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Optimum Water Production Rate from an Aquifer Overlain by a Gas Reservoir

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Abstract

In most parts of the world, where there are hydrocarbon production activities, water availability is often a major problem. Sometimes when the water is available, it is contaminated with hydrocarbons, thus limiting its usage.

However, to produce clean water from an aquifer overlain by a gas cap, using a horizontal well, water production rates have to be monitored to avoid gas breakthrough into the wellbore. In this paper, an aquifer is modeled as having an active gas cap. If it is intended to produce water from the same aquifer without gas contamination, an optimum water production rate from the typical aquifer is derived.

The derivation involves the use of Green's and source functions for deriving a general pressure distribution expression. The pressure distribution expression is then solved to obtain a critical water production rate beyond which there will be contaminated water production and below which there will be clean water production. Measures are also suggested for achieving an enhanced critical water production rate to meet demand.

Keywords: Aquifer-Gas Formation, Horizontal Well, Optimum Water Production Rate.

1.0 Introduction

In Nigeria today, water availability for both domestic and industrial uses is a major problem. As one of the world's leading oil and gas producers, Nigeria water supply

problem has been most protested in the oil producing areas. This is one of the main reasons for the formation of environmental consciousness groups[1-2] now rising in number in these oil producing areas.

In almost every oil and gas reservoir, there is an aquifer zone underlying it, formed by at least, the process of the decay of organic matter. This is also true even for formations in arid land. When water underlying a gas zone is produced, the gas expands by virtue of the reduction in its pressure. The rapidity of expansion depends on (1) the composition of the gas (2) water production rate and (3) the vertical permeability of the aquifer. Since water has a much lower compressibility than gas, more volumes of gas become mobile than the corresponding volume of water produced. This leads to encroachment of the wellbore perforations by the gas causing gas breakthrough. If this is allowed to continue, the gas pressure becomes prevalent and the aquifer level must therefore fall due to sharp density difference between water and gas, both of which now exist in one "container". Thus, for this reason, aquifer sources are usually considered as infinite even though the aquifer structure may be bounded. This property of infinity is further attributed to aquifer connectivity to far and larger aquifer sources, like underground rivers, etc, with which a single pressure regime is prevalent. For this reason, most aquifers underlying gas reservoirs may be produced for a life span even longer than that of gas, if they were subjected to the same production schedule.

Should simultaneous or separate production of water and or gas be contemplated, this paper derives the optimum water production (critical) rate that will lead to only water production through a horizontal well. The approach involves a solution to a 3D-diffusivity equation describing flow of water in horizontal wells. Source and Green's functions are selected to represent the boundaries of the aquifer and the gas cap. The aquifer pressure distribution is then derived. Finally, this distribution is utilized in deriving a relationship for the critical water production rate. Factors affecting this critical rate are investigated with a view to providing a possibility for increasing the critical water production rate, even without gas production.

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2.0 Aquifer Model Description

Fig.1 shows an aquifer bounded on top by a gas cap and at the bottom by a no-flow (impermeable) layer. Laterally, the aquifer is assumed to have infinite dimensions. A horizontal well of length L, (along the x-axis), width y_w,, (along the y-axis) and stand-off z_w, (along the z-axis) is drilled to produce water at rate q, high enough but not greater than a critical rate q_{wc}, so that gas will not be produced. The aquifer pressure at equilibrium (before production) was p_i. If the aquifer is isotropic, a relationship is desired between the aquifer, wellbore and water properties and time.

It is further assumed that the water in the aquifer has a small but constant compressibility; i.e., the water is substantially free of gas contamination. Therefore, water flow energy is derived strictly from the aquifer. The pressure distribution relationship to be derived shall assume that water is produced through the wellbore only and under a pressure drop Δp psi.

