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Selecting the Optimum Cycle Length for Delayed Deteriorating Inventory Model with Constrained Retailer's Capital

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Abstract

This paper presents an inventory model on the selection of the best cycle length for a delayed deteriorating inventory items where the supplier allows some period within which to settle for the goods supplied. The supplier does not charge interest if payment is made within the allowed period, interest is charged on the unsold inventory only if payment is made after the period. The model considers three different scenarios depending on where the permissible or allowed period falls. Numerical examples on the application of the model are provided.

Keywords: Inventory; Delayed Deterioration; Permissible Delay in Payment.

1.0 Introduction

The depletion of Inventory in real situation is considered to be as a result of demand or deterioration or both. The demand for the item could be a constant, linear, exponential or stock dependent. The deterioration could be in form of lost of value or quality of the inventory over a period of time which is mostly the case with the commonly used items like fruits, vegetables, meat, perfumes, blood in blood banks and so on. The deterioration in this case is mainly due to the age of the inventory or in some cases due to the failure or lack of suitable storage facility especially in the case of items with high rate of deterioration. For example, a large quantity of blood donated and stored at a Specialist Hospital in Kano got spoiled as a result of the failure of the storage facility in the Hospital.

There are other cases of deterioration that occur due to obsolescence. This refers to the declining value of items as a result of the rapid changes in technology or the introduction of a new product by a competitor. This is mostly the case with styled goods like electronics, aircraft, mobile phones, computers and cars. Each of the listed items becomes obsolete with the introduction of a replacement model.

It is a common practice in business transactions nowadays for the supplier to offer the retailer permissible period within which to pay for the items delivered. The retailer is not charged interest when the account is settled on or before the permissible period. He is only charged interest if the replenishment account is not settled until after the permissible period.

The development of the deteriorating inventory model was pioneered by Ghare and Shrader [1] who developed a model with a constant rate of deterioration. Goyal [2] developed an EOQ model under the condition of permissible delay in payments. The work of Goyal [2] was extended by Aggarwal and Jaggi [3] to consider deteriorating items. This work was extended by Jamal et al. to allow for shortages. Meddah et al [4] investigated the effect of permissible delay in a periodic review environment. Salameh et al [5] developed an inventory model under permissible delay in payment in a continuous review situation. Chen and Chen [6] developed an inventory model for deteriorating items in a periodic review situation with shortages. Musa and Sani [7] constructed an Inventory policies model for delayed deteriorating items with permissible delay in payment.

In this paper, an attempt is employed to construct a model on the selection of the best cycle length for a delayed deteriorating inventory items with constrained retailer's capital or permissible delay in payments.

2.0 Mathematical Formulation

The following Notation and Assumptions are employed in the mathematical formulation:

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2.1 Notation

σ_1 The rate of demand before deterioration sets in
σ_2 The rate of demand after deterioration sets in
T The length of inventory cycle
C The unit cost of the item
C_O The ordering cost per order
i The inventory holding or carrying charge
P Interest to be paid per cycle
I_P The interest paid per investment in stocks per cycle length
I_e The interest that can be earned per investment in stocks per cycle length
D_P The permissible delay in settlement of the account.
θ The deterioration rate
E_1 Interest earned in a cycle length, T
E_2 Interest earned in the period, $M - T$
$I_d(t)$ Inventory level at any time *t* after the setting in of deterioration
I_d Inventory level at the time the deterioration sets in
$I(t)$ Inventory level at any time *t* before deterioration sets in
T_2 Difference between the cycle length T and the time when the deterioration sets in.
T_1 Time when deterioration sets in
I_0 Initial inventory

2.2 Assumptions

(a) Instantaneous Replenishment (b) Lead time is zero (c) Constrained Retailer's Capital

