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Convective Heat Transfer And Heat Source In MHD Flow Over A Flate Plate

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Abstract

This papers consider the thermal boundary layers under influence of uniform magnetic field with temperature dependent heat source over a flate plate.The system of coupled partial differential equations is first transformed into a system of coupled non-linear ordinary differential equations using the method of similarity transformation, which is solved numerically using the shooting iteration technique together with fourth order Runge-kutta integration scheme .The variations in dimensionless surface temperature characteristics for different values of Prandtl number (Pr)***,local Grashof number*** (Gr_x)***, Magnetic field strength*** (M)***,heat source parameter*** (α) ***and Biot number*** (a_x) ***are presented graphicallyand tabulated.***

Keywords: Convective heat transfer,MHD,heat source parameter and Flate plate.

1.0 Introduction

Convective heat transfer studies are very important in the processing industries, it is often necessary to pump fluids over long distances,and there may be a substantial drop in pressure in both the pipeline and individual units. Intermediate products are often pumped from one factory site to another, and raw materials such as natural gas and petroleum products may be pumped for a long distance to domestic or industrial consumers. It is necessary therefore, to consider the problems concerned with calculating the power requirements for pumping, designing the most suitable flow system, estimating the most economical sizes of pipes, measuring the rate of flow and frequently controlling this flow at a steady rate. Fluid flow may take place at high pressures, when process streams are fed into a reactor. For instance, or at low pressures when, for example, vapour leaves the top of a vacuum distillation column processes involving high temperatures such as gas turbines, nuclear plants,thermal energy storage, etc.

The classical problem (i.e., fluid flow along a horizontal, stationary surface located in a uniform freestream)was solved for the first time in 1908 by Blasius [1]; it is still a subject of current research [2,3] and, moreover, further study regarding this subject can be seen in most recent papers[4,5]. Moreover, Bataller [6] has presented a numerical solution for the combined effects of thermal radiation and convective surface heat transfer on the laminar boundary layer about a flat-plate in a uniform stream of fluid (Blasius flow) and about a moving plate ina quiescent ambient fluid (Sakiadis flow). Aziz [7] investigated a similarity solution for laminar thermal boundary layer over a flat-plate with a convective surface boundary condition. Mankinde and Olanrewaju [8] extended the work of Aziz to include the combined effects of buoyancy force and a convective heat exchange at the plate surface on the boundary.Numerous studies such as Refs. [9–11] considered different variations in temperature and heat flux at the plate.

In this present paper,we extend the recent work of Mankinde and Olanrewaju [8] to include the effect of magnetic field and temperature dependent source. The numerical solutions of the resulting momentum and the thermal similarity equations are reported for representative values of thermophysical parameters characterizing the fluid convection process.

2.0 Mathematical Analysis

We consider two-dimensional steady incompressible fluid flow with heat transfer by convection over a flat plate. A stream of cold fluid at temperature T_∞ moving over the right surface of the plate with a uniform velocity U_∞ while the left

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surface of the plate is heated by convection from a hot fluid at temperature T_f , which provides a heat transfer coefficient h_f under the influence of magnetic field , temperature dependent heat source and density variation due to buoyancy effects.

The continuity, momentum and energy equations describing the flow can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)-\frac{\sigma\mu_\infty H_0^2 u}{\rho} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+Q(T-T_\infty) \tag{3}$$

where *u, v* are the velocity components, *p* the pressure, σ the electrical conductivity, μ_∞ the magnetic permeability, *k* the thermal conductivity, ρ the density of the fluid, β the coefficient of volume expansion, Cp the specific heat at constant pressure. T_∞ is the equilibrium temperature and other symbols have their usual meanings.
The velocity boundary conditions relevant to the problem are taken as

$$u(x,0)=v(x,0)=0 \tag{4}$$

$$u(x,\infty)=T_\infty \tag{5}$$

We assume the bottom surface is heated by convection from a hot fluid at temperature T_f which provides a heat transfer coefficient h_f .The boundary conditions at the plate surface and far into the cold fluid may be written as

$$-k\frac{\partial T}{\partial y}(x,0)=h_f\left[T_f-T(x,0)\right] \tag{6}$$

$$T(x,\infty)=T_\infty \tag{7}$$

A similarity solution of Equations (1)-(5) is obtained by defining an independent variable η and a dependent variable variable f in terms of the stream function ψ as