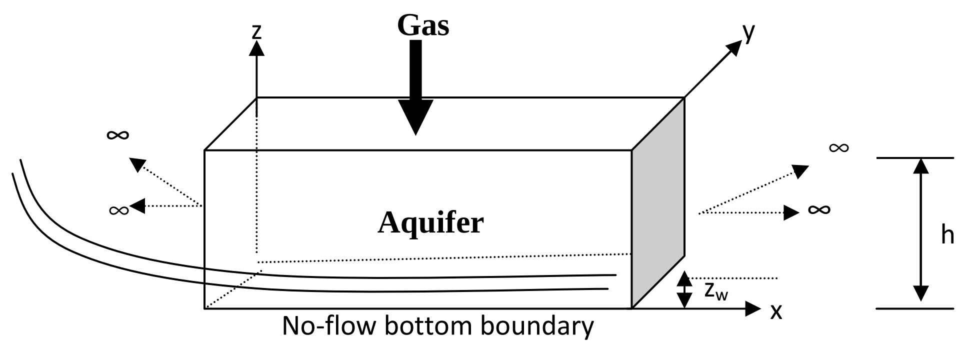

Fig. 1: Aquifer Model showing Gas and Horizontal Well Position**s**

Although, a constant water production rate is considered, if the well experiences successive different rates, the total historical rate can be computed by superposition technique at the wellbore. The effects of wellbore partial penetration in all the directions of the aquifer, wellbore, skin, produced water salinity and temperature are, however, not considered. Gas displacement of the aquifer is considered to unstable.

3.0 Mathematical Model Description

Water flow from the aquifer is governed chiefly by the 3D diffusivity equation[4-5] (analogous to the heat conduction equation[6]) given as follows for anisotropic aquifer:

$$k_x \frac{\partial^2 p}{\partial x^2} + k_y \frac{\partial^2 p}{\partial y^2} + k_z \frac{\partial^2 p}{\partial z^2} = \phi \mu_w c_w \frac{\partial p}{\partial t} \quad (1)$$

The following dimensionless parameters based on the wellbore half-length are introduced:

$$i_D = \frac{2i}{L}\sqrt{\frac{k}{k_i}} \quad (2)$$

where i = x, y, or z.

$$p_D = \frac{kh\Delta p}{141.2 B_w q_w \mu_w} \quad (3)$$

$$h_D = \frac{2h}{L}\sqrt{\frac{k}{k_z}} \quad (4)$$

$$L_D = \frac{L}{2h}\sqrt{\frac{k}{k_x}} \quad (5)$$

$$t_D = \frac{4x2.64x10^{-4} kt}{\phi \mu c_t L^2} \quad (6)$$

Substituting Eq. (2) to (6) into Eq.(1), we have a completely dimensionless diffusivity equation to be solved as follows:

$$\frac{\partial^2 p_D}{\partial x_D^2}+\frac{\partial^2 p_D}{\partial y_D^2}+\frac{\partial^2 p_D}{\partial z_D^2}=\frac{\partial p_D}{\partial t_D}\quad\text{.....}(7)$$

The dimensionless source functions are selected according to References [6] and [7] as follows:

(1) x-axis

Along the x-axis, the wellbore experiences a source strength from an infinite slab in an infinite aquifer occupying the region $-\sqrt{k/k_x} \le x_D \le \sqrt{k/k_x}$, given by integration of the Green's function:

$$G(x_D,t_D)=\frac{1}{2}\int_{-k/k_x}^{k/k_x}\frac{1}{\sqrt{\pi t_D}}\exp(-(x_D-x_D')^2/4t_D dx_D'\quad\text{.....}(8)$$

This gives the source function $s(x_D, t_D)$ as follows:

$$s(x_D,t_D)=\frac{1}{4}\left[erf\,(\frac{\sqrt{k/k_x}+x_D}{2\sqrt{\pi t}_D})+erf\,(\frac{\sqrt{k/k_x}-x_D}{2\sqrt{\pi t}_D})\right]\quad\text{.....}(9)$$

This is predicated on the assumption that the aquifer is infinite laterally and that the well is located in the middle of the x-axis.