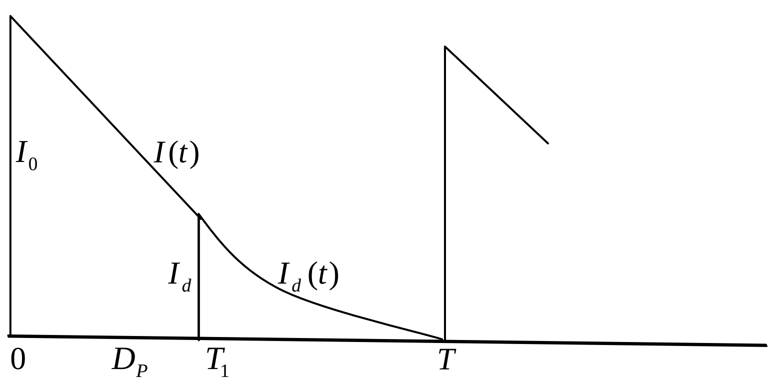

Figure 1: Inventory movement in a review period $0 \le D_P \le T_1$

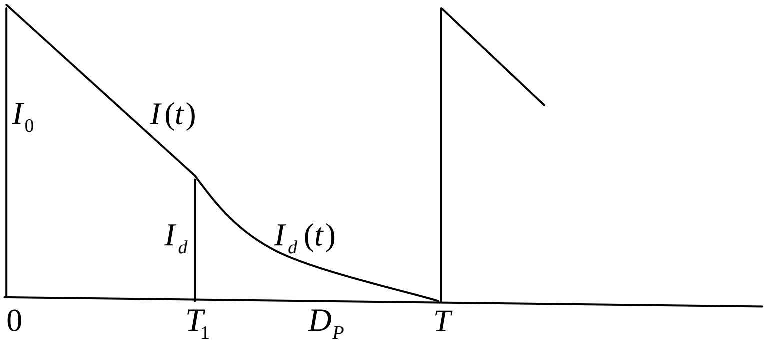

Figure 2: Inventory movement in a review period $T_1 \le D_P \le T$

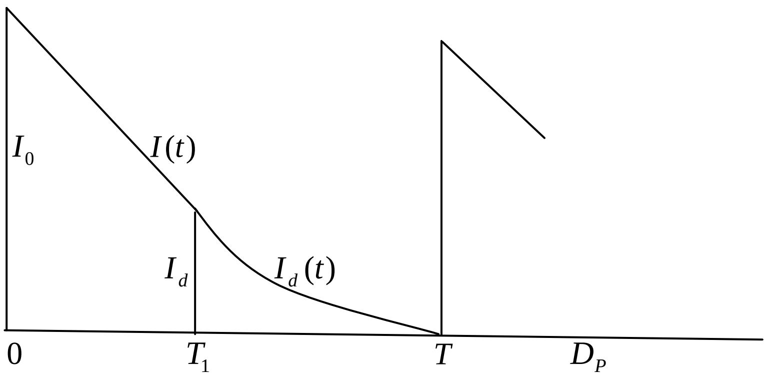

Figure 3: Inventory movement in a review period $T_1 \le T \le D_P$

The movement of the inventory in the interval $0 \le t \le T_1$ is described by the differential equation:

$$\frac{dI(t)}{dt} = -\sigma_1 \tag{1}$$

The equation is solved to give: $I(t) = -\sigma_1 t + K_1$ (2)

Where K_1 is an arbitrary constant. We apply the boundary conditions at $t = 0$, $I(t) = I_0$, in equation (2) to have $I_0 = K_1$, so that we get from (2)

$$I(t) = -\sigma_1 t + I_0 \tag{3}$$

Moreover, applying the boundary condition $t = T_1$, $I(t) = I_d$ in (3) yields:

$$I_0 = I_d + \sigma_1 T_1 \tag{4}$$

Substituting equation (4) into (3) gives:

$$I(t) = I_d + (T_1 - t)\sigma_1 \tag{5}$$

The movement of the inventory in the interval $T_1 \le t \le T$ is described by the differential equation:

$$\frac{dI_d(t)}{dt} + \theta I_d(t) = -\sigma_2 \ , \tag{6}$$

The solution of equation (6) after using a suitable integrating factor is given as :

$$I_d(t) = -\frac{\sigma_2}{\theta} + K_2 e^{-\theta t} \tag{7}$$

Applying the boundary conditions at $t = T_1$, $I_d(t) = I_d$ in equation (7) gives:

$$I_d = -\frac{\sigma_2}{\theta} + K_2 e^{-\theta T_1}, \text{ where } K_2 = I_d e^{\theta T_1} + \frac{\sigma_2}{\theta} e^{\theta T_1} \quad (8)$$