$$\eta=y\sqrt{\frac{U_\infty}{\upsilon x}}\quad,\ f(\eta)=\frac{\psi}{U_\infty\sqrt{\frac{\upsilon x}{U_\infty}}}\quad,u=U_\infty\frac{df}{d\eta}\ ,v=\frac{1}{2}\sqrt{\frac{U_\infty\upsilon}{x}}\left(\eta\frac{df}{d\eta}-f\right) \tag{8}$$

and the dimensionless temperature θ, Prandtl number $\Pr$, magnetic parameter M,Grashof number Gr and heat source parameter γ

$$\theta(\eta)=\frac{T-T_\infty}{T_f-T_\infty}\ ,\ \Pr=\frac{k}{\alpha},M=\frac{\sigma H_0^2 x}{\rho U_\infty}\ ,\ Gr_x=\frac{g\beta x^3(T_f-T_\infty)}{\upsilon^3},\alpha=\frac{Qx}{kU_\infty} \tag{9}$$

the continuity equation is identically satisfied and the momentum and heat transfer equations reduced to

$$f'''+\frac{1}{2}ff''-Mf'+Gr_x\theta=0 \tag{10}$$

$$\theta''+\frac{1}{2}\Pr f\theta'-\alpha\theta=0 \tag{11}$$

The transformed boundary conditions are

$$f(0)=0,\ f'(0)=0\ ,\quad \theta'(0)=-a_x\left[1-\theta(0)\right] \tag{12}$$

$$f'(\infty)=1,\theta(\infty)=0 \tag{13}$$

where

$$a_x=\frac{hf}{k}\sqrt{\frac{\upsilon x}{U_\infty}} \tag{14}$$

For the momentum and the energy equations to have a similarity solution, the parameter Gr_x and Bi_x must be constants and not functions of x as in Equation(9) and (14).This condition can be met if the thermal expansion coefficient β is proportional to x^{-1} [8] and the heat transfer coefficient h_f is proportional to $x^{-\frac{1}{2}}$ [7] .We therefore assume

$$h_f=cx^{-\frac{1}{2}}\quad ,\beta=mx^{-1} \tag{15}$$

where c and m are constants.when substituted Equation(15) into Equation(14),we obtained

$$Gr_x=\frac{\upsilon mg(T_f-T_\infty)}{U_\infty^2}\quad ,a_x=\frac{c}{k}\sqrt{\frac{\upsilon}{U_\infty}} \tag{16}$$

With a_x and Gr by Equation (16), the solutions of Equations (10)–(13)yield the similarity solutions. However, the solutions generated arethe local similarity solutions whenever a_x and Gr_x are defined as in Equation (9)and (14).

3.0 Results And Discussions

The governing partial differential equations are transformed into non-linear ordinary differential equations with appropriate boundary conditions by similarity variable. Finally, the systems of similarity equations with boundary conditions are solved numerically by employing using the shooting iteration technique together with fourth order Runge-kutta integration scheme built in Maple 17. The computations have been carried out for various values of magnetic field strength M,Grashof

number Gr,Prandtl number,heat source parameter and Biot number.The edge of the boundary layer depending on the values of $\eta=10$.

Figure 1(a) shows the numerical results for different values of Magnetic field strength . It is observed that as the magnetic strength increases, the velocity decreases. Furthermore, the velocity boundary layer thickness becomes thinner. This shows that the rate of transport is considerably reduced by the presence of the magnetic field. This is due to the fact that the variation of M leads to the variation of Lorentz force due to the magnetic field and Lorentz force produces more resistance to the transport phenomena. In Figure(b),magnetic field has no effect on the thermal boundary layer thickness.

Figure 2(a) shows the numerical results for different values of Grashof number , an increase in Grashof number ,also increase the velocity and thicken velocity boundary layer .This shows that the rate of transport increases by the presence of Grashof number but it has no effect on thermal boundary layer thickness in Figure2(b).

Figure 3(a) shows that for different values of Prandtl number ,an increase in Prandtl number has no effect on the velocity boundary layer thickness and has no effect on the velocity boundary layer thickness in Figure 3(b).