(2) y-axis

The well is an infinite plane in an infinite reservoir since the wellbore width is too small compared with the aquifer expanse along this direction. Hence, the appropriate source function is:

$$s(y_D,t_D)=\frac{1}{2\sqrt{\pi t}_D}e^{-(y_D-y_{wD})^2/4t_D}\quad\text{.....}(10)$$

(3) z-axis

On top of the aquifer is a gas bearing zone exerting a constant pressure while at the bottom there is a no-flow boundary, according to Fig. 2. This is a mixed boundary situation with a prescribed pressure (pressure recharge) at the top. Therefore, the source function is written as:

$$s(z_D,t_D)=\frac{2}{h_D}\sum_{n=1}^{\infty}\exp(-\frac{(2n+1)^2\pi^2 kt_D}{4h_D^2k_x})\cos((2n+1)\frac{\pi z_{wD}}{h_D})\cos((2n+1)\frac{\pi z_D}{h_D})\quad\text{.....}(11)$$

It should be noted that $z_D = z_{wD} + r_{wD}$.

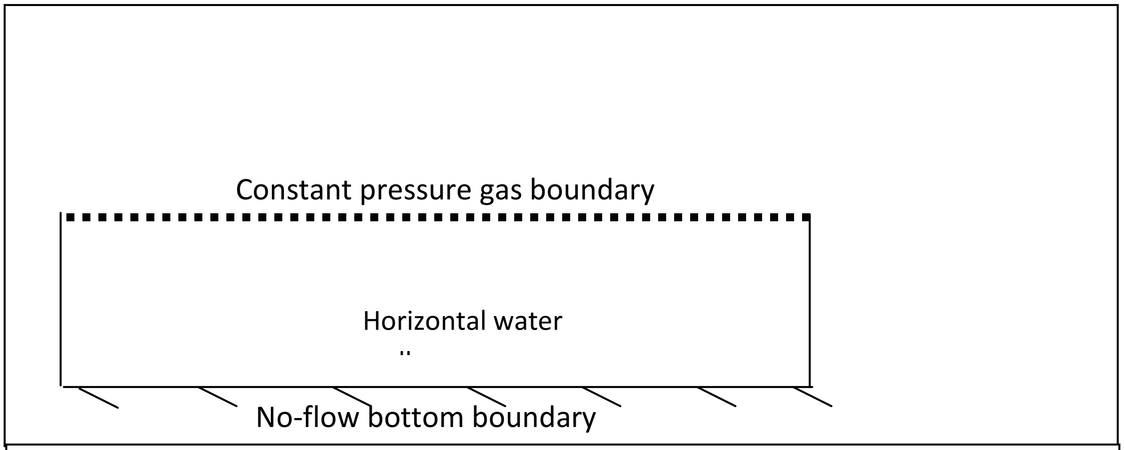

Fig. 2: x-z-plane showing a mixed boundary situation

In this dimensionless form the pressure drop is written as the product of the 3D sources according to Newman's product[3,6,8] method as:

$$p_D(x_D,y_D,z_D,\tau)=2\pi h_D\int_0^{t_D}s(x_D,\tau)\bullet s(y_D,\tau)\bullet s(z_D,\tau)d\tau\quad\text{.....}(12)$$

for a constant production rate.
Substituting Eqs.(9) to (11) into Eq. (12) we have

$$p_D(x_D,y_D,z_D,\tau)=\frac{\sqrt{\pi}}{2}\sqrt{{}^{k}\!/_{k_y}}\int_0^{t_D}\left[erf\,(\frac{\sqrt{{}^{k}\!/_{k_x}}+x_D}{2\sqrt{\tau}})+erf\,(\frac{\sqrt{{}^{k}\!/_{k_x}}-x_D}{2\sqrt{\tau}})\right]\bullet\frac{e^{-\frac{(y_D-y_{wD})^2}{4\tau}}}{\sqrt{\tau}}\bullet \sum_{n=1}^{\infty}\exp(-\frac{(2n+1)^2\pi^2k\tau}{4h_D^2k_x})\cos(\frac{(2n+1)\pi z_{wD}}{h_D})\cos(\frac{(2n+1)\pi z_D}{h_D})d\tau \quad (13)$$

4.0 Computation of Well Responses

Eq. (13) is the required dimensionless pressure distribution when the horizontal well feels the external vertical boundaries of the aquifer. It produces a steady state effect in the wellbore eventually. If the time at which steady state is reached is denoted by t_{Dss} , then the corresponding critical water production rate is q_{wc}, which is the critical water production rate; and the prevailing wellbore flowing pressure will be p_{wfss}. Meanwhile, before t_{Dss} is reached, an important event had taken place: a pressure drop history had been in existence in the wellbore until after the uppermost wellbore boundary was felt. The pressure history was contributed by pressure transients due to different reservoir and well boundaries and is summed up by superposition of all pressure drops occasioned by the boundaries.

