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Use of Adomain Decomposition Method to a General Riccati Equation

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Abstract

In this present study, the Adomain decomposition method is applied to the solution of General Riccati Differential Equations. The method is applied to some examples and the results show that the method is reliable, accurate and converges rapidly.

Keywords: Riccati Differential Equations, Adomain Decomposition Method, Lipschitz condition and linear and nonlinear ordinary differential equations

1.0 Introduction

Recently a great deal of interest has been focused on the application of Adomian's decomposition method for the solution of many different problems. For example in [1 - 8] boundary value problems, algebraic equations and partial differential equations are considered. The Adomian decomposition method, which accurately computes the series solution, is of great interest to applied sciences. The method provides the solution in a rapidly convergent series with components that are elegantly computed. The main advantage of the method is that it can be applied directly for all types of differential and integral equations, linear or nonlinear, homogeneous or inhomogeneous, with constant coefficients or with variable coefficients. Another important advantage is that the method is capable of greatly reducing the size of computation work while still maintaining high accuracy of the numerical solution. In this present article, we present the theoretical analysis and practical application of ADM. Furthermore, we present a further insight into the use of (ADM) for solving general Riccati differential equation. We compare the result obtained with the exact or theoretical solutions.

2.0 The Basic Concepts of Adomain Decomposition Method

For our construction, we shall refer general Riccati differential equation of the form

$$\frac{dx}{dt} = T(t)x + U(t)x^2 + V(t), \quad x(0)= W(t) \tag{1}$$

Where T(t), U(t), V(t) and W(t) are scalar functions. To solve we further assume that x(t) is sufficiently differentiable and that the solution of (1) exists and satisfies the Lipschitz condition. ADM usually defines an equation in an operation form by considering the highest- ordered derivative in the problem.

In an operator form, equation (1) can be written as

$$S\, x(t)= T(t)x + U(t)x^2 + V(t), \tag{2}$$

Where the differential operator S is given as

$$S = \frac{d}{dt} \tag{3}$$

The inverse operator S^{-1} is considered a one fold integral operator defined by

$$S^{-1} = \int_0^1 dt \tag{4}$$

If we operate S^{-1} on the right hand side of (2) and use initial condition $x_o(0) = W(t)$, we have

$$X(t) = x_o + S^{-1}\left(U(t)x^2 + T(t)x + V(t)\right) \tag{5}$$

Let $f(t, x) = U(t)x^2 + T(t)x + V(t)$

Then, equation (5) becomes

$$X(t) = x_o + S^{-1} f(t, x) \tag{6}$$

The ADM introduce the x (t) in an infinite as

$$x(t) = \sum_{n=0}^{\infty} X_n(t) \tag{7}$$

Where the components $Xn(t)$ will be determined recursively. Moreover, the method defined the non linear function f(t, x) by the infinite series of the form.

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f(t, x) $=\sum_{n=0}^{\infty} A_n$ (8)

If we put equation (7) and (8) in (6), we obtain

$\sum_{n=0}^{\infty} X_n(t) = \sum_{n=0}^{\infty} A_n$ (9)

The next step is to seek a way to determine the component $X_n(t)$ for which n ≥ 0. We first identify the zero components $X_0(t)$ by all terms that arise from the initial conditions. The remaining component is determined by using the preceding component.

Each term of the series (7) is given by the recurrent relation.

$X_0(t)$ = W (t)

$X_{n+1}(t)$ = S^{-1} (A_n), n ≥ 0 (10)

$X_{n+1}(t)$ = S^{-1} (U (t) x_n^2 +T (t)x_n + V (t))

We must state here that in practice all term of the series in (7) cannot be determined and the solution will be approximated by series of the form.

$\varphi_n(t) = \sum_{n=0}^{n-1} X_n(t)$ (11)

With (11), we obtain series solution for our system (1). The method reduces significantly the massive computation which may arise if discretisation methods are used for the solution of non- linear problems.

Numerical Examples

Example 1

We consider the system

X'(t) = x^2 x(0) = 1

T(t) =0, U(t) =1,V(t) = 1 and W(t) = 1 (12)

With the theoretical solution given as

X(t) = $\frac{1}{1-t}$

We apply ADM operator to equation (12) to produce

Sx = x^2 (13)

Operating S^{-1} on both sides of (13) and use the initial condition we obtain

X (t)=x(0) + S^{-1} (x^2)

X_0 = 1 + t (14)

X_{n+1}=1 + S^{-1} (x_n^2)

X_1 = 1 + $S^{-1}(1 + 2t + t^2)$

X_1= 1+t +t^2+ $\frac{t^3}{3}$

X_2= 1+t +t^2+ t3+$\frac{2t^4}{3}$+$\frac{t^5}{3}$+$\frac{t^6}{9}$ + $\frac{t^7}{63}$

In the same manner, the rest of the component of the iteration formulae (14) can be obtained using the maple package.

