Journal of the Nigerian Association of Mathematical Physics
Volume **24 (July, 2013), pp 433 – 440**

A Theoretical Investigation of Specific Heat and Gap Equations in A Two-Band Model.

[1]Iyorzor Ben. E., [1]Babalola M. I., [2]Okhiria J. I., [3]Enukpere E. E., [1]Idiodi J. O. A.

[1]Department of Physics,
University of Benin, Benin City.
[2]College of Agriculture, Iguoriakhi, Benin City.
[3]College of Education, Warri.

Abstract

We consider a model which takes into consideration electron-electron repulsion, formulated in the Hubbard model along with the electron – electron attraction due to electron – phonon interaction in the BCS formulation. A two – band Hamiltonian model was used to derive the superconducting gap equations. The specific heat jump Δ_C at the critical temperature T_c was obtained directly from the BCS gap equation. The specific heat versus temperature curve has been found to have almost similar features to that in the work of Kishore and Llamba, and that of Eliashberg theory of superconductivity.

1.0 Introduction

Specific heat had traditionally been regarded as a critical test tool in the development of the theory of superconductivity and in the quest for new superconducting materials. The magnitude of the specific heat jump at the transition temperature Tc, and the exponentially vanishing specific heat at low temperature, unveiling the existence of a gap in the spectrum of electronic excitations, contributed to establish the validity of the BCS theory [1, 2]. It also informs us about the nature of the phase transition and the symmetry of the pairing state [3,4]. The specific heat is specially suitable to study the BCS superconductors [3]. For example all the parameters in the BCS formula $T_c = 1.14\,\theta_D^{-1/\gamma}$ can in principle be determined by a single specific heat measurement. It gives the critical temperature, T_c, from the position of the jump, the Debye temperature, θ_D, from the slope of the specific heat versus T^3 in the limit $T_c \to 0$, and γ from the ratio of high and low temperature values of the sommerfeld constant.

2.0 BCS background

The BCS theory and its subsequent refinements based on the Eliashberg equations shows that high critical temperatures in phonon – mediated superconductors are favoured by high phonon frequencies, and by a large density of states at the Fermi level [1]. The Allen – Dynes formula [5], an interpolation based on numerical solutions of the Eliashberg equations valid over a wide range of coupling strengths and for various shapes of the phonon spectrum, provides us with the formula

$$T_c = \frac{\langle W_{\ln} \rangle}{1.20} \exp\left[-\frac{1.04\left(1+\lambda_{ep}\right)}{\lambda_{ep} - \mu^* - 0.62\lambda_{ep}\ \mu^*} \right] \tag{2.1}$$

where

$$\lambda_{ep} \equiv 2\int_o^\infty \alpha^2 F(W)\, d\ \ln W \ = \ \frac{N(0)\langle I^2 \rangle}{m\ \overline{w}_2^2} \tag{2.2}$$

Corresponding author: ***Iyorzor Ben. E.,*** E-mail: beniyorzor@yahoo.com, Tel.: +2347035400919

$$\overline{w}_2 \equiv \left[\frac{2}{\lambda_{ep}} \int_o^\infty w^2 \alpha^2 F(w) d \ln w \right]^{1/2} \tag{2.3}$$

$$W_{\ln} \equiv \exp\left(\frac{2}{\lambda_{ep}} \int_o^\infty \ln w \alpha^2 F(w) d \ln w \right) \tag{2.4}$$

Where λ_{ep} is the dimensionless electron – phonon coupling, $\mu^* \cong 0.1$ is the retarded coulomb repulsion, $F(W)$ is the normalized phonon density of states (PDOs), $\alpha^2 F(w)$ is the spectral electron – phonon interaction function, $N(0)$ is the density of states at the Fermi level per spin and per atom (EDOs), $\langle I^2 \rangle$ is the properly averaged electron-ion matrix element squared, and m is the average atomic mass. $N(0)\langle I^2 \rangle \equiv \eta$ is the so-called Hop field electronic parameter, whereas, $M\overline{w}_2^2$ is an average force constant.