Early Radial Flow Dimensionless Pressure Distribution

During the early radial flow period the well behaves as a fully penetrating vertical well of thickness L in an infinite aquifer. The dimensionless pressure during the early radial flow is derived from Eq. 13 as:

$$p_D(x_D,y_D,z_D,t_D)=-\frac{\alpha h_D}{8}Ei\left[-\frac{(y_D-y_{wD})^2+(z_D-z_{wD})^2}{4t_D}\right]\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(14)$$

because during this period all the source functions have infinite origin[8-11] and the effect lasts for all t_D until the external gas boundary acts to stabilize it. The exponential integral, Ei(-x) is read from Table[12]. In Eq.(14), $\alpha=2\sqrt{k/k_x}$ if $x_D<\sqrt{k/k_x}$, 1 if $x_D=\sqrt{k/k_x}$ and 0 if $x_D>\sqrt{k/k_x}$.

Early Linear Flow Dimensionless Pressure Distribution

If the bottom no-flow boundary of the reservoir is felt first before the top boundary, then a linear flow would persist. This period is characterized by equal change in p_D with t_D. the length of existence of this period is determined by vertical permeability, length of well, well standoff and water withdrawal rate. The total dimensionless pressure distribution for this period is written by superposition as:

$$p_D(x_D,y_D,z_D,\tau)=-\frac{\alpha h_D}{8}Ei(-\frac{(y_D-y_{wD})^2+(z_D-z_{wD})^2}{4t_D}+$$
$$\frac{\sqrt{\pi}}{2}\sqrt{{}^{k}\!/_{k_y}}\int_{t_{De}}^{t_D}\left[erf(\frac{\sqrt{{}^{k}\!/_{k_x}}+x_D}{2\sqrt{\tau-t_{De}}})+erf(\frac{\sqrt{{}^{k}\!/_{k_x}}-x_D}{2\sqrt{\tau-t_{De}}})\right]\bullet\frac{e^{-\frac{(y_D-y_{wD})^2}{4\tau-t_{De}}}}{\sqrt{\tau}-t_{De}}\bullet \sum_{n=1}^{\infty}\exp(-\frac{(2n+1)^2\pi^2k\tau-t_{De}}{4h_D^2k_x})\cos(\frac{(2n+1)\pi z_{wD}}{h_D})\cos(\frac{(2n+1)\pi z_D}{h_D})d\tau \quad (15)$$

where

$$t_{De}\geq(\sqrt{{}^{k}\!/_{k_x}}-x_D)^2/8\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(16)$$

Eq. (15) can be solved analytically [8] if the flow periods are known with certainty. Eq. (16) gives an approximation. A plot of p_D against log t_D gives a better value of t_{De}.

We are not interested in the flow beyond t_{Dss}, because at t_{Dss}, the gas is ready to be discharged into the wellbore. Therefore, the actual expression for t_{Dss} must be derived. For this purpose, noting that at steady state, change in pressure with time is zero. That is,

$$\frac{\partial p_D(x_D, y_D, z_D, t_D = t_{Dss})}{\partial t_D} = 0 \qquad (17)$$

Eq. (13) is solved analytically for $t_D = t_{Dss}$. Therefore, applying Eq. (13) in Eq. (16) and solving, retaining only the first term in the summation and the first real value of t_D in the resulting exponential series, we have the minimum dimensionless time for attainment of steady state as:

$$t_{Dss} \geq \frac{4}{\pi^2 L_D^2} \qquad (18)$$

considering n = 0, i.e., wellbore vicinity. Eq. (18) is mere approximation. Much better estimates are obtained from a well test plot of p_D versus log of t_D.

