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Ordering Policies of Delayed Deteriorating items with Unconstrained Retailer's Capital, Linear Trend in Demand and Shortages

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Abstract

The constant demand rate may not always be suitable for inventory items such as milk, meat, vegetable, highly volatile liquid etc. where the age of the inventory item has a negative impact on demand which is consequent to the loss of consumer confidence on the quality of such products and physical loss due to spoilage, depletion and deterioration. In this paper, we developed an inventory ordering policy model for delayed deteriorating items with unconstrained retailer's capital where payment is effected on receipt of the replenishment inventory. The model is constructed on the assumptions of linear trend in demand and shortages. Numerical examples on the application of the model are provided.

Keywords: Inventory, Delayed Deterioration, Trended Demand, Shortages

1.0 Introduction

In general, deterioration of inventory can be defined as damage, spoilage, decay, obsolescence, evaporation, pilferage and so on which lead to the decrease in usefulness or value of the original inventory item. The decrease or depletion of inventory due to deterioration is always a function of the on hand inventory. The deterioration rate in items such as steel, glassware, toys etc. is very much negligible and not worthy of consideration when computing the economic lot size. The deterioration is remarkably high in other inventory items such as blood in blood banks, fish, fruits, petrol, radioactive chemical, drugs and grains.

The constant demand rate may not always be suitable for many inventory items. These include milk, meat, vegetable, highly volatile liquid etc. where the age of these inventory items has a negative impact on demand which is consequent due to the loss of consumer confidence on the quality of such products and physical loss due to spoilage, depletion and deterioration.

The development of inventory replenishment policy model with a linear trend in demand was pioneered by Donaldson [1]. Henery [2] developed a model that considers increasing linear demand pattern. Ritchie [3] developed a simple optimal solution for an EOQ model with linear increasing demand. Cox et al. [4] developed a model on the determination of order quantities with a linear trend in demand. Goyal [5] constructed a model on the determination of economic replenishment intervals for linear trend in demand. Chung and Tsai [6] constructed an algorithm to determine the EOQ for deteriorating items with shortages and a linear trend in demand.

Yang et al. [7] developed a model on the exact solution of inventory replenishment policy for a linear trend in demand. Musa and Sani [8] developed an EOQ model of delayed deteriorating inventory with linear time dependent holding cost.

In this paper, we developed an inventory ordering policy model for delayed deteriorating items with unconstrained retailer's capital where payment is effected on receipt of the replenishment inventory. The model is built on the assumption that items in stock have a delayed deterioration meaning that they do not start deteriorating immediately they are stocked until after some time. The demand rate before deterioration sets in is a linear function of time, whereas when deterioration sets in, the demand rate is constant and it is assumed to remain so up to when the inventory is completely depleted. The model allows for shortages where orders placed during the stock out period are satisfied first when replenishment is received.

2.0 Assumptions and Notation

The following assumptions and notation are employed in the development of the model.

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Assumptions

(i) Unconstrained retailer's capital (ii) Shortages are allowed
(iii) Instantaneous replenishment (iv) Linear trend in demand (v) lead time is zero

Notation

$\lambda_1(t)$ The demand rate (unit per unit time) during the period before deterioration sets in where

$$\lambda_1(t) = \gamma_1 + \gamma_2 t$$

λ_2 The constant demand rate (unit per unit time) after the start of deterioration

