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An EPQ Model for Items That Exhibit Delay in Deterioration with Reliability Consideration

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Abstract

The classical inventory deteriorating model assumes that deteriorating items start deteriorating immediately after such items are kept in stock. This is not always the case as there are some items (which include tomatoes, cassava, bread, cakes, etc) whose deterioration does not start immediately they are put in stock. Moreover the economic production quantity (EPQ) model assumes that items produced are always of perfect quality, however in real situation, product quality is never perfect. Everything depends on reliability (quality assurance) of such items. In this paper, we construct an EPQ model for deteriorating items whose deterioration does not start immediately they are stocked but rather after some period of time and the unit cost of production is directly related to the process reliability and inversely related to the rate of demand. Numerical examples are given to illustrate the developed model.

1.0 Introduction

The problem of determining the most desirable order quantity under stable conditions is commonly known as the classical economic order quantity (EOQ) problem. The equivalent of this problem in production is the economic production quantity (EPQ) model or lot size inventory model. Over the years, a voluminous amount of research work on this subject has been done and many interesting results have been reported in the literature. Inventory depletion is generally considered to be as a result of demand rate only but this is not the general situation in real life. There are cases of inventory depletion that are not due to demand only, but also due to deterioration or other factors. The decaying inventory problem was first analyzed by Ghare and Shrader [1] who developed an EOQ model with constant rate of decay. Covert and Philip [2] extended the work of Ghare and Shrader [1] and obtained an EOQ model for variable rate of deterioration by assuming a two parameter weibull distribution. Misra [3] developed a deteriorating model with finite replenishment rate. Heng et al. [4] considered lot size inventory system with finite replenishment rate, constant demand rate and exponential decay. Sarker et al. [5] considered lot size inventory model with inventory level dependent demand and deterioration. Arcelus et al. [6] also developed a model on retailer's pricing, credit and inventory policies for deteriorating items in response to temporary price/credit incentives. Yang [7] considered the pricing strategy for deteriorating items using quantity discount when demand is price sensitive. An EOQ model for deteriorating items under inflation and time discounting was developed by Moon et al. [8]**.** Manna and Chaudhuri [9] considered an EOQ model with ramp type demand rate, time dependent deterioration rate, unit production cost and shortages.

In all the above literature, the authors assumed that deterioration of the items start from the instance of their arrival in stock. As a matter of fact, many items (for example, firsthand vegetables, fruits, and some items produced in industry like bread, cakes, etc) have a span of maintaining fresh quality or original condition. During that period, there is no deterioration occurring. Thus it is necessary to consider inventory problems for non-instantaneous deteriorating items. Ouyang et al. [10] developed an EOQ model for non-instantaneous deteriorating items with permissible delay in payments and where the demand before deterioration starts is the same as that after deterioration begins. Sugapriya and Jeyaraman [11] developed a model to determine a common production cycle time for an economic production quantity model of non instantaneous

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deteriorating items allowing price discount and permissible delay in payments. Musa and Sani [12] developed an inventory model for some tropical items that exhibit delay in deterioration. Geetha and Uthayakumar [13] developed an EOQ model for non-instantaneous deteriorating inventory items with partial backlogging**.** Monika and Shon [14] developed an EOQ model for two levels of storage for non-instantaneous deteriorating items with stock dependent demand, time varying partial backlogging and under permissible delay in payments. Baraya and Sani [15] developed an EPQ model for delayed deteriorating items with stock-dependent demand rate and linear time dependent holding cost. Musa and Sani [16] developed an Inventory model for delayed deteriorating items under permissible delay in payments but where the demand before deterioration starts is different from that after deterioration starts. Thus, this paper is a generalization of Ouyang et al. [10].

Many authors modified inventory policies by considering the issue of process reliability and quality improvement in EOQ/EPQ models. Tapiero et al. [17] developed an EOQ model for reliability, pricing and quality control. Cheng [18] developed an economic production quantity model with process reliability and quality assurance considerations. Tripathy et al. [19] developed an EOQ model with process reliability consideration. Tripathy and Pattnaik [20] developed an EOQ model for optimal inventory policy with reliability consideration and instantaneous receipt under imperfect production process.