The sommerfeld constant $\gamma = (\lim_{T\to 0} C/T)$ is another important parameter given by specific heat. $C_n(T)$, the normal – state specific heat, can be measured using a field larger than the upper critical field $H_{c^2}(0)$ (if available). Furthermore, γ is proportional to the EDOs renormalized by the electron – phonon interaction:

$$\gamma = \frac{2}{3} \Pi^2 K_B^2 \; N(0)\left(1+\lambda_{ep}\right) \tag{2.5}$$

where K_B is the Boltzman constant and N(0) is the EDOs for one spin direction. Expressing $N(0)$ in states per eV per atom and per spin direction, and γ in $mJ/K^2 gat$ (gat ≡ gram – atom), equation (2.5) becomes

$$N(0)\left(1+\lambda_{ep}\right) = 0.212\gamma . \tag{2.6}$$

The lattice specific heat provides information on the phonons. The slope of the specific heat, $\beta_3 = \lim_{T\to 0} d(C/T)/d(T^2)$, gives the initial Debye temperature

$$\theta_D = \left(\frac{12 R \pi^4}{5\beta_3} \right)^{1/3}$$

which depends on the sound velocities; R is the ideal gas constant.

3.0 Theoretical framework of the specific heat

Here we consider a model which in addition to having electron – phonon induced attractive interaction between electrons, also takes into account a repulsive coulomb interaction, formulated in the Hubbard model. This model has recently been considered by Hocquet et al [6] to study the critical temperature and the isotope effect. Within the Bogoliubov – Valatin [7] approximation for the above model, one obtains the BCS gap equation for one band as:

$$\Delta_k = -\frac{1}{2} \sum_{k'} \frac{\left(V_{kk'} + \frac{u}{N}\right)\Delta_{k'}}{\sqrt{E_{k'}^2 + \Delta_k^2}} \tanh\left(\frac{\sqrt{E_{k'}^2 + \Delta_k^2}}{2KT} \right) \tag{3.1}$$

Considering the energy gap equation for the two-band model, equation (3.1) becomes

$$\begin{aligned} \Delta_{1k}(T) = &- \sum_{k'} \frac{(V_{11kk'} + U)\Delta_{1k'}}{\sqrt[2]{\in_{1k'}^2 + \Delta_{1k'}^2}} \tanh\left(\frac{\sqrt{\in_{1k'}^2 + \Delta_{1k}^2}}{2T} \right) \\ &- \sum_{k'} \frac{(V_{12kk'} + U)\Delta_{2k'}}{\sqrt[2]{\in_{2k'}^2 + \Delta_{2k'}^2}} \tanh\left(\frac{\sqrt{\in_{2k'}^2 + \Delta_{2k'}^2}}{2T} \right) \end{aligned} \tag{3.2}$$

$$\Delta_{2k}(T) = -\sum_{k'} \frac{(V_{22kk'} + U)\Delta_{2k'}}{\sqrt[2]{\in_{2k'}^2 + \Delta_{2k'}^2}} \tanh\left(\frac{\sqrt{\in_{2k'}^2 + \Delta_{2k'}^2}}{2T}\right)$$

$$- \sum_{k'} \frac{(V_{12kk'} + U)\Delta_{1k'}}{\sqrt[2]{\in_{1k'}^2 + \Delta_{1k'}^2}} \tanh\left(\frac{\sqrt{\in_{1k'}^2 + \Delta_{1k'}^2}}{2T}\right) \qquad (3.3)$$

where $V_{12kk'}$ is scattering potential of a pair with both mates in band 1 into band 2 and vice versa.

In (3.1), $E_k = \in_k + Un/2 - \in_f$, is the Hartree Fock one particle energy, U is the repulsive intra-atomic interaction of the Hubbard model, N is the total number of sites, n is the average number of electrons per site, $\in_k$ is the bare particle energy in the band, and $\in_f$ is the Fermi energy. Following BCS, the scattering matrix element $V_{kk'}$ due to phonon mediated interaction is assumed to have a non vanishing value $-V/N$ with $V > 0$ only if both $|E_k|$ *and* $|E_{k'}|$ are smaller than the Debye energy $\hbar w_D$.