Figure 4(a) shows the numerical result for different values of heat source parameter,anincreases in heat source parameter, the velocity decreases and the velocity boundary layer thickness becomes thinner. This shows that the rate of transport is reduced by the presence of the heat source parameter and In Figure 4(b) the thermal boundary layer thickness becomes thinner and also decreases the wall temperature.

Figure 5(a) shows the numerical results for different values of Biot number number, increase in the Biot number thickens the thermal boundary layer thickness and also increases the velocity and thickens the thermal boundary layer thickness and also increases the wall temperature in Figure 5(b).

From table 1, we see that the local Skin Friction Coefficient at the surface decreases by increasing of Magnetic field strength but has no effect on the Nusselt Number. The results presented demonstrate quite clearly that M,which is an indicator of the viscosity with temperature, has a substantial effect on the drag and heat transfer characteristics.From Table 2, we see that the local skin friction coefficient at the surface increases byincreasing of the Grashof number but has no local Nusselt number.

Table 3 shows that an increase in Prandtl number has no significant on both local skin friction coefficient at the surface and the local Nusselt number.Table 4 shows that the local Skin Friction Coefficient at the surface decreases by increasing of heat source but the Nusselt Number decreases.

Table 5,show that an increase in Biot number has significant increseases on both local skin friction coefficient at the surface and the local Nusselt number.

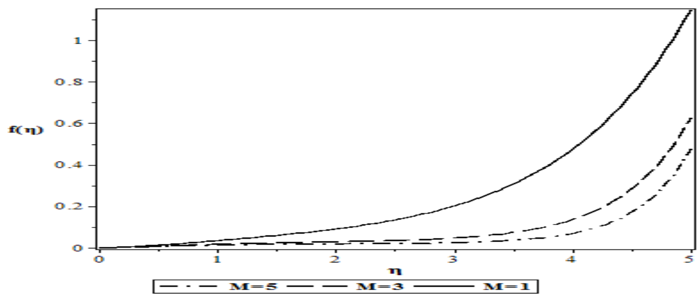

Figure 1(a): Effects of magnetic parameter on the velocity profiles for G = 10,Pr =0.72, $\alpha = 5$, anda=0.1

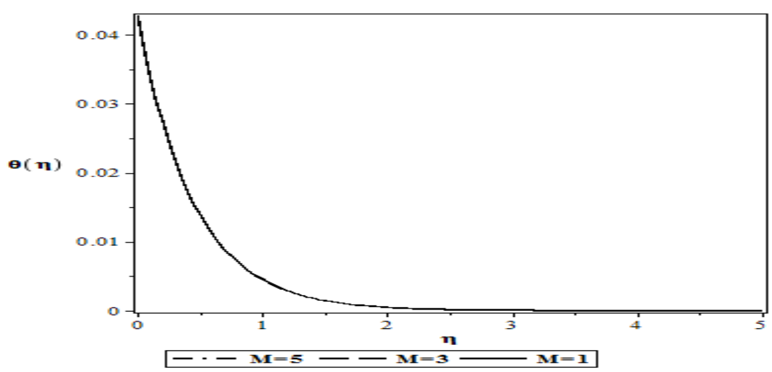

Figure 1(b): Effects of magnetic parameter on the temperature profiles for G = 10,Pr =0.72, $\alpha = 5$, anda=0.1

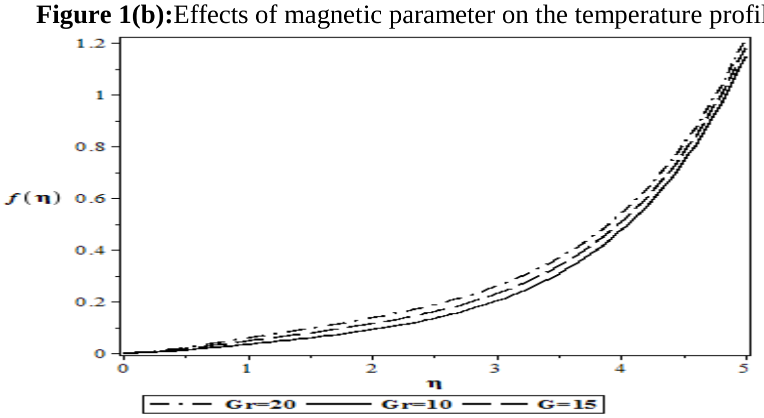

Figure 2(a): Effects of Grashof number on the velocity profiles for M= 1,Pr =0.72, $\alpha = 5$, and a=0.1