Substituting K_2 into (7) gives

$$I_d(t) = -\frac{\sigma_2}{\theta} + \left(I_d e^{\theta T_1} + \frac{\sigma_2}{\theta} e^{\theta T_1} \right) e^{-\theta t} = \frac{\sigma_2}{\theta}(e^{(T_1 - t)\theta} - 1) + I_d e^{(T_1 - t)\theta} \quad (9)$$

Also applying the boundary condition at $t = T$, $I_d(t) = 0$, we get from equation (9)

$$I_d = \frac{-\sigma_2}{\theta}(1 - e^{-(T_1 - T)\theta}) \quad (10)$$

Substituting (10) into (9) gives:

$$I_d(t) = \frac{\sigma_2}{\theta}(e^{(T-t)\theta} - 1) \quad (11)$$

Substituting equation (5) into (10) yields:

$$I(t) = -\frac{\sigma_2}{\theta}(1 - e^{-(T_1 - T)\theta}) + (T_1 - t)\sigma_1 \quad (12)$$

Total demand between T_1 and *T*= the demand rate at the onset of deterioration x time period when the item deteriorates $= \sigma_2 T_2$

The number of items that deteriorate during the time interval $T_1 \le t \le T$ is computed from:

$d(T_2)$ =The amount that remains from the quantity ordered at the start of deterioration

-The total demand between T_1 and $T = I_d - \sigma_2 T_2$ (13)

Substituting equation (10) into (13) gives:

$$d(T_2) = -\frac{\sigma_2}{\theta}(1 - e^{-(T_1 - T)\theta} + T_2\theta) \quad (14)$$

3.0 Inventory Scenarios

There are three clear inventory scenarios as given below:

(a) $0 \le D_P \le T_1$, where the permissible period within which to settle for the replenishment account is less than the time the deterioration begins. This Scenario is represented by Figure 1.

(b) $T_1 \le D_P \le T$, where the permissible period is greater than the time the deterioration begins but less than the inventory cycle length. This is described in Figure 2.

(c) $T_1 \le T \le D_P$, where the permissible period is greater than both the cycle length and the time the deterioration sets in. This situation is described in Figure 3.

3.1 Case 1 $(0 \le D_P \le T_1)$

The customer in this case uses the revenue obtained from the sale of items in stock and continues to earn interest from the accrued revenue up to the permissible period, D_P, the customer only pays interest if payment is made beyond the permissible period .

3.2 Evaluation of the cost functions: The total inventory cost is a function of ordering cost, Inventory Carrying Cost, cost of deteriorated items, interest payable and interest earned. The costs are computed individually thus:

(a) The inventory ordering cost is given as C_O

(b) The inventory carrying cost C_H which is the cost associated with the storage of the inventory until it is depleted is given as:

$$C_H = iC\int_0^{T_1} I(t)dt + iC\int_{T_1}^{T} I_d(t)dt$$

$$= iC\int_0^{T_1} \left\{ -\frac{\sigma_2}{\theta}(1 - e^{-(T_1 - T)\theta}) + (T_1 - t)\sigma_1 \right\} dt + iC\int_{T_1}^{T} \left\{ \frac{\sigma_2}{\theta}(e^{(T-t)\theta} - 1 \right\} dt$$

$$= \left(\left(1 + \frac{1}{\theta T_1} \right) e^{-(T_1 - T)\theta} + \frac{T_1 \sigma_1 \theta}{2\sigma_2} - \frac{1}{\theta T_1} - \frac{T}{T_1} \right) \frac{iC\sigma_2 T_1}{\theta} \quad (15)$$