The specific heat jump, ΔC, at the critical temperature T_c is related to the temperature derivative of the square of the gap parameter by the expression [8],

$$\Delta C = -\sum_k \left(-\frac{\partial f_k}{\partial \in_k}\right) \left[\frac{d\Delta_k^2}{dT}\right]_{T=T_c} \qquad (3.4)$$

The electronic specific heat C_e of a superconductor is calculated using the following expression

$$C_e = \frac{d}{dT}\int dk \in_k \; f_k$$

where $\in_k = \sqrt{(\in - \in_f)^2 + \Delta(k)^2}$ is the quasi – particle energy spectrum near the Fermi energy $\in_f$,

$f_k = \dfrac{1}{1 + \exp\left(\dfrac{E}{K_\beta T}\right)}$ is the Fermi – Dirac distribution function and $\beta = \dfrac{1}{KT}$.

From the simple BCS form of the scattering matrix element, (see equation (3.1)), the solutions of the BCS gap equation has the following structure

$$\left.\begin{aligned} \Delta_k &= \Delta_1 \; if \; |E_k| < \hbar w_D \\ &= \Delta_2 \; if \; |E_k| > \hbar w_D \end{aligned}\right\} \qquad (3.5)$$

Substituting equation (3.3) into equations (3.1) and (3.2), one obtains the equations for $\Delta C,\ \Delta_1$ *and* Δ_2 *as*:

$$\Delta C = \left[\frac{d\Delta_1^2}{dT}\right]_{T=T_c} \langle \eta(\in)\rangle_D + \left[\frac{d\Delta_2^2}{dT}\right]_{T=T_c} \langle \langle \eta(\in)\rangle_w - \langle \eta(\in)\rangle_D \rangle \qquad (3.6)$$

$$\left.\begin{aligned} \Delta_1 &= (V - U)\Delta_1 F_1^D - U\Delta_2\left(F_2^W - F_2^D\right), \\ \Delta_2 &= -U\,\Delta_1 F_1^D - U\Delta_2\left(F_2^W - F_2^D\right) \end{aligned}\right\} \qquad (3.7)$$

where

$$\left.\begin{aligned} F_{\alpha}^{D} &= \int_{-\hbar W_D}^{\hbar W_D} d\in \frac{\eta(\in+\in_f)\tanh\left(\dfrac{\sqrt{\in^2+\Delta_\alpha^2}}{2kT}\right)}{\sqrt[2]{\in^2+\Delta_\alpha^2}} \\ F_{2}^{W} &= \int_{-W-\in_f}^{w-\in_f} d\in \frac{\eta(\in+\in_f)\tanh\left(\dfrac{\sqrt{\in^2+\Delta_2^2}}{2kT}\right)}{\sqrt[2]{\in^2+\Delta_2^2}} \end{aligned}\right\} \quad (3.8)$$

$\eta(\in)$ is the electronic density of states (DOS) per spin, $\alpha = 1, 2,$ and $\langle\eta(\in)\rangle_D$ and $\langle\eta(\in)\rangle_W$ are the thermally averaged DOS, given as

$$\left.\begin{aligned} \langle\eta(\in)\rangle_D &= \int_{-\hbar W_D+\in_f}^{\hbar W_D+\in_f} d\in\eta(\in)\left(-\frac{\partial f}{\partial\in}\right), \\ \langle\eta(\in)\rangle_W &= \int_{-W}^{W} d\in\eta(\in)\left(-\frac{\partial f}{\partial\in}\right) \end{aligned}\right\} \quad (3.9)$$

We make the assumption that each pairing interaction potential consisted of two parts: an attractive electron – phonon interaction V_p and an attractive non electron – phonon interaction (i.e. coulomb electronic part) U_c [9]. With this assumption, the interaction potential $V_{kk'}$ can be written as:

$$V_{kk'} = \begin{Bmatrix} -V_p + V_c & , & |\in_k - \in_{k'}| < \hbar w_D \\ U_c & , & |\in_k - \in_{k'}| < \hbar w_C \end{Bmatrix} \quad (3.10)$$

where w_D *and* w_C are the characteristics energy cut off of the Debye phonon and non – phonon respectively.