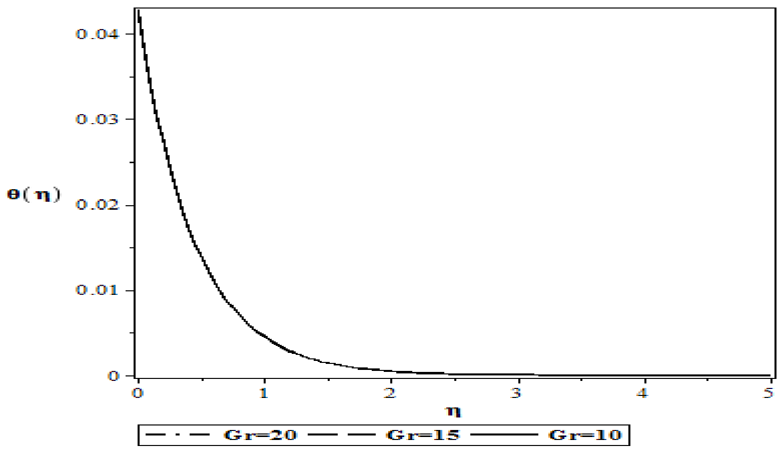

Figure 2(b): Effects of Grashof number on the temperature profiles for M= 1,Pr =0.72, $\alpha = 5$, anda=0.1

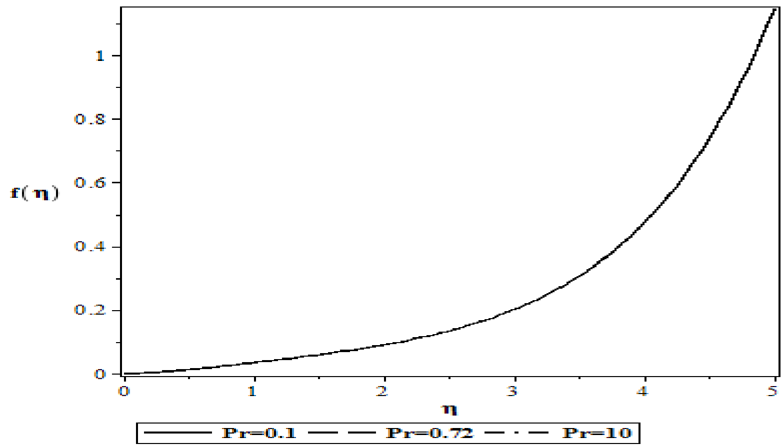

Figure 3(a): Effects of Prandtl number on the velocity profiles for M= 1,Gr=10, $\alpha = 5$, anda=0.1

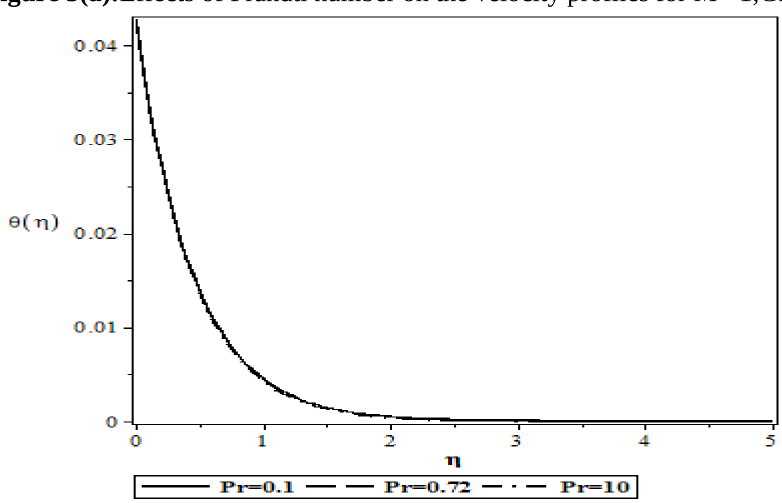

Figure 3(b): Effects of Prandtl number on the velocity profiles for M= 1,Gr=10, $\alpha = 5$, anda=0.1