In this paper, we construct an EPQ model for items whose deterioration does not start immediately they are stocked but rather after some period of time. It is also assumed in the paper that the unit cost of production is directly related to the process reliability and inversely related to the rates of demand. Before deterioration begins, depletion of inventory is dependent only on demand. From the time deterioration begins up to the end of the cycle, depletion of inventory will depend on both deterioration and demand. Process reliability in this paper goes hand in hand with the unit cost of a product in such a way that the higher the reliability of a product, the higher the cost of production and vice versa. This is usually expected in real life because the higher the price of an item, the more reliable it will be meaning that the process reliability is nearly equal to 1. In many cases of real life, the unit cost of production of an item is directly related to it's reliability (quality assurance). After developing the model, we have given some numerical examples to illustrate the applications of the developed model.

THE MATHEMATICAL MODEL

In developing the model, the following notation and assumptions are used:

NOTATION

μ_1 The demand rate (units per unit time) during the period before deterioration sets in
μ_2 The demand rate (units per unit time)after deterioration sets in
Q The production quantity (units in a production run)
X The production cycle length (time unit)
C The unit cost of the item (in Naira)
A The set-up cost per production run (Naira per production run)
i The inventory carrying charge (excluding interest charges)
β The rate of deterioration
X_1 The time deterioration sets in
X_2 The difference between the production cycle length and the time deterioration sets in
Y_0 The initial inventory
Y_d The inventory level at the time deterioration begins
H The holding cost or inventory carrying cost in a production cycle
r Process Reliability of the items $(0 \le r \le 1)$

ASSUMPTIONS

(i) Instantaneous production
(ii) Unconstrained suppliers capital
(iii) Backorders not allowed
(iv) All items are inspected and defective ones are discarded
(v) Demand of product exceeds its supply
(vi) The unit cost of production of the item C is directly related to the process reliability r (quality assurance) and inversely related to the demand rates μ_1 & μ_2 . This relationship is specifically assumed to be:

$$C \propto r^k \left(\mu_1 \frac{X_1}{X} + \mu_2 \frac{X_2}{X} \right)^{-b}$$

$$\Rightarrow\ C \propto (1-r)^{-k}\left(\mu_1\frac{X_1}{X}+\mu_2\frac{X_2}{X}\right)^{-b}$$

$$\therefore\quad C = a(1-r)^{-k}\left(\mu_1\frac{X_1}{X}+\mu_2\frac{X_2}{X}\right)^{-b} \tag{1}$$

Where a, b, k are non-negative real numbers to be chosen based on some known data so as to provide the best fit of the estimated cost function.

