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An EOQ Model for Ameliorating Items with Constant Demand and Linear Time Dependent Holding Cost

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Abstract

In this paper, an economic order quantity (EOQ) model is developed for ameliorating items in which the demand rate is constant and the holding cost is a linear function of time. Items that incur a gradual increase in quality, quantity or both while in inventory are referred to as ameliorating items. Fruits, grains, and other foodstuff provide good examples. When these items are in the production centre, they undergo amelioration and our objective is to develop a model that determines the optimal replenishment cycle time, such that the total variable cost is minimized. Unlike many EOQ models where the holding cost is considered a constant, in this paper, we consider the holding cost to be a linear function of time. Numerical examples are given to illustrate the developed model.

Keywords: Deterministic Finite State Automata, Malicious Signatures database and Model checking.

1.0 Introduction

The assumption that inventory items always preserve their physical characteristics is not always true because some items are subject to risks of breakage, damage, spoilage, evaporation, obsolescence, etc,. The decay that prevents items from being used for their original purpose is termed as deterioration. Extensive literature has evolved over the last three decades on controlling the inventory of deteriorating items. It all started with Ghare and Schrader [1] who developed a simple economic order quantity model with a constant rate of decay. Later, researchers focused their attention on different types of models involving decaying inventories. Extensive literature on deteriorating items could be seen in the paper by Goyal and Giri [2] and also the one by Raafat [3] both of whom presented a thorough survey on the subject.

It is observed however, that some items when in inventory undergo tremendous increase in quantity or quality or both. Generally, fast growing animals like fishes, poultry, cattle, etc, provide good examples. Some fruit merchants in Nigeria invest huge amount of money in buying large plantations of orange, banana, pineapple, etc and keep such farms for months waiting for the arrival of times of festivities when the demand for these items increase exponentially. Within this period, it is certain that these items (in the farm) undergo increase in quantity and quality. The items that exhibit such properties are referred to as ameliorating items.

The existing literature on inventory seems to ignore or give little attention to the ameliorative nature of inventory. It was until the year 1997 that Hwang [4], for the first time studied two inventory models that is, the economic order quantity model (EOQ) and partial selling quantity model (PSQ) in connection with ameliorating items under the assumption that the ameliorating time follows the Weibull distribution. Again, in 1999, Hwang [5] developed inventory models for both ameliorating and deteriorating items separately under the LIFO and FIFO issuing policies. Later in the year 2005, Moon et al [6-7], developed an EOQ model for ameliorating/deteriorating items under inflation and time discounting. The model studied inventory models with zero-ending inventory for fixed order intervals over a finite planning horizon allowing shortages in all but in the last cycle and developed another model with shortages in all cycles taking into account the effects of inflation and time value of money. In 2005 Mondal et al [8] developed a partial selling inventory model for ameliorating items under profit maximization. In the year 2011 Singh et al [9] used genetic algorithm to develop an optimal replenishment

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policy for ameliorating items with shortages under inflation and time value of money. Optimum replenishment policy for all the components of a supply chain was determined by considering all costs of the system simultaneously and assuming demand rate to be a function of time. Shortages were allowed and the back-ordering function was considered to be a decreasing function of waiting time. Attempt to solve the harvest and sale decision problem of fresh agricultural items by incorporating both the amelioration of field items and deterioration of stored items into inventory model was made by Chen [10]. Chen developed profit models of the farmer with ripeness and price dependent demand over a finite planning horizon for two situations viz: when the fresh agricultural items are harvested at maturing point and when they are harvested at critical ripeness point. A model for both ameliorating and deteriorating items under the influence of inflation and time value of money was developed by Mishra et al [11]. The model was used to test for maximization of total average cost when an extra inventory is added into or removed out of the on the on-hand inventory. An EOQ model for ameliorating items with constant demand was studied by Gwanda and Sani [12] in the year 2011. The model determined an optimum order quantity for ameliorating items in which the demand rate, the amelioration rate and the holding costs are constants.

Mishra and Singh [13] observed that even though holding costs are in most inventory models considered as being a constant function of time, in reality the costs vary with time. Mishra and Singh then developed a deterministic inventory model in which both the demand and the holding costs are linearly dependent on time, and backlogging rate is a variable and dependent on the length of the next replenishment. Shortages are also allowed.

A deterministic inventory model was developed by Roy [14] where the deterioration rate is time dependent, demand rate is a function of selling price and holding cost is time dependent.