In the bulk limit, the sums over K' in equations (3.2) and (3.3) can be converted into energy integrals and also applying the condition in equation (3.10) together with the introduction of energy density of states $N(0)$, under a weak coupling $(\lambda = N(0)V << 1)$ approximation (using $K_\beta = \hbar = 1$) we get

$$\begin{aligned} \Delta_{1k} = &- N(0)\int(V_{1kk'})\frac{\Delta_{1k'}}{2\in_{1k'}}\tanh\left[\frac{\beta\in_{1k''}(T)}{2}\right]d\in_{1k'} \\ &- N(0)\int(V_{12kk'})\frac{\Delta_{2k'}}{2\in_{2k'}}\tanh\left[\frac{\beta\in_{2k''}(T)}{2}\right]d\in_{2k'} \end{aligned} \quad (3.11)$$

$$\begin{aligned} \Delta_{2k} = &- N(0)\int(V_{2kk'})\frac{\Delta_{2k'}}{2\in_{2k'}}\tanh\left[\frac{\beta\in_{2k''}(T)}{2}\right]d\in_{2k'} \\ &- N(0)\int(V_{12kk'})\frac{\Delta_{1k'}}{2\in_{1k'}}\tanh\left[\frac{\beta\in_{1k''}(T)}{2}\right]d\in_{1k'} \end{aligned} \quad (3.12)$$

Applying the condition in equation (3.10), the interaction potential $(V_{kk'})$ and the superconducting gap in equation (3.11) could further be written for the phonon part, Δ_1, and the coulomb electronic part, Δ_1', respectively as

$$\Delta_1 = (V_1 - U_1)\Delta_1 F_1^D - U_1\Delta_1'\left(F_2^W - F_2^D\right) + (V_{12} - U_{12})\Delta_2 F_2^D - U_{12}\Delta_2'\left(F_2^W - F_2^D\right) \quad (3.13)$$

and

$$\Delta_1' = -U_1\Delta_1 F_1^D - U_1\Delta_1'\left(F_2^W - F_2^D\right) - U_{12}\Delta_2 F_2^D - U_{12}\Delta_2'\left(F_2^W - F_2^D\right) \quad (3.14)$$

where

$$F_\alpha^D = \int_{-\hbar w_D}^{\hbar w_D} d\in \frac{\eta\left(\in + \in_f\right)\tanh\left(\frac{\sqrt{\in^2 + \Delta_\alpha^2}}{2KT}\right)}{2\sqrt{\in^2 + \Delta_\alpha^2}} \text{ where } \alpha = 1, 2,$$

Similarly, the phonon part, Δ_2, and the coulomb electronic part, Δ_2', of equation (3.12) could be written respectively as

$$\Delta_2 = \left(V_2 - U_2\right)\Delta_2 F_2^D - U_2\Delta_2'\left(F_2^W - F_2^D\right) + \left(V_{12} - U_{12}\right)\Delta_1 F_1^D - U_{12}\,\Delta_1'\left(F_2^W - F_2^D\right) \qquad (3.15)$$

and

$$\Delta_2' = -U_2\Delta_2 F_2^D - U_2\Delta_2'\left(F_2^W - F_2^D\right) - U_{12}\Delta_1 F_1^D - U_{12}\,\Delta_1'\left(F_2^W - F_2^D\right) \qquad (3.16)$$

Substituting equation (3.14) from equation (3.13) we obtain:

$$\Delta_1 - \Delta_1' = V_1\Delta_1 F_1^D + V_{12}\Delta_2 F_2^D,$$

further simplification gives

$$\Delta_1' = \left(1 - V_1F_1^D\right)\Delta_1 - V_{12}\Delta_2 F_2^D \qquad (3.17)$$

Similarly, solving eqns. (3.15) and (3.16) in the same way, yields

$$\Delta_2' = \left(1 - V_2F_2^D\right)\Delta_2 - V_{12}\Delta_1 F_1^D \qquad (3.18)$$

Adding equations (3.17) and (3.18) we obtain eqn. (3.19) and also from equations (3.14) and (3.16) we obtain eqn. (3.20)

$$\Delta_1' + \Delta_2' = \left(\left(1 - V_1F_1^D\right)\Delta_1 - V_{12}\Delta_2F_2^D\right) + \left(\left(1 - V_2F_2^D\right)\Delta_2 - V_{12}\Delta_1F_1^D\right) \qquad (3.19)$$

and

$$\Delta_1' + \Delta_2' = \left[-U_1\Delta_1F_1^D - U_1\Delta_1'\left(F_2^W - F_2^D\right) - U_{12}\Delta_2F_2^D - U_{12}\Delta_2'\left(F_2^W - F_2^D\right)\right]$$
$$+\left[-U_2\Delta_2F_2^D - U_2\Delta_2'\left(F_2^W - F_2^D\right) - U_{12}\Delta_1F_1^D - U_{12}\Delta_1'\left(F_2^W - F_2^D\right)\right] \qquad (3.20)$$