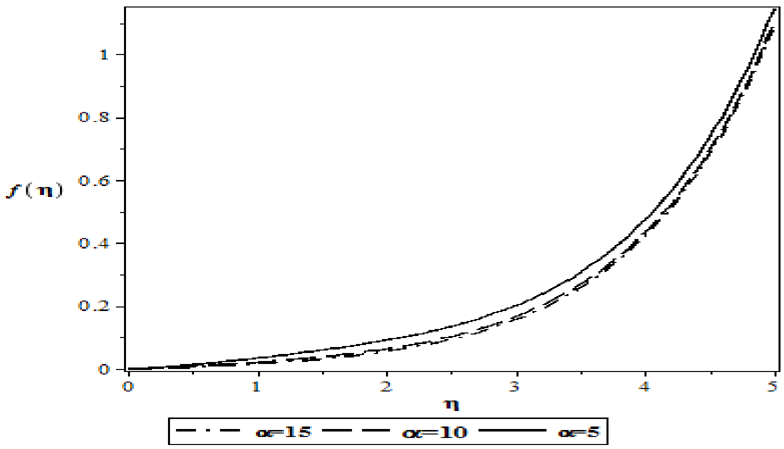

Figure 4(a): Effects of heat source parameter on the velocity profiles for M= 1,Gr=10,Pr=0.72, anda=0.1

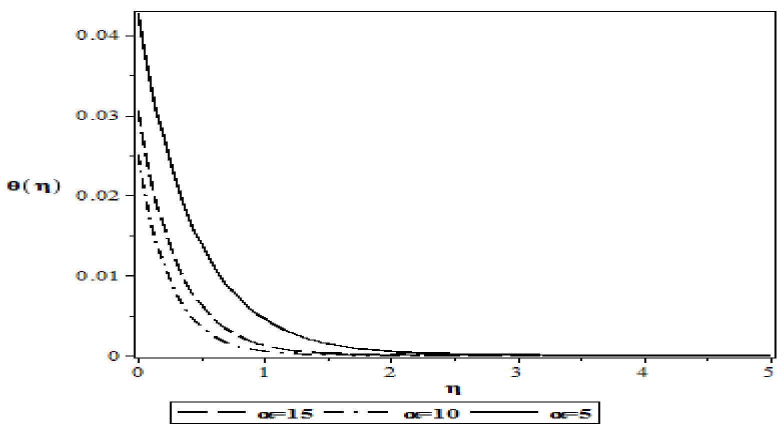

Figure 4(b): Effects of heat source parameter on the velocity profiles for M= 1,Gr=10,Pr=0.72, anda=0.1

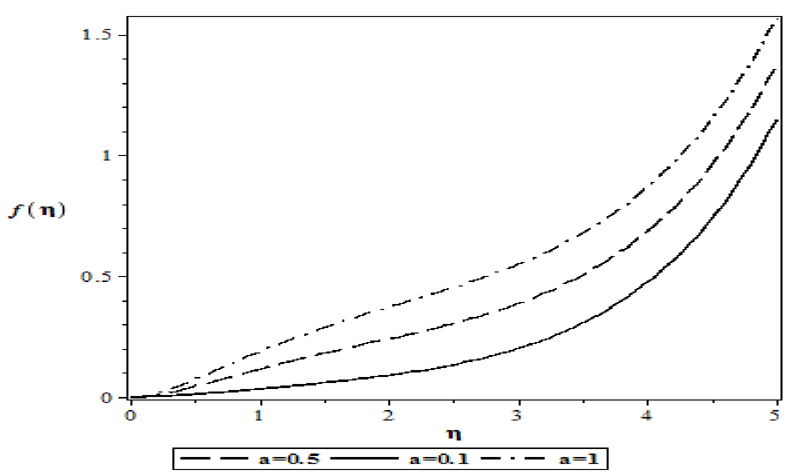

Figure 5(a): Effects of Biot number on the velocity profiles for M= 1,Gr=10,Pr=0.72, and $\alpha = 5$