(c) The interest payable per cycle is given by:

$$P = CI_p \int_{D_P}^{T_1} I(t)dt + CI_p \int_{T_1}^{T} I_d(t)dt$$

$$= \left(\left(1 - \frac{D_P}{T_1} \right) e^{-(T_1 - T)\theta} + \frac{(D_P - T)}{T_1} + \frac{1}{\theta T_1} (e^{(T - T_1)\theta} - 1) \right) + CI_p T_1 \sigma_1 \left(\frac{T_1}{2} + \frac{D_P (D_P - 2T_1)}{2T_1} \right) \frac{CI_p \sigma_2 T_1}{\theta} \quad (16)$$

If $D_P = T_1$, the interest payable from (16) becomes:

$$P = \left(1 - \frac{T}{T_1} + \frac{1}{\theta T_1} (e^{(T - T_1)\theta} - 1) \right) \frac{CI_p \sigma_2 T_1}{\theta} \quad (17)$$

(d) The interest earned per cycle which is the interest earned during the positive stock of the inventory is given by:

$$E_I = CI_e \int_0^{T_1} \sigma_1 t dt + CI_e \int_{T_1}^{T} \sigma_2 t dt = ((\sigma_1 - \sigma_2) T_1^2 + \sigma_2 T^2) \frac{CI_e}{2} \quad (18)$$

(e) The cost of deteriorated items is given as: $Cd(T_2) = -\frac{C\sigma_2}{\theta}(1 - e^{-(T_1 - T)\theta} + (T - T_1)\theta)$ (19)

The total inventory cost per cycle length , $TC_{11}(T) = \frac{1}{T}$ (Inventory ordering cost + Cost of deteriorated items + inventory carrying cost + Interest payable per cycle -Interest earned during the cycle).

$$TC_{11}(T) = \frac{1}{T}(C_O + Cd(T_2) + C_H + P_I - E_I)$$

$$= \frac{C_O}{T} + \frac{C}{T}\left(-\frac{\sigma_2}{\theta}(1 - e^{-(T_1 - T)\theta} + T_2\theta \right) + \left(\left(1 + \frac{1}{\theta T_1} \right) e^{-(T_1 - T)\theta} + \frac{T_1 \sigma_1 \theta}{2\sigma_2} - \frac{1}{\theta T_1} - \frac{T}{T_1} \right) \frac{iC\sigma_2 T_1}{\theta T}$$

$$+ \left(\left(1 - \frac{D_P}{T_1} \right) e^{-(T_1 - T)\theta} + \frac{(D_P - T)}{T_1} + \frac{1}{\theta T_1} (e^{(T - T_1)} - 1) \right) \frac{CI_p \sigma_2 T_1}{\theta T} + \left(\frac{T_1}{2} + \frac{D_P (D_P - 2T_1)}{2T_1} \right) \frac{CI_p T_1 \sigma_1}{T}$$

$$- \left((\sigma_1 - \sigma_2) T_1^2 + \sigma_2 T^2 \right) \frac{CI_e}{2T} \quad (20)$$

We evaluate $\frac{dTC_{11}(T)}{dT} = 0$ to determine value of $T = T_{11}$ which gives the minimum total inventory cost after simplification as follows:

$$-C_O + \frac{C\sigma_2}{\theta}\left(1 + (\theta T - 1)e^{-(T_1 - T)\theta} \right) - C\sigma_2 T_1 + \left(\left(1 + \frac{1}{\theta T_1} \right) (\theta T - 1) e^{-(T_1 - T)\theta} - \frac{T_1 \sigma_1 \theta}{2\sigma_2} + \frac{1}{\theta T_1} \right) \frac{iC\sigma_2 T_1}{\theta}$$

$$+ \left(\left(1 - \frac{D_P}{T_1} \right) (\theta T - 1) e^{-(T_1 - T)\theta} - \frac{D_P}{T_1} + (\theta T - 1) e^{(T - T_1)\theta} + 1 \right) \frac{CI_p \sigma_2 T_1}{\theta} - \left(\frac{T_1^2 + D_P (D_P - 2T_1)}{2T_1} \right) \Bigg) CI_p T_1 \sigma_1$$

$$+ \left((\sigma_1 - \sigma_2) T_1^2 - T^2 \sigma_2 \right) \frac{CI_e}{2} = 0 \quad (21)$$

