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Response Analysis of Viscously Damped Euler- Bernoulli Beam Carrying Linearly Varying Distributed Moving Loads

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Abstract

A theory describing the dynamics behavior of a viscously damped Bernoulli -Euler Beam traversed by a linearly varying distributed moving load is investigated, the governing equation of fourth order partial differential equation wassolved by assumed solution in series form to reduce the coupled equation to ordinarysecond order differential equation. The obtained differential equation was solved with Mathematical software (Maple). The numerical results arepresented in plotted curves which show that the response amplitude of the beam increase as the value of the damping decreases. And the maximum amplitude wasattained at ω_b= 2:0, and that for a fixed value of ω_b (dampingcoefficient) and for various values of the length (L).Increase in the length produces maximum amplitude.

Keywords: dynamic responses, Bernoulli - Euler Beam, linearly varying distributed, loads.

1.0 Introduction

Several investigations have been carried out on the problems of the dynamic response of structures to moving loads [1 – 13]. Such studies are of importance in the field of transportation and in designing space station facilities and machine parts. In fact, a moving load induces larger deflections and stresses on the structure on which it moves then does an equivalent static load. As there are several different structures on which loads move so are many types of load. Consequently, moving load problems did and still continues to draw attention of researchers [1 – 13]. In the field of engineering, applied mathematics and physics. For simplicity, moving loads can be assumed to be approximated or concentrated otherwise they are distributed. The problems that involved the former, that is concentrated moving load problems have been the most common subject of investigating among researchers. This is perhaps due to the fact that it is a simplified formulation for moving load problems. On the other hand, a more realistic approach is to assume that the load is distributed over a length or contact area as it moves. In the case of a distributed load, a load that is distributed with constant magnitude is referred to as uniformly distributed, while load's distribution of the formular$C_1 + C_2x$; where C_1 and C_2 are constants and x is a variable is said to be linearly distributed. While there is a very limited number of publications on beam with distributed loads, most of these few publication focused on uniformly distributed problems [2, 5, 7].

The idea behind assuming a uniform distribution is encouraging in that it results in a considerable simplification of moving load's distribution problems. However, in the practical sense, in the area of road transports, designing of machine parts and aerospace engineering,

uniform distribution is just a specific case and the simplest. Hence it is more practically useful to consider the load as linearly distributed as opposed to a uniformly distributed.

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In this context, this study focuses on the response of linearly varying distributed moving load on the deflection of a beam and also comparison with or without the damping co-efficient, length of beam and velocity.

Numerical example involving a simply supported beam is presented.

2.0 Mathematical Formulation and Simplification of the Governing Equation

The vibration of a beam as described by Bernoulli-Euler's differential equation, based on the assumption that the theory of small deformations, Hooke's law and Navier's hypothesis is being applied. Further assumptions are as follows. The beam is of constant cross - section and constant mass per unit length, the moving mass moves at constant speed from left to right and beam damping is proportional to the velocity of the vibration.

Under the above assumptions, the governing equation of motion is described by the following equation.

$$EI\frac{\partial^4 u(x,t)}{\partial x^4}+\mu\frac{\partial^2 u(x,t)}{\partial t^2}+2\omega_b\mu\frac{\partial u(x,t)}{\partial t}=q(x,t)----------------(1)$$

Where x is the length coordinate with the origin at the left hand end of the beam.

T is the time coordinate with the origin at the instant of the force arriving on the beam.

$W(x,t)$ is the beam deflection at the point x and time t, measured from the equilibrium position when the beam is located with its own weight.

E is the Young's modulus of the beam.

I is the constant moment of inertia of the beam cross section.

μ is the constant mass per unit length of the beam.

ω_b is the circular frequency of the damping of the beam.

$q(x,t)$ is the applied force, and is defined by macaulay notation as the load.

$$q(x,t)=W_1\langle x-\alpha_1\rangle^0+\frac{W_2-W_1}{d}\langle x-\alpha_1\rangle^1+W_2\langle x-\alpha_1\rangle+W_2\langle x-\alpha_1\rangle^1-\frac{W_2-W_1}{d}\langle x-\alpha_1\rangle----------(2)$$

Where

$w_1=M_1g$ And $w_2=M_2g$ are the forces produced by masses M_1 and M_2 respectively at the end points of the load. As shown in Figure 1,

$d=a_2+a_2$ is the length of the load,

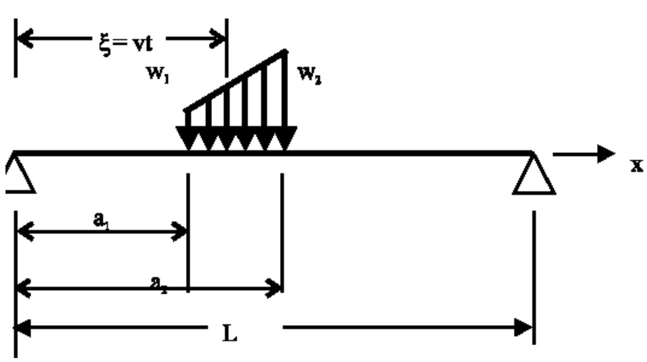

Fig 1: Schematic diagram showing linearly varying distributed moving loads on Beam.