T The inventory cycle length

C The unit cost of the item

T_1 The time the deterioration begins

T_2 The length of time with the positive stock of the inventory

T_3 The period during which deterioration occurs

A The ordering cost per order

i The inventory carrying charge

θ The rate of deterioration

$D(T_2)$ The number of items that deteriorate during the time $[T_1, T_2]$

H_C The inventory holding cost per cycle

I_0 The initial inventory

$I(t)$ The inventory level at any time t before deterioration begins

I_d The inventory level at the time deterioration begins

$I_d(t)$ The inventory level at any time t after deterioration sets in

T_d The total demand between T_1 and T_2

$C[d(T_2)]$ The cost of deteriorated items

q The order quantity

q_1 The quantity sold at time T_2

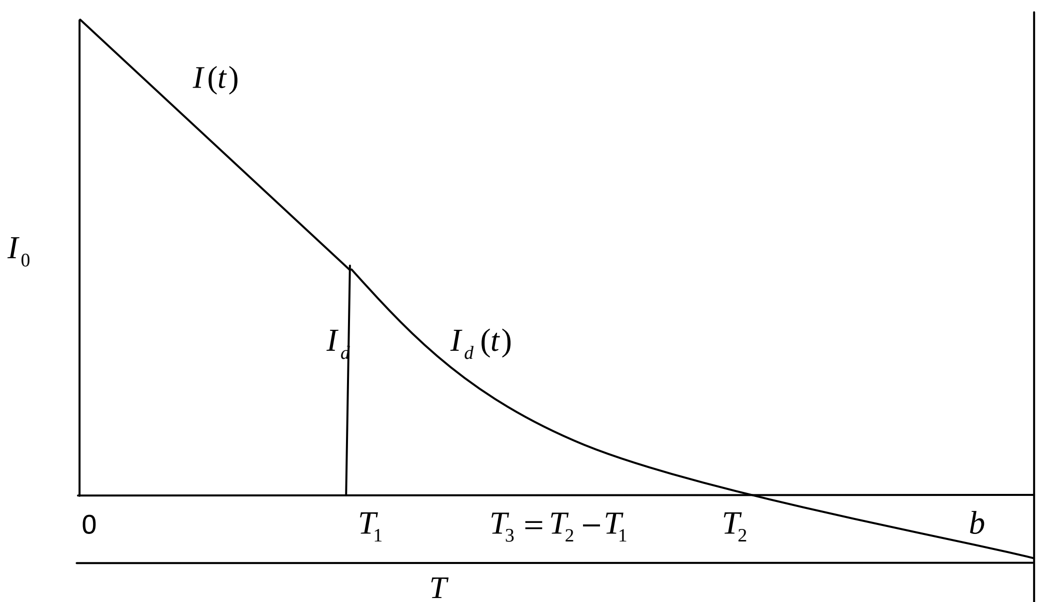

Figure 1: Delayed deteriorating inventory situation with shortages

3.0 The Mathematical Model

The inventory level in the interval $[0, T_1]$, before deterioration begins where depletion occurs only due to demand is described by the following differential equation:

$$\frac{dI(t)}{dt} + \lambda_1(t) = \frac{dI(t)}{dt} + (\gamma_1 + \gamma_2 t) = 0 \tag{1}$$

Separating the variables in equation (1) and solving yields:

$$\int dI(t) = -\int (\gamma_1 + \gamma_2 t)dt \;\; \Rightarrow \;\; I(t) = -\gamma_1 t - \frac{\gamma_2 t^2}{2} + \alpha_1 \tag{2}$$

Where α_1 is an arbitrary constant. At $t = 0$, $I(t) = I_0$, we get from equation (2) $I_0 = \alpha_1$, which is substituted in equation (2) to get

$$I(t) = -\gamma_1 t - \frac{\gamma_2 t^2}{2} + I_0 \tag{3}$$

Also at $t = T_1$, $I(t) = I_d$, we get from equation (3)

$$I_d = -\gamma_1 T_1 - \frac{\gamma_2 T_1^2}{2} + I_0 = -\left(\gamma_1 + \frac{\gamma_2 T_1}{2}\right)T_1 + I_0 \tag{4}$$

