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Determining the extent of stabilizing of a mathematical model undergoing changing final time and initial conditions

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Abstract

Experts have observed that in the uncontrolled case we cannot guarantee where the arbitrary unstable steady-state solution will converge to because it is unstable. However, in the controlled case they have reported that the arbitrary unstable steady state solution can converge to the given steady-state solution. Our point of departure in this study is to investigate the extent of stabilizing the trivial and nontrivial unstable steady-state solutions when the final time and the initial conditions undergo certain fundamental changes.

1.0 Introduction

The theory of stabilizing a mathematical model of population systems has been recently defined, analysed and illustrated with four practical application problems [2]. It is on this basis that we have verified that in the uncontrolled case, we cannot guarantee where the arbitrary unstable steady-state solution will converge to because it is unstable. Other techniques of stabilizing other partial differential equation models can be studied in the interesting publications of [3], [4], [5]. However, we have also determined numerically that the arbitrary unstable steady-state solution can converge to a given steady-state solution over a long period of time. But stabilizing a mathematical model which undergoes changing final time and initial conditions remains an open problem. In this study, we have tackled this challenging and interesting scientific and mathematical problem drawing on an example from crop science which involves the competition interaction between two legumes in the Niger Delta Region of Nigeria [1].

2.0 Mathematical Formulation

A Lotka-Volterra type system of continuous nonlinear first order ordinary differential equations is considered:

$$\frac{dC(t)}{dt} = \alpha_1 C(t)(1 - \beta_1 C(t) - \gamma_1 G(t) \qquad (1)$$

$$\frac{dG(t)}{dt} = \alpha_2 G(t)(1 - \gamma_2 G(t) - \beta_2 C(t) \qquad (2)$$

with initial starting biomasses $C(0) = 0.1$ grams per area of growth and $G(0) = 0.2$ grams per area of growth. Here, we have considered these parameter values: $\alpha_1 = 0.168$,
$\beta_1 = 0.002,\ \gamma_1 = 0.004, \alpha_2 = 0.164, \beta_2 = 0.004\ and\ \gamma_2 = 0.002$[1].

3.0 Method and Motivation

In this study, we have utilized a simulation approach [2] to stabilize the above system of model ODEs. As a result of its inherent difficulty of stabilizing this model using an incorrect set of initial conditions, we propose to analyse this problem with the initial starting biomasses which will produce both competition theory and stabilization of an unstable co-existence steady-state solution $(26.66, 28.66)$ having two eigenvalues 0.0553 and -0.1659. These two legumes will co-exist under

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these specified biomasses but may not survive together because $\alpha_{12} = \frac{\gamma_1}{\beta_1} = 2 > 1.0244 = \frac{K1}{K2}$ and $\alpha_{21} = \frac{\beta_2}{\gamma_2} = 2 > 0.9762 = \frac{K2}{K1}$. The arbitrary steady-state solution (N_{1e}, N_{2e}) is said to be unstable, that is, (N_1, N_2) is not convergent to (N_{1e}, N_{2e}) when the time variable approaches infinity. How do we stabilize this unstable steady-state solution? Following Yan and Ekaka-a [2], we consider the following numerical steps

Step 1: Find the linearized problem about the arbitrary steady-state solution (N_{1e}, N_{2e}).

Step 2: Find the positive definite matrix P_i from the Riccati equation.

Step 3: Apply P_i in the nonlinear equations, then (N_1, N_2) is convergent to (N_{1e}, N_{2e}).

4.0 Results and Discussions

The application of the above method clearly shows that a biological interaction called competition is a dominant type of interaction which describes the deterministic dynamics between cowpea and groundnut over changing duration of growth and initial conditions or initial biomasses. This first set of results which complements a biological theory of competition is displayed in Table 1:

Table 1: Determining the type of interaction between cowpea and groundnut under changing Tfinal or duration of growth and initial conditions (ICs)

Example	Tfinal (days)	ICs (grams)	C_b (grams)	C_{bi} (grams)	G_b (grams)	G_{bi} (grams)
1	40	0.1, 0.2	13.02	41.75	32.98	51.93
2	50	0.2, 0.3	16.90	76.77	38.58	76.30
3	60	0.3, 0.4	17.23	83.03	39.60	81.12
4	70	0.4, 0.5	17.55	83.86	39.36	81.86
5	80	0.5, 0.6	19.43	83.98	36.10	81.98
6	90	0.6, 0.7	25.96	84.00	29.40	82.00
7	95	0.7, 0.8	40.06	84.00	17.28	82.00

C_b represents the biomass of cowpea in interaction with groundnut, C_{bi} represents the biomass of cowpea growing in isolation of C_b, $G_b$represents the biomass of groundnut in interaction with cowpea, G_{bi}represents the biomass of groundnut growing in isolation of G_b

On the basis of a popular ecological theory [6], Table 1 clearly shows that a dominant competition is in place between two legumes such as cowpea and groundnut.