In our present research, we focus our attention on ameliorating inventory where both the rate of amelioration and demand are constants while the holding cost is linearly dependent on time.

Assumptions and notation:

The proposed ameliorating inventory model is developed under the following assumptions and notation:

The inventory system involves only one single item and one stocking point.

Replenishments are instantaneous with a constant lead time.

Shortages are not allowed.

Amelioration occurs when the items are effectively in stock.

The cycle length is T.

The initial stock level is I_o.

The inventory carrying cost per unit per unit time is a linear function of time $C_h(t) = \lambda_1 + \lambda_2 t$

The unit cost of the item is a known constant C, and the replenishment cost is also a known constant C_o per replenishment.

The demand rate per unit time, R, is a constant.

The total demand in a cycle is R_t.

The rate of amelioration, A, is a constant.

The ameliorated amount when considered in terms of value (say, weight) in a cycle is A_m.

The Mathematical Model:

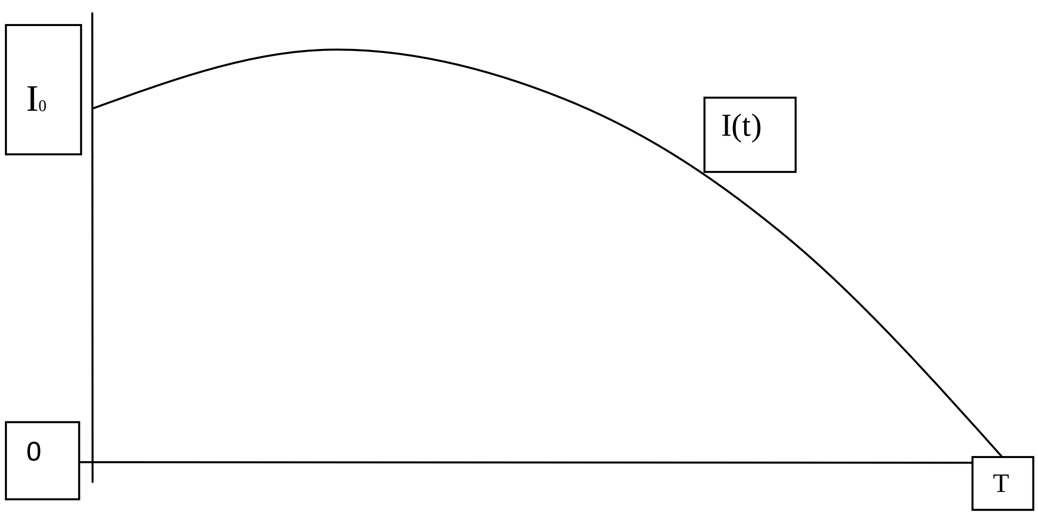

Fig.1: Inventory movement in a constantly ameliorating inventory

During the time interval ($0 \leq t \leq T$) amelioration occurs at constant rate of A and when the demand rate occurs also at a constant rate of R per unit time the differential equation that describes the state of inventory level I(t) is given by

$$\frac{dI(t)}{dt} - AI(t) = -R \tag{1}$$

The solution of the above equation is

$$I(t) = \frac{R}{A} + ke^{At} \tag{2}$$

where k is an arbitrary constant. Applying the boundary condition, at t = 0, $I(t) = I_o$, and substituting in equation (2) we obtain

$$I_o = \frac{R}{A} + k$$

from which k is obtained as

$$k = I_o - \frac{R}{A}$$

Substituting the value of k in equation (2) yields

$$I(t) = \frac{R}{A} + \left(I_o - \frac{R}{A} \right) e^{At}$$

$$\Rightarrow I(t) = \frac{R}{A}\left(1 - e^{At}\right) + I_0 e^{At} \tag{3}$$

Also when $t = T$, $I(t) = 0$. Then equation (3) becomes

$$0 = \frac{R}{A}\left(1 - e^{AT}\right) + I_o e^{AT}$$

That is, $I_o = -\frac{\mathrm{Re}^{-AT}}{A}\left(1 - e^{AT}\right)$

$$= \frac{R}{A}\left(1 - e^{-AT}\right) \tag{4}$$

Substituting equation (4) into equation (3) gives

$$I(t) = \frac{R}{A}\left(1 - e^{At}\right) + \left[\frac{R}{A}\left(1 - e^{-AT}\right)\right] e^{At}$$

$$= \frac{R}{A} - \frac{\mathrm{Re}^{At}}{A} + \frac{\mathrm{Re}^{At}}{A} - \frac{\mathrm{Re}^{A(t-T)}}{A}$$

$$= \frac{R}{A}\left(1 - e^{A(t-T)}\right) \tag{5}$$

The total demand within the interval ($0 \leq t \leq T$) = demand rate $\times$ time period.