If the well produces water under a dimensionless draw down of $p_{D,}$ the dimensionless critical water production rate $q_{D,}$, is written as follows:

$$q_{Dc} = \frac{1}{p_D} \qquad (19)$$

where

$$q_{Dc} = \frac{141.2 q \mu B}{kh\Delta p} \qquad (20)$$

according to Ref. [13], but using pressure drop instead of density difference. p_D in Eq. (19) may be obtained from Eq. (14) or (15) according to flow time as discussed above.

5.0 Results and Discussion

Dimensionless critical production rates were computed for several dimensionless well lengths L_D (equal to the reciprocal of dimensionless reservoir thickness h_D ,for isotropic reservoir), and radii. In each case, uniform flux, i.e. $x_D = 0.0$, was assumed. Wellbore stand off , $z_{wD} = 0.5$ (central location) and well width $y_D = 0.0$ (line source well). $z_D = z_{wD} + r_{wD}$. The results obtained are shown in **Tables 1** and **2**.

Table 1: Dimensionless Critical Production Rates for $r_{wD} = 2.828\text{x}10^{-3}$

t_D	Dimensionless Lengths				
	0.10	**0.25**	**0.50**	**1.00**	**10.00**
10^{-6}	8.20	20.40	40.80	81.70	818.00
10^{-5}	0.30	0.80	1.60	3.20	32.70
10^{-4}	0.10	0.30	0.60	1.20	11.90
10^{-3}	0.07	0.20	0.30	0.70	7.10
10^{-2}	0.04	0.10	0.20	0.50	4.60
10^{-1}	0.04	0.10	0.10	0.20	4.20
1	0.04	0.10	0.10	0.20	4.20
10	0.04	0.10	0.10	0.20	4.20
100	0.04	0.10	0.10	0.20	4.20
1000	0.04	0.10	0.10	0.20	4.20

Table 2: Dimensionless Critical Production Rates for $r_{wD}=2.828\text{x}10^{-4}$

t_D	Dimensionless Length				
	0.10	**0.25**	**0.50**	**1.00**	**10.00**
10^{-6}	0.11	0.30	0.60	1.20	12.00
10^{-5}	0.07	0.18	0.30	0.70	7.00
10^{-4}	0.04	0.12	0.20	0.50	5.00
10^{-3}	0.03	0.10	0.20	0.40	4.00
10^{-2}	0.30	0.10	0.20	0.30	3.00
10^{-1}	0.30	0.10	0.20	0.30	3.00
1	0.30	0.10	0.20	0.30	3.00
10	0.30	0.10	0.20	0.30	3.00
100	0.30	0.10	0.20	0.30	3.00
1000	0.30	0.10	0.20	0.30	3.00

Effects of Wellbore Pressure Drop, Δp

From Eq. 20, small pressure draw down in the wellbore is conducive for higher daily water production. It is easy to achieve such draw down with small and inexpensive pumps. This is a sand face pressure drop and can be optimized if (1) the wellbore condition is good (2) pump station is close enough to the wellhead (3) the pump is in a sound mechanical state and (4) the pump is correctly selected.

Effects of Wellbore Length L, and width, y_w

For a constant draw down, results in Tables 1 and 2 show that longer wells would produce the effects of large daily water production. This increases the sweep area of the aquifer by the wellbore thus preventing early gas breakthrough. Short wells would cause high capillary effects, which promotes coning of the gas into the wellbore. Narrower wells, i.e., smaller y_{wD} or r_{wD}, can also boost the critical water production rate.

Effects of Water Properties

Water flow properties, such as viscosity and formation volume factor, do not affect the critical water production rate significantly because they are fairly constant for fresh water. Fresh water produced at the surface with that in the wellbore are the same because the water neither expand nor shrink at the surface. This is further due to the little effect of temperature on fresh water. However, highly viscous water will reduce the critical rate and, therefore, the volume of water that can be produced for a particular time. Dissolved solids and high depth of aquifer are the main causes of elevated viscosity. Water compressibility produces almost insignificant effects on the critical rate, because there is no gas dissolved in the fresh water.

