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Stabilizing a mathematical model of competition-colonization: context, performance deterioration and prospects

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Abstract

In this study, we will attempt to stabilize the multiple steady states of the well-known competition-colonization model. The implications of this sophisticated analysis for delayed and full stabilizations have been made. The results which we have achieved and have not seen elsewhere are presented and discussed quantitatively.

1.0 Introduction

The well-known competition-colonization model [1] is an important educational model in mathematical ecology. The key feature of this model is that it has multiple steady-state solutions that require further stabilization [2]. The theory and the practical application examples which demonstrate the stabilization of interacting population systems have already been reported in the literature [2 – 5]. Our present stabilization analysis will be based on this theory.

2.0 Mathematical formulation

Following [1], we consider the following model equations:

$$\frac{du_1}{dt} = \lambda_1 u_1(1-u_1) - u_1 \qquad (1)$$

$$\frac{du_2}{dt} = \lambda_2 u_2(1-u_1-u_2) - u_2 - \lambda_1 u_1 u_2 \qquad (2)$$

where$u_i(t)$ is the density of species *i* at time *t*. As in most ecological model, λ_1 and λ_2 are positive constants which reflect some biological insights [1]. In this scenario, λ_1 is called the birth rate of the density of species 1whereas λ_2 is the birth rate of the density of species 2. The above system of model equations has two multiple steady-state solutions namely

$u_{1e} = 1 - \frac{1}{\lambda_1}andu_{2e} = \frac{\lambda_2 - \lambda_1^2}{\lambda_1\lambda_2}$ provided $\lambda_2 > \lambda_1^2$ where $\lambda_1 > 1$.

3.0 Methodology

It is clear that the competition-colonization model will have multiple steady-state solutions which will vary as values of λ_1 and λ_2 also vary. By using the standard theory of linearization in the neighborhood of an arbitrary steady-state solution and calculating the Jacobian elements, we can characterize whether each steady-state solution is either stable or unstable. For the steady-state solutions which are unstable under the changing values of $\lambda_1$and λ_2, we follow the procedure of [2] to develop a controller based on Euler discretization scheme which was used to stabilize the unstable steady-state solutions or further stabilize the stable steady-state solution.

4.0 Results and Discussions

For the purpose of clarity, we shall consider various scenarios of stabilizing the steady-state solutions of a competition-colonization model. Our results of this analysis will be clearly presented in this section.

Result 1

If $\lambda_1 = 2$and $\lambda_2 = 5$, such that $u_{1e} = 0.5$ and $u_{2e} = 0.1$ and using the initial conditions of 2 and 10 in the appropriate units of the interacting populations when the step length $k = 0.01$, we have obtained the following results

We observe from Table 1 that convergence starts to set in when the value of Tfinal is 10 up till when the value of Tfinal is30.

Result 2

In this scenario, we consider when the values of λ_1 and λ_2 are 1.5 and 3 respectively such that $u_{1e} = 0.3333$and $u_{2e} = 0.1667$. Using the same initial conditions and step length as used in Result 1 analysis, we have observed a dominant

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Table 1: Stabilization of steady-state solution: case 1

Tfinal	Estimated u_{1e}	Estimated u_{2e}	Extent of stabilization
2	0.5500	0.0213	Delayed stabilization
4	0.5170	0.0577	Delayed stabilization
6	0.5077	0.0813	Delayed stabilization
8	0.5033	0.0921	Delayed stabilization
10	0.5014	0.0986	Delayed stabilization
12	0.5006	0.0994	Delayed stabilization
14	0.5002	0.0998	Delayed stabilization
16	0.5001	0.0999	Near stabilization
18	0.5000	0.1000	Full stabilization
20	0.5000	0.1000	Full stabilization
22	0.5000	0.1000	Full stabilization
24	0.5000	0.1000	Full stabilization
26	0.5000	0.1000	Full stabilization
28	0.5000	0.1000	Full stabilization
30	0.5000	0.1000	Full stabilization

occurrence of performance deterioration in the output of the steady-state solution particularly indicating negative coordinates of a steady-state solution. A similar observation has been made for the scenario when $\lambda_1 = 2$ and $\lambda_2 = 6$ such that $u_{1e} = 0.5$and $u_{2e} = 0.1667$. Therefore, our choices of steady-state solutions which are based on the chosen values of the model parameters do not guarantee effective stabilization of these steady-state solutions.

Result 3

In this scenario, $\lambda_1 = 2$ and $\lambda_2 = 4.5$ such that $u_{1e} = 0.5000$and $u_{2e} = 0.0556$. We obtain the following results:

Table 2: Stabilization of steady-state solution: case 3

Tfinal	Estimated u_{1e}	Estimated u_{2e}	Extent of stabilization
2	0.5474	0.0247	Delayed stabilization
4	0.5134	0.0408	Delayed stabilization
6	0.5054	0.0494	Delayed stabilization
8	0.5022	0.0530	Delayed stabilization
10	0.5009	0.0545	Delayed stabilization
12	0.5004	0.0551	Delayed stabilization
14	0.5002	0.0554	Delayed stabilization
16	0.5001	0.0555	Near stabilization
18	0.5000	0.0555	Stabilization
20	0.5000	0.0556	Full stabilization
22	0.5000	0.0556	Full stabilization
24	0.5000	0.0556	Full stabilization
26	0.5000	0.0556	Full stabilization
28	0.5000	0.0556	Full stabilization
30	0.5000	0.0556	Full stabilization

We also observe from Table 2 that near stabilization occurs when the value of Tfinal is 16.

5.0 Conclusion

The original formulation of a competition-colonization model lacks the application of stabilization of its steady-state solutions. In this study, we have successfully demonstrated that $u_1 \rightarrow u_{1e}$ and $u_2 \rightarrow u_{2e}$ as t or Tfinal increases. Therefore, the multiple steady-state solutions of a competition-colonization model which were not previously stabilized have been stabilized.In summary, we have been able to construct a controller which was used to stabilize the steady-state solutions of a competition-colonization model. It is worth mentioning that the competition-colonization model exhibits performance deterioration as popularly called in the applied control theory literatures.

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