$$\left.\begin{aligned} a_1 &= \frac{vt - d,}{2} \\ a_2 &= \frac{vt + d,}{2} \end{aligned}\right\} ------------------------(3)$$

V is the velocity of load, g is the acceleration due to gravity and the Macaulay Notation is defined as

$$< x - \alpha >^n = \begin{cases} 0 & x \le \alpha_{i,} \\ (x - \alpha_i) & x \ge \alpha_i \end{cases} ---------------(4)$$

With the boundary conditions;

$$\left.\begin{aligned} w(0,t) = w^{ll}(L,t) = 0 \\ w(0,t) = w^{ll}(L,t) = 0 \end{aligned}\right\} ---------------------------------(5)$$

and the initial conditions

$$w(x,0) = \frac{\partial w(x,t)}{\partial t} = --(6)$$

3.0 Method of Solution

Evidently, a closed form solution of the partial differential equation (5) does not exist. The assumed solution of the form

$$w(x,t) = \sum_{n=1}^{\infty} X_n(x)T_n(t) ----------------------(7)$$

Were considered, where*x* and *t* are the known Eigenfunctions of the beam and n is the number of contributed modes.

The differentials of equation (6) with respect to x and t yields

$$\left.\begin{aligned} w^{l}(x,t) &= \sum_{n=1}^{\infty} X_n^{l}(x)T_n(t) = \frac{\partial w(x,t)}{\partial x} \\ w^{ll}(x,t) &= \sum_{n=1}^{\infty} X_n^{ll}(x)T_n(t) = \frac{\partial^2 w(x,t)}{\partial x^2} \\ w^{lll}(x,t) &= \sum_{n=1}^{\infty} X_n^{lll}(x)T_n(t) = \frac{\partial^3 w(x,t)}{\partial x^3} \\ w^{llll}(x,t) &= \sum_{n=1}^{\infty} X_n^{llll}(x)T_n(t) = \frac{\partial^4 w(x,t)}{\partial x^4} \end{aligned}\right\} -------- (8)$$

$$\left.\begin{aligned} \dot{W}(x,t) &= \sum_{n=1}^{\infty} X_n(x)\dot{T}_n(t) = \frac{\partial w(x,t)}{\partial t} \\ \ddot{W}(x,t) &= \sum_{n=1}^{\infty} X_n(x)\ddot{T}_n(t) = \frac{\partial^2 w(x,t)}{\partial t^2} \end{aligned}\right\} ---------(9)$$

Substituting equations (8) and (9) into equation (1), we obtain

$$\left.\begin{aligned}&EI\sum_{n=1}^{\infty}X_n^{iiii}(x)T_n(t)+\mu\sum_{n=1}^{\infty}X_n(x)\ddot{T}_n(t)+2\omega_b\mu\sum_{n=1}^{\infty}X_n(x)\dot{T}_n(t)=W_1\langle x-\alpha_1\rangle^0+\frac{W_2-W_1}{d}\langle x-\alpha_1\rangle^1-\\&W_2\langle x-\alpha_2\rangle^0-\frac{W_2-W_1}{d}\langle x-\alpha_2\rangle\end{aligned}\right\}--(10)$$

Applying the boundary condition and the initial condition of equations (5) and (6) into equation (10).
The natural modes satisfy the homogenous differential equation

$$EIX_n^{iiii}(x)-\mu\omega_n^2X_n(x)=0-----------------(11)$$

The n[th] normal mode of vibration of a uniform beam gives

$$X_n(x)=A_nSin\frac{\lambda_nX}{L}+B_nCos\frac{\lambda_nX}{L}+C_nSinh\frac{\lambda_nX}{L}+D_nCosh\frac{\lambda_nX}{L}--------------(12)$$

Where A_n, B_n, C_n and D_n are constants and λ_n are determined. The natural circular frequency is given by using the desired ends support condition.

$$\omega_n^2=\frac{n^4n^4EI}{\mu L^4}-----------------------------------(13)$$

multiplying both sides of equation (10) by $X_{k\,(x)}$ and integrating along the entire length of the beam we have

$$\int_O^L\sum_{n-1}^{\infty}\mu Wn^2X_n(x)T_n(t)X_k(n)dx\int_O^L\sum_{n-1}^{\infty}\mu Xn(x)\overline{T_n}(t)X_k(x)dx\int_O^L\sum_{n-1}^{\infty}2\mu\omega_bX_n(x)\overline{T_n}(x)$$

$$X_{k(n)}{}^{dx}=\left[\int_O^LW_1\langle x-\alpha_1\rangle^0+\frac{W_2-W_1}{d}\langle x-\alpha_1\rangle^1-W_2\langle x-\alpha_1\rangle^0-\frac{W_2-W_1}{d}\langle x-\alpha_2\rangle X_k(x)\right]dx------(14)$$