3.3 Case 2 $(T_1 \le D_P \le T)$

In this case the permissible period for the settlement of the replenishment is greater than the time the deterioration sets in.

3.4 Evaluation of the Cost functions

In this case the ordering cost, the cost of deteriorated items, the interest earned per cycle and the inventory holding cost are same as in case 1.

(a) The interest to be paid per cycle is given by:

$$P = CI_p \int_{D_P}^{T} I_d(t)dt = CI_p \frac{\sigma_2}{\theta}\int_{D_P}^{T}(e^{(T-t)\theta}-1)dt = \left(-\frac{1}{\theta}(1-e^{(T-D_P)\theta}) - (T-D_P)\right)\frac{CI_p\sigma_2}{\theta} \qquad (22)$$

If the permissible period coincides with the cycle length i.e $D_P = T$, we get from equation (22)

$P = \frac{CI_p\sigma_2}{\theta}\left(-\frac{1}{\theta}(1-e^{(D_P-D_P)\theta})-(D_P-D_P)\right)$ =0 which clearly shows that at the end of the cycle, no interest is payable since the inventory is completely depleted.

On the other hand, if $D_P = T_1$, equation (22) yields:

$$P = \frac{CI_p\sigma_2}{\theta}\left((T_1-T)+\frac{1}{\theta}(e^{(T-T_1)\theta}-1)\right) \qquad (23)$$

Equation (23) coincides with equation (17), which is due to the fact that both equations describe a situation where $D_P = T_1$. Moreover, If $D_P = 0$, equation (16) becomes:

$$P = \left(e^{-(T_1-T)\theta} - \frac{T}{T_1} + \frac{1}{\theta T_1}(e^{(T-T_1)\theta}-1)\right)\frac{CI_p\sigma_2 T_1}{\theta} + \frac{CI_p T_1^2\sigma_1}{2} \qquad (24)$$

i.e. $0 \le P \le \frac{CI_p\sigma_2 T_1}{\theta}\left(e^{-(T_1-T)\theta} - \frac{T}{T_1} + \frac{1}{\theta T_1}(e^{(T-T_1)\theta}-1)\right) + \frac{CI_p T_1^2\sigma_1}{2}$ (25)

The total inventory cost per cycle length is given by:

$$TC_{12}(T) = \frac{1}{T}(C_O + Cd(T_2) + C_H + P_I - E_I)$$

$$= \frac{C_O}{T} + \frac{C}{T}\left(-\frac{\sigma_2}{\theta}(1-e^{-(T_1-T)\theta}+T_2\theta)\right) + \frac{1}{T}\left(\left(1+\frac{1}{\theta T_1}\right)e^{-(T_1-T)\theta} + \frac{T_1\sigma_1\theta}{2\sigma_2} - \frac{1}{\theta T_1} - \frac{T}{T_1}\right)\frac{iC\sigma_2 T_1}{\theta}$$

$$+\frac{1}{T}\left(-\frac{1}{\theta}(1-e^{(T-D_P)\theta}-(T-D_P)\right)\frac{CI_p\sigma_2}{\theta} - \frac{1}{T}\left((\sigma_1-\sigma_2)T_1^2+\sigma_2 T^2\right)\frac{CI_e}{2} \qquad (26)$$