Rearranging eqn. (3.20) we have:

$$\Delta_1' + \Delta_2' + U_1\Delta_1'\left(F_2^W - F_2^D\right) + U_{12}\Delta_2'\left(F_2^W - F_2^D\right) + U_2\Delta_2'\left(F_2^W - F_2^D\right)$$
$$+ U_{12}\Delta_1'\left(F_2^W - F_2^D\right) = -U_1\Delta_1F_1^D - U_{12}\Delta_2F_2^D - U_2\Delta_2F_2^D - U_{12}\Delta_1F_1^D \qquad (3.21)$$

Assuming that the two bands Δ_1 *and* Δ_2 are identical (degenerate case),

$$\Delta_1 = \Delta_2 = \Delta,\;\; \Delta_1' = \Delta_2' = \Delta'$$
$$U_1 = U_2 = U,\;\; F_1^D = F_2^D = F^D$$
$$\therefore\;\; \Delta_1 + \Delta_2 = 2\Delta\,,\quad \Delta_1' + \Delta_2' = 2\Delta', \text{ and}$$
$$U_1 + U_2 = 2U,\;\; F_1^D = F_2^D = 2F^D \;\; [10]$$

Applying these assumptions in equation (3.21), we get

$$2\Delta'\left[1 + U\left(F_2^W - F_2^D\right) + U_{12}\left(F_2^W - F_2^D\right)\right] = -\,2\Delta UF^D - 2\Delta U_{12}F^D \qquad (3.22)$$

Also allowing these assumptions to hold for the intraband interaction i.e. $V_1 = V_2 = V$, then equation (3.19) reduces to

$$2\Delta' = 2\Delta\left(1 - VF^D - V_{12}F^D\right) \qquad (3.23)$$

Substituting equation (3.23) into (3.22) we have

$$1 = \left(V - \frac{U}{1 + U\left(F_2^W - F_2^D\right) + U_{12}\left(F_2^W - F_2^D\right)} - \frac{U_{12}}{1 + U\left(F_2^W + F_2^D\right) + U_{12}\left(F_2^W - F_2^D\right)} + V_{12}\right)F^D$$

Considering the limits $U_{12} \to 0$ *and* $V_{12} \to 0$, we have

$$1 = \left(V - \frac{U}{1 + U\left(F_2^W - F_2^D\right)}\right)F^D \qquad (3.24)$$

also equations (3.17) and (3.18) reduce to

$$\Delta_1' = \left(1 - V_1F_1^D\right)\Delta_1 \text{ and } \Delta_2' = \left(1 - V_2F_2^D\right)\Delta_2 \qquad (3.25)$$

For a single band model, equations (3.24) and (3.25) reduce to

$$1 = \left(V - \frac{U}{1 + U\left(F_2^W - F_2^D\right)} \right) F_1^D \tag{3.26}$$

and

$$\Delta_2 = \left(1 - VF_1^D\right)\Delta_1 \tag{3.27}$$

We shall use equations (3.26) and (3.27) to calculate the temperature derivative of the square of the gap parameters Δ_1 *and* Δ_2 at the critical temperature T_c in order to obtain the specific heat jump Δ_C. Differentiating equation (3.27) with respect to T and taking the limit $\Delta_1 \rightarrow 0$ as $T \rightarrow T_c$ we get

$$\left[\frac{d\Delta_2^2}{dT}\right]_{T=T_c} = \left(1 - VF_D\right)^2 \left[\frac{d\Delta_1^2}{dT}\right]_{T=T_c}, \tag{3.28}$$