Solving for I_0 from equation (4) gives:

$$I_0 = I_d + \left(\gamma_1 + \frac{\gamma_2 T_1}{2}\right)T_1 \tag{5}$$

We substitute equation (4) into (3) to get,

$$I(t) = -\left(\gamma_1 t + \frac{\gamma_2 t^2}{2}\right) + I_d + \left(\gamma_1 + \frac{\gamma_2 T_1}{2}\right)T_1$$

$$= I_d - \gamma_1 t - \frac{\gamma_2 t^2}{2} + \gamma_1 T_1 + \frac{\gamma_2 T_1^2}{2}$$

$$= I_d + (T_1 - t)\gamma_1 + (T_1^2 - t^2)\frac{\gamma_2}{2} \tag{6}$$

Depletion of inventory in the interval $[T_1, T_2]$ after deterioration sets in will be as a result of the combined effect of demand and deterioration which is represented by the differential equation:

$$\frac{dI_d(t)}{dt} + \theta I_d(t) + \lambda_2 = 0 \;, \tag{7}$$

From equation (7) we get the integrating factor $I(t, I_d) = e^{\int \theta dt} = e^{\theta t}$. We multiply equation (7) through by the factor and integrate to get:

$\int \frac{d}{dt}(I_d(t)e^{\theta t}) = -\lambda_2 \int e^{\theta t}dt$, simplifying, gives:

$$I_d(t) = -\frac{\lambda_2}{\theta} + C_2 e^{-\theta t} \tag{8}$$

where C_2 is an arbitrary constant. We apply the condition at $t = T_1$, $I_d(t) = I_d$ which yields from equation (8)

$$I_d = -\frac{\lambda_2}{\theta} + C_2 e^{-\theta T_1} \quad \Rightarrow C_2 = \left(I_d + \frac{\lambda_2}{\theta}\right)e^{\theta T_1} \tag{9}$$

Substituting C_2 into equation (8) to get

$$I_d(t) = \frac{\lambda_2}{\theta}(e^{(T_1 - t)\theta} - 1) + I_d e^{(T_1 - t)\theta} \tag{10}$$

Now at $t = T_2$, $I_d(t) = 0$, equation (10) becomes

$$0 = \frac{\lambda_2}{\theta}(e^{(T_1-T_2)\theta} - 1) + I_d e^{(T_1-T_2)\theta} \Rightarrow I_d = \frac{-\lambda_2}{\theta}(1 - e^{(T_2-T_1)\theta}) \tag{11}$$

Substituting equation (11) into (10) gives:

$$I_d(t) = \frac{\lambda_2}{\theta}(e^{(T_1-t)\theta} - 1) - \frac{\lambda_2}{\theta}(1 - e^{(T_2-T_1)\theta})e^{(T_1-t)\theta} = \frac{\lambda_2}{\theta}(e^{(T_2-t)\theta} - 1) \tag{12}$$

Substituting I_d from equation (11) into (6) to get

$$I(t) = (T_1 - t)\gamma_1 + (T_1^2 - t^2)\frac{\gamma_2}{2} - \frac{\lambda_2}{\theta}(1 - e^{(T_2-T_1)\theta}) \tag{13}$$

The product of the demand rate at the beginning of deterioration and the time period when the item deteriorates gives the total demand for the inventory, T_d between T_1 and T_2 . Hence, $T_d = \lambda_2 T_3$

and the number of items that deteriorate during the interval $[T_1, T_2]$ is given by:

$$d(T_2) = I_d - \lambda_2 T_3 \tag{14}$$

Substituting I_d from equation (11) into equation (14) yields

$$d(T_2) = -\frac{\lambda_2}{\theta}(1 - e^{(T_2-T_1)\theta}) - \lambda_2 T_3 = -\frac{\lambda_2}{\theta}(1 - e^{(T_2-T_1)\theta} + (T_2 - T_1)\theta) \tag{15}$$

4.0 The total Variable (Inventory) Cost:

This is made up of the sum of the inventory carrying cost, ordering cost, backorder cost and the cost due to deterioration of materials. The costs are as given below.