By solving the equation $\frac{dTC_{12}(T)}{dt} = 0$, the value of $T = T_{12}$ which minimizes the total variable cost per unit time could be obtained in a simplified form as follows:

$$-C_O + \frac{C\sigma_2}{\theta}\left(1+(\theta T-1)e^{-(T_1-T)\theta}\right) - C\sigma_2 T_1 + \left(\left(1+\frac{1}{\theta T_1}\right)(\theta T-1)e^{-(T_1-T)\theta} - \frac{T_1\sigma_1\theta}{2\sigma_2} + \frac{1}{\theta T_1}\right)\frac{iC\sigma_2 T_1}{\theta}$$

$$+\left(\frac{1}{\theta}\left(1-e^{(T-D_P)\theta}\right)+Te^{(T-D_P)\theta}-D_P\right)\frac{CI_p\sigma_2}{\theta} + \left((\sigma_1-\sigma_2)T_1^2-\sigma_2T^2\right)\frac{CI_e}{2} = 0 \qquad (27)$$

3.4 Case 3 $(D_P > T)$

Interest is not paid by the customer in this case since $D_P > T$ meaning that the inventory is completely depleted, however, he continues to earn interest on sales revenue up to the permissible period .The interest earned is a combination of that earned in a cycle, T , plus that earned in $D_P - T$.

3.5 Evaluation of the Cost functions

In this case the ordering cost, the cost of deteriorated items and the inventory holding cost are same as in cases 1 and 2.

(a) **Interest Earned in a Cycle, *T*, plus that earned in** $D_P - T$

Let the interest earned in a cycle, *T*, be E_1 and that earned in $D_P - T$ be E_2 .Then

$$E_1 = CI_e\sigma_1\int_0^{T_1} t dt + CI_e\sigma_2\int_{T_1}^{T} t dt = \frac{CI_e}{2}\left((\sigma_1-\sigma_2)T_1^2 + T^2\sigma_2\right) \qquad (28)$$

The interest earned during $D_P - T$ i.e. beyond the cycle length and up to the permissible period is given by:

$$E_2 = CI_e(\sigma_1 T_1 + \sigma_2(T - T_1))(D_P - T) \tag{29}$$

The total interest earned, E_T is obtained by combining equations (28) and (29) as follows:

$$E_T = E_1 + E_2 = \frac{CI_e}{2}\left((\sigma_1 - \sigma_2)T_1^{\,2} + T^2\sigma_2\right) + CI_e(\sigma_1 T_1 + \sigma_2(T - T_1))(D_P - T)$$

$$= \frac{CI_e}{2}\left\{(\sigma_1 - \sigma_2)T_1^{\,2} + T^2\sigma_2 + 2\sigma_1 T_1 D_P - 2\sigma_1 T_1 T + 2\sigma_2 TD_P - 2\sigma_2 T^2 - 2\sigma_2 T_1 D_P + 2\sigma_2 T_1 T)\right\}$$

In this case, the total inventory cost per cycle length *T* is given by:

$$TC_{13}(T) = C_O + Cd(T_2) + C_H - E_T$$

$$= \frac{C_O}{T} + \frac{C}{T}\left(\left(-\frac{\sigma_2}{\theta}\right)\left(1 - e^{-(T_1 - T)\theta} + T_2\theta\right)\right) + \frac{1}{T}\left(\left(1 + \frac{1}{\theta T_1}\right)e^{-(T_1 - T)\theta} + \frac{T_1\sigma_1\theta}{2\sigma_2} - \frac{1}{\theta T_1} - \frac{T}{T_1}\right)\frac{iC\sigma_2 T_1}{\theta}$$

$$- \frac{1}{T}\left((\sigma_1 - \sigma_2)T_1^{\,2} + \sigma_2 T^2 + 2\sigma_1 T_1 D_P - 2\sigma_1 T_1 T + 2\sigma_2 TD_P - 2\sigma_2 T^2 - 2\sigma_2 T_1 D_P\right.$$

$$\left. + 2\sigma_2 T_1 T\right)\frac{CI_e}{2} \tag{30}$$

The value of $T = T_{13}$ which minimizes $TC_{13}(T)$ can be obtained by solving the equation $\frac{dTC_{13}(T)}{dT} = 0$ and simplifying to yield:

$$-C_O + \frac{C\sigma_2}{\theta}\left(1 + (\theta T - 1)e^{-(T_1 - T)\theta}\right) - C\sigma_2 T_1 + \left(\left(1 + \frac{1}{\theta T_1}\right)(\theta T - 1)e^{-(T_1 - T)\theta} - \frac{T_1\sigma_1\theta}{2\sigma_2} + \frac{1}{\theta T_1}\right)\frac{iC\sigma_2 T_1}{\theta}$$

$$+ \left((\sigma_1 - \sigma_2)T_1^2 + 2T_1 D_P(\sigma_1 - \sigma_2)\sigma_2 T^2\right)\frac{CI_e}{2} = 0 \tag{31}$$

At $T = D_P$, the cost function $TC_{12}(T) = TC_{13}(T)$ which is denoted by $TC(D_P)$ and given by:

$$TC(D_P) = \frac{C_O}{D_P} + \frac{C}{D_P}\left(\left(-\frac{\sigma_2}{\theta}\right)\left(1 - e^{-(T_1 - M)\theta} + T_2\theta\right)\right) + \frac{1}{D_P}\left(\left(1 + \frac{1}{\theta T_1}\right)e^{-(T_1 - M)\theta} + \frac{T_1\sigma_1\theta}{2\sigma_2} - \frac{1}{\theta T_1} - \frac{D_P}{T_1}\right)\frac{iC\sigma_2 T_1}{\theta}$$

$$- \frac{1}{D_P}\left((\sigma_1 - \sigma_2)T_1^{\,2} + \sigma_2 D_P^{\,2}\right)\frac{CI_e}{2} \tag{32}$$

4.0 Selection criteria for the best cycle length T

From the last section, we considered three inventory scenarios as follows:

(i) $0 \le D_P \le T_1 < T$ (ii) $T_1 \le D_P \le T$ (iii) $T_1 \le T \le D_P$

As already indicated in the last section, T_{11}, T_{12} and T_{13} be the periods associated with the three categorized inventory scenarios above, then:

(1) If $D_P \le T_1$ and $D_P \le T_{11}$, compare $TC_{11}(T_{11})$ and $TC(D_P)$, then go to (5).

(2) If $D_P > T_1$, $D_P \le T_1$ and $D_P \ge T_{13}$ compare $TC_{12}(T_{12})$, $TC_{13}(T_{13})$ and $TC(D_P)$, then go to (5).

(3) If $D_P > T_1$, $D_P \le T_1$ but $D_P < T_{13}$, compare $TC_{12}(T_{12})$ and $TC(D_P)$, then go to (5)

(4) If $D_P > T_1$, $D_P > T_{12}$ but $D_P \ge T_{13}$, compare $TC_{13}(T_{13})$ and $TC(D_P)$, then go to (5).

(5) To find the best cycle length, select that cycle length associated with the least cost.

(6) If $D_P \le T_1$ but $D_P > T_{11}$ or $D_P > T_1$ but $D_P > T_{12}$ and $D_P < T_{13}$ then the optimum cycle length will be D_P

5. 0 Numerical Examples

Three examples are considered as above, so as to get the best period *T*, depending on the category in which the example falls.

Table 1: Parameter values and the optimal cycle length for the inventory model with constrained retailer's capital

S/N	C_O	C	σ_1	σ_2	i	D_P	T_1	θ	I_e	I_p	$T=T_{11}$	$T=T_{12}$	$T=T_{13}$
1	250	70	3000	500	0.11	0.0384	0.0575	0.4	0.18	0.09	0.3288	-	-
2	500	50	2000	200	0.12	0.0575	0.0384	0.2	0.13	0.12	-	0.1671	0.4493
3	300	150	2000	500	0.12	0.1534	0.0192	0.7	0.14	0.11	-	-	0.0767

Table 2: Selection of the best cycle length corresponding to the least overall inventory cost