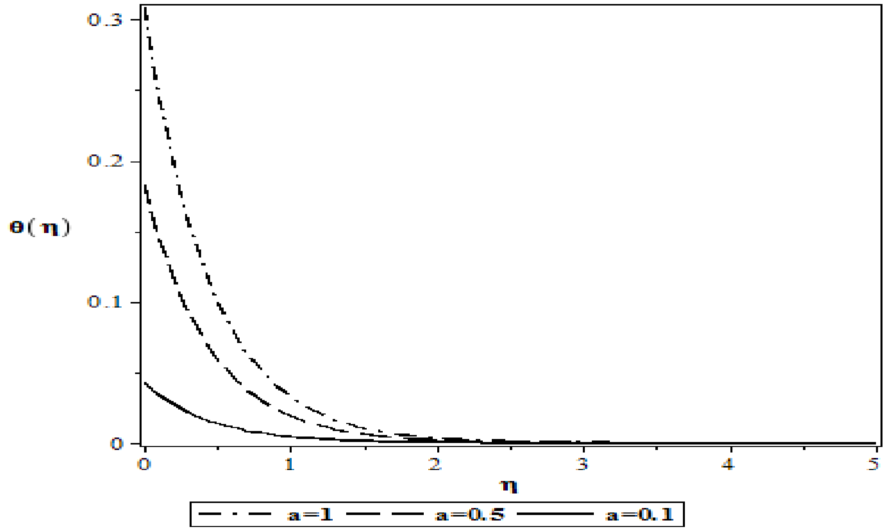

Figure 5(b): Effects of Biot number on the temperature profiles for M= 1,Gr=10,Pr=0.72, and $\alpha = 5$

Table 1: Values of $f''(0)$, $\theta(0)$ and $-\theta'(0)$ for Gr=10, Pr=0.72, $\alpha = 5$ and $a_x = 0.1$

M	$f''(0)$	$\theta(0)$	$-\theta'(0)$
1	0.14866167	0.04279224	0.09572077
2	0.10846297	0.04279831	0.09572016
3	0.09574633	0.04279995	0.09572001

Table 2: Values of $f''(0)$, $\theta(0)$ and $-\theta'(0)$ for M=1, Pr=0.72, $\alpha = 5$ and $a_x = 0.1$

Gr	$f''(0)$	$\theta(0)$	$-\theta'(0)$
10	0.14866169	0.04279922	0.09572077
15	0.21489877	0.04278628	0.09572137
20	0.28104515	0.04278033	0.09572197

Table 3: Values of $f''(0)$, $\theta(0)$ and $-\theta'(0)$ for M=1,Gr=10, $\alpha = 5$ and $a_x = 0.1$

Pr	$f''(0)$	$\theta(0)$	$-\theta'(0)$
0.1	0.14879890	0.04280492	0.09571980
0.72	0.14866169	0.04279224	0.09572077
1	0.14668389	0.04260650	0.09573934

Table 4: Values of $f''(0)$, $\theta(0)$ and $-\theta'(0)$ for M=1, Gr=10,Pr=0.72 and $a_x = 0.1$

α	$f''(0)$	$\theta(0)$	$-\theta'(0)$
5	0.14879890	0.04280492	0.09571950
10	0.08982605	0.03065308	0.09693469
15	0.06772539	0.02516987	0.09748301

Table 5: Values of $f''(0)$, $\theta(0)$ and $-\theta'(0)$ for M=1, Pr=0.72 and $\alpha = 5$

a_x	$f''(0)$	$\theta(0)$	$-\theta'(0)$
0.1	0.14879890	0.04280492	0.09571951
0.5	0.58211904	0.18271679	0.40864416
1	0.92171882	0.30895282	0.69104717

4. 0 Conclusion

Convective heat transfer and heat source in MHD flow over a flate plate are studied. It is observed that the magnetic field strength increases, the velocity decreases and velocity boundary becomes thinner . An increase in Grashof number , increases the velocity and thicken velocity boundary layer.Both have no effect on thermal boundary layer thickness.However,Prandtl number has no effect on the velocity boundary and thermal boundary layer thickness. The heat source parameter increases, the velocity decreases and the velocity boundary thickness becomes thinner and thermal boundary layer thickness becomes thinner and also decreases the wall temperature. Biot number increases, thickens the thermal boundary layer thickness , increases the velocity, thickens the thermal boundary layer thickness and also increases the wall temperature.Moreso, This result qualitatively agrees with the expectations, since an increment in Biot number exerts accelerating force on the flow.

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