac{-\eta^2}{r - r\cos(\eta\xi) - \eta\sin(\eta\xi)}\right) + \frac{\alpha_0\beta}{\mu}$$

$$u_{22}(\xi) = \alpha_1\left(\frac{-\eta^2[\sec(\eta\xi) \pm \tan(\eta\xi)]}{r\cos(\eta\xi) + \eta[\pm 1 + \sin(\eta\xi)]}\right) + \alpha_0$$

$$v_{22}(\xi) = \frac{\alpha_1\beta}{\mu}\left(\frac{-\eta^2[\sec(\eta\xi) \pm \tan(\eta\xi)]}{r\cos(\eta\xi) + \eta[\pm 1 + \sin(\eta\xi)]}\right) + \frac{\alpha_0\beta}{\mu}$$

$$u_{23}(\xi) = \alpha_1\left(\frac{-\eta^2}{r \mp r\cos(\eta\xi) \mp \eta\sin(\eta\xi)}\right) + \alpha_0$$

$$v_{23}(\xi) = \frac{\alpha_1\beta}{\mu}\left(\frac{-\eta^2}{r \mp r\cos(\eta\xi) \mp \eta\sin(\eta\xi)}\right) + \frac{\alpha_0\beta}{\mu}$$

$$u_{24}(\xi) = \alpha_1\left(\frac{-\eta^2}{r - r\cos(\eta\xi) - \eta\sin(\eta\xi)}\right) + \alpha_0$$

$$v_{24}(\xi) = \frac{\alpha_1\beta}{\mu}\left(\frac{-\eta^2}{r - r\cos(\eta\xi) - \eta\sin(\eta\xi)}\right) + \frac{\alpha_0\beta}{\mu}$$

Family 3: More solutions to equation (6), when $p = 0,\ rs \neq 0$ and g_1 is an arbitrary constant are:

$$\alpha_0 = \frac{-a\beta + 2b\mu \mp ra\mu^2}{a^2\mu} \qquad \alpha_1 = \pm\frac{2\mu}{a}$$

$$\xi = \mu x + \beta y + \left[\frac{-12ab\mu\beta + 12b^2\mu^2 + 9a^2\beta^2 - (r^2 + 8sp)a^2\mu^4}{2\mu a^2}\right]t$$

$$u_{25}(\xi) = \alpha_1\left(\frac{r}{1 + e^{r\xi}g_1}\right) + \alpha_0$$

$$v_{25}(\xi) = \frac{\alpha_1\beta}{\mu}\left(\frac{r}{1 + e^{r\xi}g_1}\right) + \frac{\alpha_0\beta}{\mu}$$

$$u_{26}(\xi) = \alpha_1\left(\frac{rg_1}{e^{r\xi} + g_1}\right) + \alpha_0$$

$$v_{26}(\xi) = \frac{\alpha_1\beta}{\mu}\left(\frac{rg_1}{e^{r\xi} + g_1}\right) + \frac{\alpha_0\beta}{\mu}$$

Family 4: Rational solutions to equation (6), when $p = r = 0,\ s \neq 0$ and d_1 is an arbitrary constant are:

$$\alpha_0 = \frac{-a\beta + 2b\mu}{a^2\mu} \qquad \alpha_1 = \pm\frac{2\mu}{a}$$

$$\xi = \mu x + \beta y + \left[\frac{-12ab\mu\beta + 12b^2\mu^2 + 9a^2\beta^2}{2\mu a^2}\right]t$$

$$u_{27}(\xi) = \alpha_1\left(\frac{-s}{s\xi + d_1}\right) + \alpha_0$$

$$v_{27}(\xi) = \frac{\alpha_1\beta}{\mu}\left(\frac{-s}{s\xi + d_1}\right) + \frac{\alpha_0\beta}{\mu}$$

4.0 Results and Discussion

Remark 1: The travelling wave solutions of the coupled Konopelchenko-Dubrovsky (KD) equation obtained using the extended generalized Riccati equation mapping method for the hyperbolic, trigonometric and rational function types are presented in $u_1 - u_{27}$ and $v_1 - v_{27}$.

Remark 2: The solutions $u_8,\ u_9,\ u_{11}$ and u_{12} are identical and can all be described by u_{11} alone. Also solutions $u_{20},\ u_{21},\ u_{23}$ and u_{24} are identical and can all be described by solution u_{23}. Hence the extended generalized Riccati equation mapping method produces 21 different classes of travelling wave solution to the KD equation.

Remark 3: All the obtained travelling wave solutions to the KD equation $u_1 - u_7,\ v_1 - v_7,\ u_{11}, v_{11},\ u_{13} - u_{19},\ v_{13} - v_{19},\ u_{23},\ v_{23},\ u_{25} - u_{27}$ and $v_{25} - v_{27}$ were checked by putting them back into equation (6) with the aid of Mathematica.

5.0 Conclusion

Twenty one different hyperbolic, trigonometric and rational function travelling wave solutions to the coupled Konopelchenko-Dubrovsky (KD) equation has been obtained using the extended generalized Riccati equation mapping method. The results have been verified by putting them back into the KD equation with the aid of Mathematica.

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