Effects of Aquifer Properties

The most important aquifer properties are the directional permeabilities. The critical water production rate increases with increases in the directional permeabilities. Critical production rate varies slightly but inversely with the aquifer thickness. Well stimulation practices, such a acidizing, fracturing and perforating, can boost near wellbore permeabilities significantly.

6.0 Conclusion

In areas where gas production activities are predominant, water production can also go on simultaneously. This study shows that in an aquifer overlain by a gas cap

(1) Long horizontal wells can boost daily water production requirement
(2) Water properties do not significantly affect water production rate
(3) Water production rate can be boosted with a clean wellbore
(4) Aquifer thickness slightly but inversely affects water production rate

Nomenclature

B_w	water formation volume factor, aquifer barrel/surface barrel
h	aquifer thickness, ft
x	distance in the x-direction, ft
y	distance in the y-direction, ft
z	distance in the z-direction, ft
L	well length, ft
z_w	well stand-off from the bottom of the aquifer, ft
k	total permeability, md
p	pressure, psi
t	time, hours
c	water compressibility, 1/psi
μ_w	water viscosity,cp
ϕ	porosity, fraction
r_w	wellbore radius, ft
τ	dummy time variable
q	production rate per unit half length of wellbore
q_{wc}	critical production rate, bbl/D
Δ	drop
G	Green’s function
S	source function

Subscript

D	dimensionless
ss	steady state
w	water
wf	wellbore flowing
e	early

Superscript

‘ arbitrary well position

References

[1] World Bank (1991) Annual Report of World Bank, vol. 1 and 2.

[2] World Bank (1995) Annual Report of World Bank, vol. 1 and 2.

[3] Newman, A.B. (1936): “Heating and Cooling Rectangular and Cylindrical Solids”, Ind. and Eng. Chem. Vol. 28, pp 545.

[4] Hubbert, M.K. (1956): “Darcy’s Law and the Field Equations of the Flow of Underground Fluids”, *Trans., AIME* Vol. 207, pp. 222-239.

[5] Matthews, C.S and Russell, D.G. (1967): *Pressure Buildup and Flow Tests in Wells*, Monograph Series, Society of Petroleum Engineers of AIME, Dallas, Vol. 1.

[6] Carslaw, H.S. and Jaeger, J.C. (1959): *Conduction of Heat in Solids*, Oxford at the Clarendon Press, pp. 33.

[7] Adewole, E.S. Rai, B.M. and Audu, T.O.K. (2006): "Mathematical Models of Pressure Distribution of Selected Reservoir Systems", *Abacus: Journal of the Mathematical Association of Nigeria,* vol. 33, No. 2B, Mathematical Series, p. 462-470.

[8] Adewole, E.S., Rai, B.M and T.O.K Audu (2001): "The Use of Gauss-Legendre Quadrature in Solving Flow Problems in Reservoirs containing Horizontal Wells", *J. of Nigerian Ass. of Mathematical Physics*, Vol. 5 pp. 89-100.

[9] Ozkan, E. and Raghavan, R. (1991): "Performance of Horizontal Wells with and without Gas Cap or Aquifer", *SPEFE, Trans*., AIME, **291**, pp. 86.

[10] Adewole, E.S., Rai, B.M. and Audu, T.O.K. (2003): "Pressure Distribution in a Layered Reservoir with Lateral Wells", *J. Nigerian Ass. of Mathematical Physics*, Vol. 7, pp.135.

[11] Adewole, E.S., Rai, B.M. and Audu, T.O.K.(2003): "Well Test Analysis of Horizontal Well Subject to Simultaneous Gas Cap and Bottom Water Drive Mechanisms using Type Curves", *Nigerian Journal for Engineering Research and Development,* Vol. 2, No. 4, pp. 45.

[12] Abramowitz, Milton and Stegun, Irene, A. (ed.) (1964) *Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables, National Bureau of Standards Applied Mathematics Series-55,* pp. 227-253.

[13] Joshi, S.D. *Horizontal Well Technology (*1991), Pennwell Books, U.S.A, pp. 301.