Similarly differentiating equation (3.26) with respect to T and taking the limit $\Delta_1, \Delta_2 \rightarrow 0$ as $T \rightarrow T_c$ we get

$$F_D\, U*^2 \left(G_W - G_D\right)\left[\frac{d\Delta_2^2}{dT}\right]_{T=T_c} + \left(V - U*\right)\frac{dF_D}{dT_C}$$

$$+ \left(V - U*\right)G_D \left[\frac{d\Delta_1^2}{dT}\right]_{T=T_c} + F_D\, U*^2\, \frac{\partial\left(F_W - F_D\right)}{\partial T_C} = 0 \tag{3.29}$$

where

$$F_D = \int_{-\hbar W_D/2kT_C}^{\hbar W_D/2kT_C} d\in \frac{\eta\left(2kT_C \in + \in_f\right)\tanh\left(\in\right)}{2\in},$$

$$F_W = \int_{\left(-W - \in_f\right)/2kT_C}^{\left(W - \in_f\right)2kT_C} d\in \frac{\eta\left(2kT_C \in + \in_f\right)\tanh\left(\in\right)}{2\in},$$

$$G_D = \int_{-\hbar W_D/2kT_C}^{\hbar W_D/2kT_C} d\in \frac{\eta\left(2kT_C \in + \in_f\right)}{\left(4kT_C\right)^2} Q\left(\in\right),$$

$$G_W = \int_{\left(-W - \in_f\right)/2kT_C}^{\left(W - \in_f\right)2kT_C} d\in \frac{\eta\left(2kT_C \in - \in_f\right)}{\left(4kT_C\right)} Q\left(\in\right),$$

$$\mathrm{Q}\left(\in\right) = \frac{\tanh^2 \in}{\in^2} + \frac{\tanh \in - \in}{\in^3} \tag{3.30}$$

and

$$U* = \frac{U}{1 + U\left(F_W - F_D\right)} \tag{3.31}$$

Upon substitution $\left[\frac{d\Delta_2^2}{dT}\right]_{T=T_C}$ from equation (3.28) into equations (3.29) and (3.6) we get

$$\left[\frac{d\Delta_1^2}{dT}\right]_{T=T_C} = -\frac{\left(V^*\right)\frac{\partial F_D}{\partial T_C} + U^{*2}F_D\, \frac{\partial\left(F_W - F_D\right)}{\partial T_C}}{\left(V^*\right)G_D + U^{*2}F_D\xi^2\left(G_W - G_D\right)}, \tag{3.32}$$

and the jump in the specific heat at T_c as

$$\Delta C = -\left[\frac{d\Delta_1^2}{dT}\right]_{T=T_C} \left\{\left\langle\eta\left(\in\right)\right\rangle_D + \xi^2\left(\left\langle\eta\left(\in\right)\right\rangle_W - \left\langle\eta\left(\in\right)\right\rangle_D\right)\right\} \tag{3.33}$$

Here $V^* = V - U^*$ and $\xi = 1 - VF_D$. And eliminating V from equation (3.32) by using the expression for T_c

which has been obtained from equation (3.26) by taking the limits $\Delta_1 \to 0$ and $\Delta_2 \to 0$, we have

$$1 - V^* F_D = 0 \tag{3.34}$$

Substituting the value U^* from equation (3.34) into equation (3.32) we get

$$\left[\frac{d\Delta_1^2}{dT}\right]_{T=T_C} = -\frac{\dfrac{\partial F_D}{\partial T_C} + (U^* F_D)^2 \dfrac{\partial (F_W - F_D)}{\partial T_C}}{G_D + (U^* F_D)^4 (G_W - G_D)} \tag{3.35}$$

For the constant DOS near Fermi energy and the thermally averaged DOS $\langle \eta(\in)\rangle_D = \eta(\in_f)$, the equation (3.30 – 3.35) show that in absence of repulsive interaction U

$$\Delta c = \frac{16\eta(\in_f) K^2 T_C \tanh(\hbar w_D/2kT_C)}{\int_{-\hbar w_D/2kT_C}^{\hbar w_D/2kT_C} Q(\in)\, d\in} \quad [2] \tag{3.36}$$

Multiplying both sides of equation (3.36) by $\frac{1}{\gamma}$ and rearranging it, we have

$$\frac{\Delta c}{\gamma T_C} = \frac{16k^2 \eta(\in_f) \tanh(\hbar w_D/2kT_C)}{\gamma \int_{-\hbar w_D/2kT_C}^{\hbar w_D/2kT_C} Q(\in)\, d\in} \tag{3.37}$$

where γ is given by the coefficient of the normal state specific heat electronic constant $(C_V - \gamma T)$ expressed as

$$\gamma = 2\Pi^2 k^2 \eta(\in_f)/3.$$

Therefore equation (3.37) becomes

$$\frac{\Delta c}{\gamma T_C} = \frac{24 \tanh(\hbar w_D/2KT_C)}{\Pi^2 F_n} \tag{3.38}$$

where $$F_n = \int_{-\hbar w_D/2kT_C}^{\hbar w_D/2kT_C} Q(\in)\, d\in$$

using the Cauchy's residue theorem, in the limit $T_C \to 0$, the equation (3.38) becomes

$$\frac{\Delta c}{T_C} = \frac{12}{\Pi^2 \int_0^\infty Q(\in)\, d\in} = 1.43\gamma \tag{3.39}$$