Next, we considered the extent of stabilizing the unstable steady-state solution (26.66, 28.66) when the initial conditions are $(1, 2)$, the step length is$k = 0.01$and the number of repeated loops is M = $\frac{Tfinal}{k}$. Our application of these data has enabled us to stabilize this unstable steady-state solution over repeated simulations. The results of this numerical method are displayed in Table 2:

Table 2: Stabilization of unstable steady-state solution (26.66, 28.66)undergoing changing final time with an initial condition (1, 2)

Example	Tfinal (days)	C_e	G_e
1	40	24.861752067947737	26.655594696401593
2	50	25.564733822280726	27.438575729867097
3	60	26.008154529079839	27.932557627675983
4	70	26.277995022272869	28.233204808305889
5	80	26.438642416953893	28.412206430853722
6	90	26.533038354111614	28.517392040720630
7	95	26.564266954811142	28.552190922406936
8	100	26.588079223471613	28.578725843747776
9	200	26.663512583446742	28.662785610884690
10	400	26.663823418954600	28.663131997603582
11	800	26.663823424115471	28.663132003354640
12	1000	26.663823424115471	28.663132003354640

Similar convergence to the steady-state solution (26.66, 28.66) has been observed for other initial conditions such as (2, 5) and (4, 10) which we cannot present the full results in this single paper.

5.0 Conclusion

On the basis of repeated simulations undergoing changes in the final time or duration of growth and initial conditions or initial biomasses, we have been able to stabilize the unstable co-existence steady-state solution. In particular, over a long period of time, that is when Tfinal is 1000 days, the starting steady-state solution (26.66, 28.66) is approached approximately. We will expect both the delayed and full stabilization outputs to provide useful insights to crop scientists who are actively involved in stability functioning and crop yields in the event of unexpected climate change as a short-term mitigation strategy. Interested mathematicians can extend our present approach to search for the instances of performance deterioration phenomena [**signs to avoid**] which crop science experts are concerned to know about in order to avoid the degeneracy of the unstable steady-state solution with the anticipation of identifying the incorrect choice of starting biomasses which are capable of affecting the correct agricultural policy in the Niger Delta Region of Nigeria in the event of climate change. In the competition theory of interacting legumes, can the performance of stabilization still be realised when the parameter values c and e otherwise called the inter-specific coefficients far outweigh the parameter values b and f also called the intra-specific coefficients? This is an open stabilization problem which has vital application in the agricultural sector.

References

[1] M.A. Ekpo and A.J. Nkanang, Nitrogen fixing capacity of legumes and their Rhizospherealmicroflora in diesel oil polluted soil in the tropics, Journal of Petroleum and Gas Engineering1(4), (2010), pp. 76-83.

[2] Yan Yubin and Enu-Obari N. Ekaka-a, Stabilizing a mathematical model of population system, Journal of the Franklin Institute, 348 (2011), pp. 2744-2758.

[3] Y. Yan, D. Coca, V. Barbu, Finite-dimensional controller design for semi-linear parabolic systems, Nonlinear Analysis: Theory, Methods and Applications 70 (2009) 4451–4475.

[4] Y. Yan, D. Coca, V. Barbu, Feedback control for Navier–Stokes equation, Nonlinear Functional Analysis and Optimization 29 (2008) 225–242.

[5] X. Zhou, J. Cui, Stability and Hopf bifurcation analysis of an eco-epidemiological model with delay, Journal of the Franklin Institute 347 (2010) 1654–1680.

[6] Ekaka-a E. N. (2009) Computational and Mathematical Modelling of Plant Species interactions in a Harsh Climate. PhD Thesis, Dept. of Mathematics, The university of Liverpool and The University of Chester, United Kingdom.