Example 2

Consider the following example

X'(t) = -x^2(t) + 1 (x) = 0 (15)

Here T(t) = 0, U(t) = -1,V(t) = 1 and W(t) = 0.

The exact solution is x (t) = $\frac{e^{2t}-1}{e^{2t}-1}$ (16)

To solve equation (15) by means of ADM, (15) becomes $s_x(t)$ = -x^2(t) + 1

X(0) = t (17)

$X_1 = t - \frac{t^3}{3}$

$X_2 = t - \frac{t^3}{3} + \frac{2t^5}{15} - \frac{t^7}{63}$

$X_3 = t - \frac{t^3}{3} + \frac{2t^5}{15} - \frac{t^7}{63} + \frac{38t^9}{2835} + \frac{134t^{11}}{51975} + \frac{4t^{13}}{12285} - \frac{t^{15}}{59535}$

In the same manner, the rest of the component of the iteration formulas (17) can be obtained using the maple package.

Example 3

Consider the following example

X'(t) = t^2+x^2(t) x(0) = 1 (18)

Here T(t)=0, U(t)=1,V(t)=t^2 and W(t)=1

We apply ADM operator to equation (18) to produce

$s_{x\,=}\,t^2$+x (19)

Operating S^{-1} on both sides of (19) and use the initial conditions, we obtain

X (t) = 1+ S^{-1} t^2+ S^{-1} x

Implies

x_0 = 1 ++ S^{-1} t^2+ S^{-1} x

$x_0 = 1 + \frac{t^3}{3}$

$X_{n+1}(t) = 1 + S^{-1}t^2 + S^{-1}x_n$ (20)

Then;

$X_1(t) = 1 + S^{-1}t^2 + S^{-1}x_0$

$X_1(t) = 1 + t + \frac{t^3}{3} + \frac{t^4}{6} + \frac{t^7}{63}$

$X_1(t) = 1 + t + t^2 + \frac{4t^3}{3} + \frac{7t^5}{15} + \frac{t^6}{18} + \frac{t^7}{7} + \frac{23t^8}{504} + \frac{5t^9}{756} + \frac{2t^{11}}{693} + \frac{t^{12}}{2268} + \frac{1}{59535}$

The rest of the component of the iteration formula (20) can be obtained using the maple package.

3.0 Numerical Result and Discussion

We now obtain numerical solution of Riccati differential equation. Table 1 shows comparison between the 2- iterate of ADM and exact solution for example 1. Table 2 shows comparison between the 3- iterate of ADM and exact solution for example 2. Table 3 shows comparison between the 2- iterate of ADM Tayor matrix, Runge Kutta, Picard and Euler for example 3

Table 1, comparison between the 2- iterate of ADM and exact solution for Example 1

T	Exact solution	2-iterate ADM	Error
0.0	1.000000000	1.000000000	0.000000000
0.1	1.111111164	1.111111164	0.000000000
0.2	1.250000000	1.249999881	0.000000119
0.3	1.428571463	1.428571224	0.000000238
0.4	1.666666627	1.666666746	0.000000119
0.5	2.000000000	2.000000000	0.000000000
0.6	2.500000238	2.500000000	0.000000238
0.7	3.333333969	3.333333969	0.000000000
0.8	5.000001907	5.000000000	0.000001907
0.9	10.000009537	10.000004768	0.000004768

Table 2, comparison between the 3- iterate of ADM and exact solution for Example 2

T	Exact solution	3-iterate ADM	Error
0.1	0.0996679946	0.0996679946	0.0000000005
0.2	0.1973753203	0.1973753160	0.0000000043
0.3	0.2913126124	0.2913124564	0.000000156
0.4	0.3799489622	0.3799469862	0.000001976
0.5	0.4621171572	0.4641033328	0.00001382
0.6	0.5370495679	0.5369833784	0.00005619
0.7	0.6043677771	0.6041244734	0.0002433
0.8	0.6640367702	0.6633009219	0.0007358
0.9	0.7162978702	0.7143823394	0.001916
1.0	0.7615941560	0.7571662670	0.00428

Table 3, comparison between the 2- iterate of ADM Tayor matrix, Runge Kutta, Picard and Euler for Example 3

Method	t=0.5	t=0.90	t=0.95
Taylor matrix	1.989580000	4.44700000	4.945000000
Picard	1.969000000	4.21300000	4.671000000
2-iterate ADM	1.964676823	4.4060704915	4.940949522

4.0 Conclusion

In this present paper, we applied ADM for solving General Riccati Differential Equation. The method is applied in a direct way without using perturbation discretisation, transformation, and linearization. It may be concluded that ADM is very powerful and efficient in finding the analytical solutions for a wide class of linear and non- linear differential equation. The method gives more realistic series solution that converges very rapidly in physical problems. It is worth mentioning that the method is capable of reducing the volume of the computational work as compared to the classical methods while still maintaining the high accuracy of the numerical result.

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