S/N	$TC_{11}(T_{11})$	$TC_{12}(T_{12})$	$TC_{13}(T_{13})$	$TC(D_P)$	Selected best cycle length
1	122,2288.9	-	-	6002.72	14days
2	-	36900.09	-	8687.62	21days
3	-	-	3308.42	5014.13	28days

6.0 Discussion on the result

(a) From the data in Table 1, row 1, it is clear that $D_P \leq T_1$ and so the value for $T = T_{11}$ which minimizes the total inventory cost is $0.3288 \approx 120$days. Therefore, we compare $TC_{11}(T_{11})$ and $TC(D_P)$ in Table 2, example (1) where $TC(D_P) \leq TC_{11}(T_{11})$. We then select $D_P = 14$ days which is associated with the least cost to be the best cycle length *T*.

(b) From row 2 in Table 2, it is clear that $D_P > T_1$. The values for $T = T_{12}$ and $T = T_{13}$ in this case are found to be $0.1671 \approx$ 61 days and $0.4493 \approx 164$ days respectively, i.e. $M \leq T_{12}$ but $D_P < T_{13}$. Therefore, we only compare $TC_{12}(T_{12})$ and $TC(D_P)$ to obtain $TC_{12}(T_{12}) = 36900.09$ and $TC(D_P) = 8687.62$. Since $TC(D_P) \leq TC_{12}(T_{12})$, we select $D_P = 21$ days to be the right cycle length T.

(c) The values above indicate that $D_P > T_1$ and so the values $T = T_{12}$ and $T = T_{13}$ are to be found so that the cycle period associated with the least cost is chosen as the best cycle length. The equation associated with $T = T_{12}$ does not have a positive root and so we have no $T = T_{12}$, on the other hand, the value for $T = T_{13}$ is $0.0767 \approx 28$days i.e. $D_P > T_{12}$ but $D_P \geq T_{13}$. We should therefore compare $TC_{13}(T_{13})$ and $TC(D_P)$ to get $TC_{13}(T_{13}) = 3308.42$ and $TC(D_P) = 5014.13$. Since $TC(D_P) \geq TC_{13}(T_{13})$, we select $T_{13} = 28$ days to be the best cycle length T.

7.0 Conclusion

In this paper, a mathematical model on the selection of the best cycle length for the inventory of delayed deteriorating items is presented. The model is built on the assumption that the demand, the deterioration rate, the inventory holding cost and other parameters are known constants.

The model considers a situation where the customer is given some allowed period within which to settle for the goods supplied. The customer is charged interest if he failed to settle the replenishment account within the permissible period. The optimal cycle length *T* in each of the three examples that gives the minimum total inventory cost was determined.

References

[1] P.M Ghare, G.F.Shrader, A Model for Exponential Decaying Inventory, Journal of Industrial Engineering 14 (1963) 238-243.

[2] S.K. Goyal, Economic Order Quantity under Conditions of Permissible Delay in Payments, Journal of Operational Research Society, 36 (1985) 335-338.

[3] S.P. Aggarwal, C.K. Jaggi, Ordering Policies of Deteriorating Items under Permissible Delay in Payment, Journal of Operational Research Society, 46 (1995) 658-662.

[4] B.S. Meddah, M.Y. Jaber , N.E. Abboud, Periodic Review (s, S) Inventory Model with Permissible Delay in Payments, Journal of Operational Research Society, 55 (2004) 147-159.

[5] M.K. Salameh, N.E. Abboud, A.N. EL-Kassar, R.E. Ghattas, Continuous Review Inventory Model with Delay in Payments, International Journal of Production Economics,85 (2003) 91-95.

[6] J.M. Cheng and L.T. Chen, Pricing and Lot Sizing for a Deteriorating item in a periodic review Inventory System with Shortages, Journal of Operational Research Society, 55 (2004) 892-901.

[7] A. Musa and B. Sani, Inventory Ordering Policies of Delayed Deteriorating Items under Permissible Delay in Payments, International Journal of Production Economics 136 (2012) 75-83.