That is, $R_T = TR$ (6)

where R_T is the total demand in a cycle T. The ameliorated amount A_m is given by

$$A_m = R_T - I_o$$

$$= TR - \left[\frac{R}{A}\left(1 - e^{-AT}\right)\right] \tag{7}$$

The linearly time dependent holding cost, $C_h(t)$, in a cycle is calculated as follows:

$$C_h = \int_0^T (\lambda_1 + \lambda_2 t) I(t)\,dt$$

$$= \int_0^T (\lambda_1 + \lambda_2 t)\left[\frac{R}{A}\left(1 - e^{A(t-T)}\right)\right] dt$$

$$= \frac{\lambda_1 R}{A}\int_0^T \left(1 - e^{A(t-T)}\right) dt + \frac{\lambda_2 R}{A}\int_0^T t\left(1 - e^{A(t-T)}\right) dt$$

$$= \frac{\lambda_1 R}{A}\left(\int_0^T dt - \int_0^T e^{A(t-T)}dt\right) + \frac{\lambda_2 R}{A}\left(\int_0^T tdt - \int_0^T te^{A(t-T)}dt\right)$$

$$= \frac{\lambda_1 R}{A}\left(T - \frac{1}{A} + \frac{e^{-AT}}{A}\right) + \frac{\lambda_2 R}{A}\left(\frac{T^2}{2} - \frac{T}{A} + \frac{1}{A^2} - \frac{e^{-AT}}{A^2}\right)$$

$$= \frac{\lambda_1 R}{A^2}\left(AT + e^{-AT} - 1\right) + \frac{\lambda_2 R}{A}\left(\frac{A^2T^2 - 2AT + 2 - 2e^{-AT}}{2A^2}\right)$$

$$= \frac{\lambda_1 R}{A^2}\left(AT + e^{-AT} - 1\right) + \frac{\lambda_2 R}{2A^3}\left(A^2T^2 - 2AT + 2 - 2e^{-AT}\right)$$

$$= \frac{R}{2A^3}\left[2A\lambda_1(AT + e^{-AT} - 1) + \lambda_2(A^2T^2 - 2AT - 2e^{-AT} + 2)\right] \quad (8)$$

The total variable cost in a cycle, *TVC* = Inventory ordering cost + holding cost – cost of ameliorated amount. That is,

$$TVC = C_o + C_h - CA_m$$

$$= C_0 + \frac{R}{2A^3}\left[2A\lambda_1(AT + e^{-AT} - 1) + \lambda_2(A^2T^2 - 2AT - 2e^{-AT} + 2)\right]$$
$$- C\left(TR - \frac{R}{A}(1 - e^{-AT})\right) \quad (9)$$

The total variable cost per unit time is then given by

$$TVC(T) = \frac{C_0}{T} + \frac{R}{2A^3T}\left[2A\lambda_1(AT + e^{-AT} - 1) + \lambda_2(A^2T^2 - 2AT - 2e^{-AT} + 2)\right]$$
$$- \frac{C}{T}\left(TR - \frac{R}{A}(1 - e^{-AT})\right) \quad (10)$$

Equation (10) is then differentiated with respect to T to obtain

$$\frac{d}{dT}\left[TVC(T)\right] = \frac{d}{dT}\left(\frac{C_O}{T}\right) + \frac{d}{dT}\left(\frac{R}{2A^3T}\left[2A\lambda_1(AT + e^{-AT} - 1) + \lambda_2(A^2T^2 - 2AT - 2e^{-AT} + 2)\right]\right)$$

$$- \frac{d}{dT}\left(\frac{C}{T}(TR - \frac{R}{A}(1 - e^{-AT}))\right)$$

$$= \frac{-C_0}{T^2} + \frac{R}{2A^3}\left[2A\lambda_1(\frac{T(-A)e^{-AT} - e^{-AT}}{T^2}) + \frac{2A\lambda_1}{T^2} + A^2\lambda_2 - 2\lambda_2(\frac{T(-A)e^{-AT} - e^{-AT}}{T^2}) - \frac{2\lambda_2}{T^2}\right]$$

$$- \frac{CR}{AT^2} - \frac{CR}{A}(\frac{T(-A)e^{-AT} - e^{-AT}}{T^2})$$

$$= \frac{-C_0}{T^2} + \frac{R}{A^3T^2}\left[(\lambda_2 - A\lambda_1)(AT + 1)e^{-AT} + A\lambda_1 - \lambda_2 + \frac{A^2T^2\lambda_2}{2}\right] + \frac{CR}{AT^2}\left[(AT + 1)e^{-AT} - 1\right]$$