Applying the orthogonality property

$$\int_L^O\mu w_n^2X_n(x)X_k(x)T_n(t)dx+\int_L^O\mu X_n(x)X_k(x)\ddot{T}_n(t)dx+\int_L^O2\mu\omega_bX_k(x)X_n(x)\dot{T}_n(t)dx=\int_O^Lq(x,t)X_k(x)\,dx--(15)$$

Since n = k =α, therefore $\int_L^OX_n(x)X_k(x)\,dx=\alpha$

$$\alpha\mu w_n^2T_n(t)+\mu\alpha\ddot{T}_n(t)+2\mu\omega_b\alpha\dot{T}_n(t)=\int_O^Lq(x,t)X_k(x)dx----------------(16)$$

$$w_n^2T_n(t)+\ddot{T}_n(t)+2\mu\omega_b\alpha\dot{T}_n(t)=\frac{1}{\alpha\mu}\int_O^Lq(x,t)X_k(x)dx-----------------(17)$$

$$\ddot{T}_n(t)+2\omega_b\dot{T}_n(t)+w_n^2T_n(t)=\frac{1}{\alpha\mu}\int_O^Lq(x,t)X_k(x)dx\;q(x,t)$$

$$=\left\{W_1\langle x,\alpha_1\rangle^0+\frac{W_2-W_1}{d}\langle x,\alpha_1\rangle^1-W_2\langle x,\alpha_2\rangle^0-\frac{W_2-W_1}{d}\langle x,\alpha_2\rangle\right\}-------------(18)$$

4.0 Numerical Method and Discussion of Results

To illustrate the foregoing analysis, the uniform Bernoulli-solar beam of length 15m were considered, mass 3kg, 6kg and 9kg, α = 1, μ = 75, EI = 2785Nm^{-1}, K = 1, 2, 3. . . V = 3.3ms^{-1}, g = 10ms^{-2}, π = 3.142, n = 1, 2, 3. . :

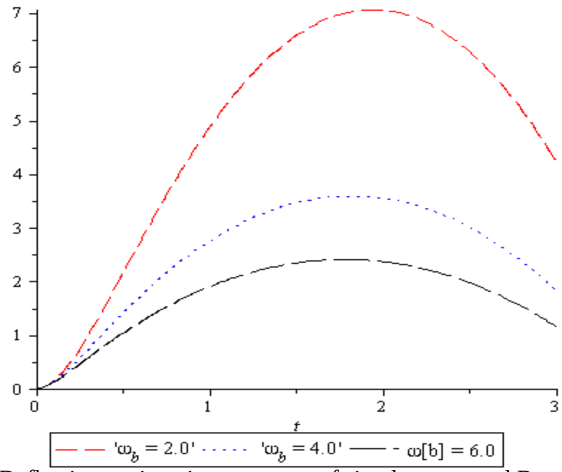

Figure 2: Deflection against time response of simply supported Bernoulli beam under the action of moving mass for the various values of damping co- efficient ω_b .

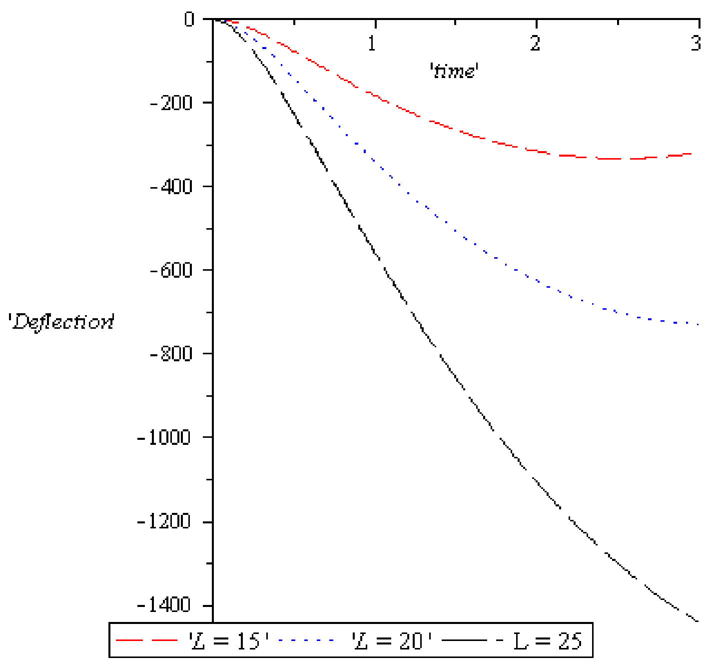

Figure 3, Graph of Deflection against time response for various value of length L.

Conclusion

The problem of assessing the Dynamic Analysis of viscously damped Euler-Bernoulli Beam carrying a linearly varying distributed moving load. The governing equation for the mathematical model is analytically simplified into a set of ordinary differential equations that are solved by a mathematical software (Maple).

Clearly, the graph (Fig. 2) shows that the response amplitude of the beam increase as the value of the damping decreases, and the maximum amplitude was attained at $\omega_b = 2{:}0$.

The results have been able to show (Fig. 3) that for a fixed value of ω_b (dampingcoefficient) and for various values of the length (L); it was observed that increase in the length of the beam gives maximum amplitude.

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