(a) The inventory ordering cost per order is given as A

(b) The inventory holding cost is computed as follows:

$$\begin{aligned}
C_H &= iC\int_0^{T_1} I(t)dt + iC\int_{T_1}^{T_2} I_d(t)dt \\
&= iC\int_0^{T_1}(T_1 - t)\gamma_1 dt + iC\int_0^{T_1}\frac{(T_1^2 - t^2)\gamma_2}{2}dt - iC\int_0^{T_1}\frac{\lambda_2}{\theta}(1 - e^{(T_2-T_1)\theta})dt + \int_{T_1}^{T_2}\frac{\lambda_2}{\theta}(e^{(T_2-t)\theta} - 1)dt \\
&= iC\gamma_1 T_1^2 - \frac{iC\gamma_1 T_1^2}{2} + \frac{iC\gamma_2 T_1^3}{2} - \frac{iC\gamma_2 T_1^3}{6} - \frac{iC\lambda_2 T_1}{\theta} + \frac{iC\lambda_2 T_1 e^{(T_2-T_1)\theta}}{\theta} - \frac{iC\lambda_2 T_2}{\theta} - \frac{iC\lambda_2}{\theta^2} + \frac{iC\lambda_2 T_1}{\theta} \\
&\quad + \frac{iC\lambda_2 e^{(T_2-T_1)\theta}}{\theta^2} \\
&= \frac{iC\gamma_1 T_1^2}{2} + \frac{iC\gamma_2 T_1^3}{3} + \frac{iC\lambda_2 T_1 e^{(T_2-T_1)\theta}}{\theta} - \frac{iC\lambda_2 T_2}{\theta} - \frac{iC\lambda_2}{\theta^2} + \frac{iC\lambda_2 e^{(T_2-T_1)\theta}}{\theta^2}
\end{aligned}$$

$$\therefore\ C_H = \left(\frac{T_1}{2} + \frac{\gamma_2\theta T_1}{3\gamma_1} - \frac{\lambda_2 T_2}{\gamma_1 T_1} - \frac{\lambda_2}{\theta\gamma_1 T_1} + \frac{\lambda_2 e^{(T_2-T_1)\theta}}{\gamma_1} + \frac{\lambda_2 e^{(T_2-T_1)\theta}}{\gamma_1 T\theta}\right)\frac{iC\gamma_1 T_1}{\theta} \tag{16}$$

(c) Cost due to deterioration of materials is given as:

$$Cd(T_2) = C\left(-\frac{\lambda_2}{\theta}(1 - e^{(T_2-T_1)\theta} + (T_2 - T_1)\theta)\right) \tag{17}$$

(d) The backorder cost per cycle, C_B is given as:

$$C_B = C_b\int_0^{T-T_2}\lambda_2 t dt = \frac{C_b\lambda_2(T - T_2)^2}{2} \tag{18}$$

The total variable cost per unit is given as:

$$TIC(T) = \frac{1}{T} \text{ (Inventory ordering cost + Inventory holding cost+ Cost of deteriorated items + Backorder cost per cycle)}$$

$$= \frac{1}{T}(A + H_C + Cd(T_2) + C_B)$$

$$TIC(T) = \frac{A}{T} + \left(\frac{T_1\theta}{2} + \frac{\gamma_2\theta T_1^2}{3\gamma_1} - \frac{\lambda_2 T_2}{\gamma_1 T_1} - \frac{\lambda_2}{\theta\gamma_1 T_1} + \frac{\lambda_2 e^{(T_2-T_1)\theta}}{\gamma_1} + \frac{\lambda_2 e^{(T_2-T_1)\theta}}{\gamma_1 T_1 \theta} \right) \frac{iC\gamma_1 T_1}{\theta T}$$

$$+ \frac{C_b\lambda_2 (T-T_2)^2}{2T} + \frac{C}{T}\left(-\frac{\lambda_2}{\theta}(1 - e^{(T_2-T_1)\theta} + (T_2 - T_1)\theta) \right) \qquad (19)$$