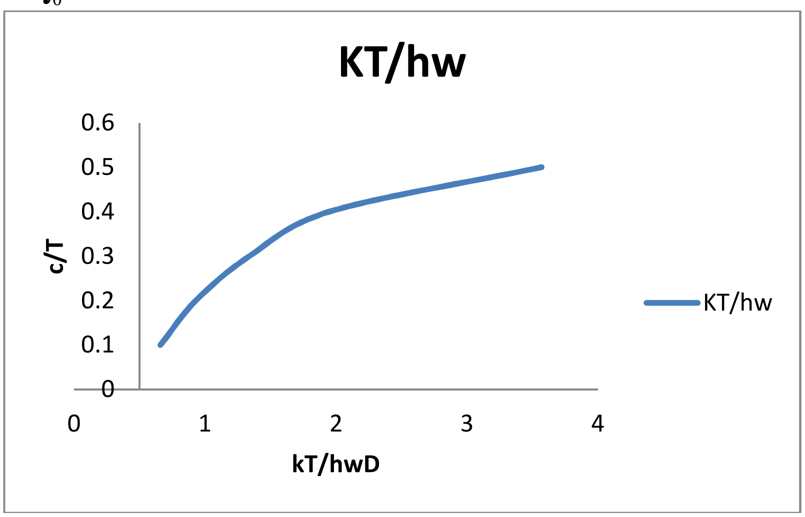

Fig. 1. $\Delta c/T_C$ versus $kT_C/\hbar w_D$ for constant density of states (from equation (3.36))

4.0 Conclusion

Within the BCS framework, the exact analytical expression for the specific heat jump Δc is given by equation (3.33) and 3.35). From our theoretical calculations it was found that the specific heat jump at T_C for a two – band superconductor is given by $\Delta_C / T_C = 1.43\gamma$. This is in agreement with existing results in the literatures [2, 11 and 12]. From Fig. 1, it was found that the behavior of Δ_C / T_C versus $kT_c / \hbar w_D$ for constant density of states was different from those of Kishore et al [2], Marsiglio et al [11] and Carbotte [12].

References

[1] A. Junod, Y. Wang, F. Bouquet, and Toulemonde, in studies of high temperature superconductors, Vol. 38, P. 179; Cond. Mat/0106394, edited by A.V. Narlikar (Nova Science Publishers, New York, 2002).

[2] R. Kishore and S. Llamba, Eur. Phys. J. B8, 161 – 164 (1999).

[3] A. Junod, in Physical Properties of high temperature superconductors II, edited by D.M. Ginsberg (World Scientific, Singapore, 1990), P. 13.

[4] J. F. Anneth, N. Goldfeld, S.R. Renn, in Physical Properties of high temperature superconductors IV, edited by D.M. Ginsberg (World Scientific, Singapore, 1994), P. 571.

[5] P.B. Allen and R.C. Dynes, Phys. Rev. B 12, 905 (1975).

[6] J. Hocquet, J.P. Jardin, P. Germain, J. Labbe, Phys. Rev. B. 52, 10330 (1995.

[7] J. Hocquet, J.P. Jardin, P. Germain, J. Labbe, J. Phys. I Francem5, 517 (1995).

[8] M. Tinkhan Introduction to Superconductivity (McGraw Hill, New York, 1975), p. 36.

[9] P. Udomsamuthirun, R. Peamsuwan, C. Kumvongsa Physica C Vol. 469, 736 (2009).

[10] S. Hong and J. Ihm, Journal of the Korean Physical Society, 24, 90 (1991).

[11] F. Marsiglio, R. Akis, J.P. Carbotte, Phys. Rev. B. 36, 5245 (1987).

[12] J.P. Carbotte, Rev Mod. Phys. 62, 1027 (1990).