For optimal cycle period T which minimizes the total variable cost per unit time, $\frac{d}{dT}[TVC(T)] = 0,$

provided $\frac{d^2}{dT^2}[TVC(T)] > 0.$ That is:

$$0 = \frac{-C_0}{T^2} + \frac{R}{A^3T^2}\left[(\lambda_2 - A\lambda_1)(AT+1)e^{-AT} + A\lambda_1 - \lambda_2 + \frac{A^2T^2\lambda_2}{2}\right] + \frac{CR}{AT^2}\left[(AT+1)e^{-AT} - 1\right]$$

which gives

$$0 = -A^3C_0 + R\left[(\lambda_2 - A\lambda_1)(AT+1)e^{-AT} + A\lambda_1 - \lambda_2 + \frac{A^2T^2\lambda_2}{2}\right] + A^2CR\left[(AT+1)e^{-AT} - 1\right] \qquad (11)$$

The analytical solution of equation (11) is not always easy to obtain but a suitable numerical method such as Newton Raphson Method cou employed to obtain a solution for T.
The economic order quantity, EOQ, is given by total demand in a cycle period minus ameliorated amount within the cycle.
That is, $EOQ = R_T - A_m$

$$= TR - \left[TR - \frac{R}{A}(1-e^{-AT})\right]$$

$$= \frac{R}{A}(1-e^{-AT}) \qquad (12)$$

which is the same as $I_{O,}$ as expected.

Numerical Examples:
Table 1 gives the solutions of ten different numerical examples with different parameters, using equations (11) and (12). In all the ten cases, the value $\frac{d^2}{dT^2}[TVC(T)] > 0,$ and so the solutions are for minimum values.

Table 1: Parameter values and the optimal cycle length for the inventory model for ameliorating items with constant demand and linear time dependent holding cost

S/No.	C_0	λ_1	λ_2	R	A	C	T	EOQ
1.	15000	10000	30	3000	0.41	200	0.0301 (11 days)	14544 units
2.	4000	2000	50	3000	0.33	200	0.0356 (13 days)	18075 units
3.	4000	1500	40	3000	0.33	200	0.0411(15 days)	18059 units
4.	4500	1000	10	2500	0.41	230	0.0630 (23 days)	12040 units
5.	3000	500	20	1000	0.35	250	0.1205 (44 days)	5596units
6.	2000	50	2	3000	0.2	100	0.2137 (77 days)	21095 units
7.	2000	50	2	3000	0.3	100	0.2603 (95 days)	19249 units
8.	1000	40	0	200	0.2	130	0.2712 (99 days)	19472 units
9.	4000	50	2	3000	0.3	100	0.3725 (136 days)	18942 units
10.	2000	50	0	3000	0.4	100	0.3836 (140 days)	13933 units

Discussion of the Results
From the numerical examples we observe that a number of factors affect the EOQ. For instance, it is clear that the higher the rate of amelioration, the lower the EOQ. This means that for items with higher rate of amelioration, the stockiest is advised to purchase less than in the case of items with lower amelioration rate. This is clear since there is a period of deterioration after amelioration. Also from the examples we notice that the EOQ diminishes with the rise in holding cost. Of course as the holding cost becomes exceedingly large, it will be more economical to reduce the number of items to be stocked in order to optimize profit. The example also conforms to common expectation that the higher the ordering cost, the higher the EOQ.
The model developed in this paper is a generalization of Gwanda and Sani [12] if $\lambda_2 = 0$ and $\lambda_1 = ic.$ The results obtained in examples 8 and 10 above correspond to the results in Gwanda and Sani [12] where the holding cost was considered constant.

Conclusion
We have presented a mathematical model on inventory of ameliorating items in which the demand rate is constant and the holding cost is linearly dependent on time. The model determines the optimum quantity to order while keeping the relevant inventory costs minimum. Numerical examples are given to illustrate the developed model and some analysis carried out on the results obtained from the examples.

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