We differentiate equation (19) with respect to T and equate to zero to obtain the value of the cycle length, T which gives the minimum total variable or inventory cost as follows:

$$\frac{dTIC(T)}{dT} = \frac{-A}{T^2} - \frac{iC\gamma_1 T_1^2}{2T^2} - \frac{iC\gamma_2 T_1^3}{3T^2} + \frac{iC\lambda_2 T_2}{\theta T^2} + \frac{iC\lambda_2}{\theta^2 T^2} - \frac{iC\lambda_2 T_1 e^{(T_2-T_1)\theta}}{\theta T^2} - \frac{iC\lambda_2 e^{(T_2-T_1)\theta}}{\theta^2 T^2}$$

$$+ \frac{C\lambda_2}{\theta T^2} - \frac{C\lambda_2 e^{(T_2-T_1)\theta}}{\theta T^2} + \frac{C\lambda_2 (T_2 - T_1)}{T^2} + \frac{C_b\lambda_2[(T-T_2)(2T-(T-T_2))]}{2T^2} \qquad (20)$$

Multiplying equation (20) through by T^2 and grouping yields:

$$-A - \left(\frac{T_1\theta}{2} + \frac{T_1^2\gamma_2\theta}{3\gamma_1} - \frac{\lambda_2 T_2}{\gamma_1 T_1} - \frac{\lambda_2}{\theta\gamma_1 T_1} - \frac{\lambda_2}{i\gamma_1 T_1} \right) \frac{iC\gamma_1 T_1}{\theta} - \left(1 + \frac{1}{\theta T_1} + \frac{\theta}{iT_1} \right) \frac{iC\lambda_2 T_1 e^{(T_2-T_1)\theta}}{\theta} + C\lambda_2 (T_2 - T_1)$$

$$+ \frac{C_b \lambda_2 (T^2 - T_2^2)}{2T^2} = 0 \qquad (21)$$

Equation (21) can be used to determine the best cycle length T which minimizes the total inventory cost. The corresponding Economic Order Quantity (EOQ) computed from the following:

$EOQ = \lambda_1 T_1 + \lambda_2 T_3 + d(T_2) + b$, where $b = q - q_1$ is the maximum backorder level allowed

$$= (\gamma_1 + \gamma_2 t)T_1 + \lambda_2 (T_2 - T_1) - \frac{\lambda_2}{\theta}(1 - e^{(T_2-T_1)\lambda} + (T_2 - T_1)\theta) + \lambda_2 (T - T_2)$$

$$= (\gamma_1 + \gamma_2 t)T_1 + \frac{\lambda_2}{\theta}(e^{(T_2-T_1)\theta} - 1) + \lambda_2 (T - T_2) \qquad (22)$$

5.0 Numerical Examples

The solutions of five different numerical examples representing the application of the model are given in Table 1.

Table 1: EOQ and Optimal Cycle length of Delayed Deteriorating Inventory Model with Linear Demand and Shortages

S/N	A	C	λ_1 (Units)	λ_2 (Units)	i	θ	γ_1	γ_2	T_1	T_2	T	$TIC(T)$	*EOQ (Units)*	I_0	b
1	30	90	700	500	0.13	0.4	100	120	0.0161	0.0384	0.0411	951.91	24	13	11
2	29	80	820	400	0.12	0.5	120	140	0.0143	0.0192	0.0219	1419.13	15	11	04
3	35	70	265	150	0.14	0.25	40	45	0.0198	0.0411	0.0438	854.83	09	05	04
4	40	75	159	145	0.11	0.30	24	27	0.0210	0.0603	0.0630	717.12	09	06	03
5	45	80	200	170	0.16	0.33	25	36	0.0145	0.0740	0.0767	778.78	14	11	03

Table 1 above gives the different cycle length T and the Economic Order Quantity (EOQ) for different parameter values. The EOQ in this case is made up of the initial inventory, I_0 and the maximum backorder level permitted which is always filled first when a new replenishment is received. The cycle length in all the five examples corresponds to the least of the overall inventory cost.

6.0 Conclusion

In this paper, we present the development of a model for the determination of inventory ordering policies of delayed deteriorating items with the assumption of linear time dependent demand and shortages. The demand before deterioration sets in is linear and when deterioration sets in, it is assumed to be a constant up to the end of the cycle. The model determines the period within which to order for inventory items and also the amount to be ordered. Five different numerical examples on the application of the model are also given.

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