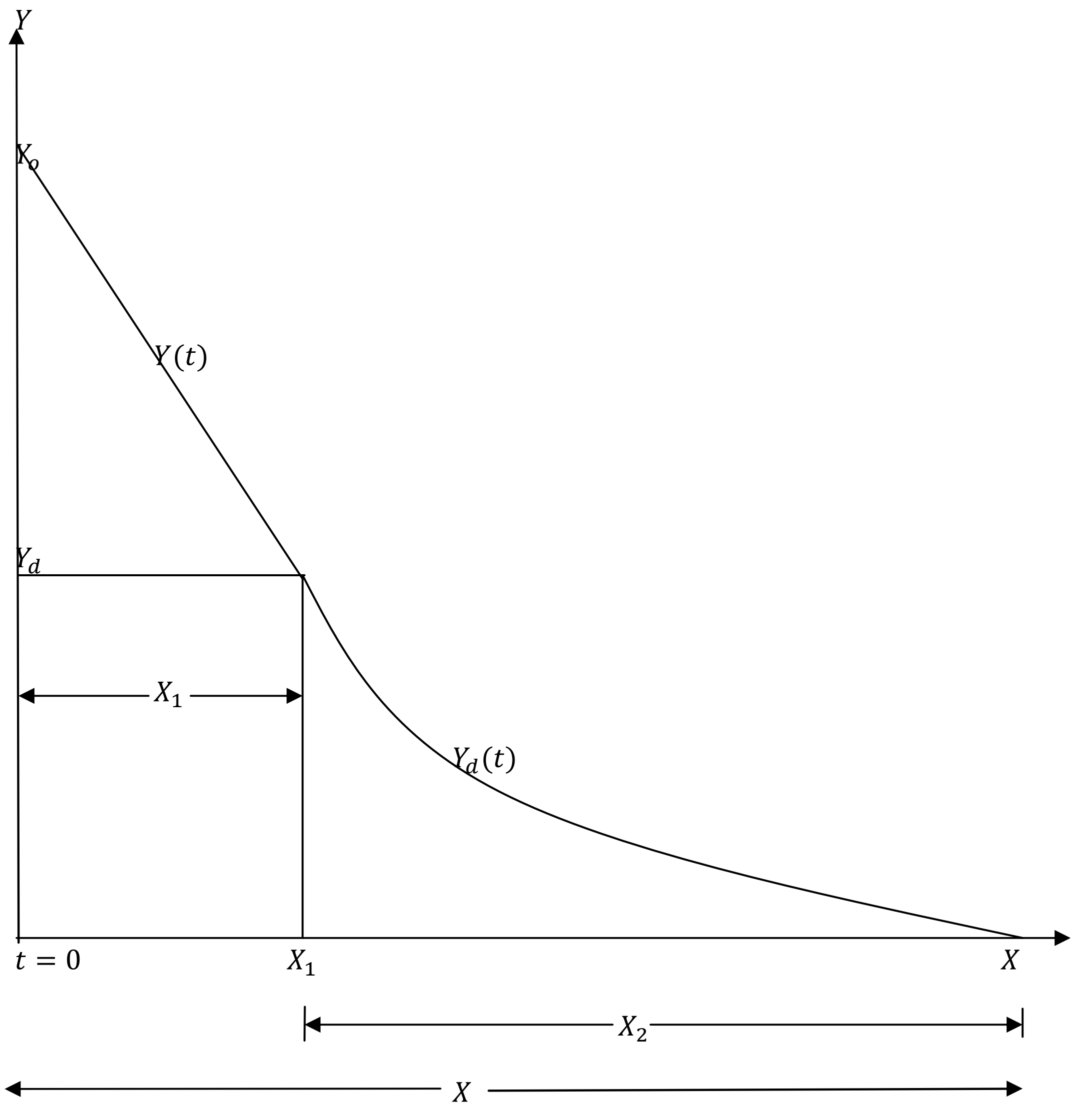

Fig1: Inventory movement in a non-instantaneous deterioration situation

Let $Y(t)$ and $Y_d(t)$ be the inventory level before and after deterioration sets in respectively. $Y(t)$ depends only on demand and the differential equation that describes the situation before deterioration sets in is given by

$$\frac{dY(t)}{dt} = -\mu_1 \quad \text{, where } 0 \le t \le X_1 \tag{2}$$

Solving (2), we get

$$Y(t) = -\mu_1 t + E \qquad \text{(where } E \text{ is a constant)} \tag{3}$$

applying the boundary condition $[Y(t) = Y_0 \text{ at } t = 0]$ in (3), we get

$$Y_0 = E \tag{4}$$

substituting (4) in (3), we obtain

$$Y(t) = -\mu_1 t + Y_0 \tag{5}$$

applying the boundary condition $[Y(t) = Y_d \text{ at } t = X_1]$ in (5), we get

$$Y_d = -\mu_1 X_1 + Y_0$$

$$Y_0 = Y_d + \mu_1 X_1 \tag{6}$$

substituting (6) in (5), we obtain

$$Y(t) = Y_d + (X_1 - t)\mu_1 \tag{7}$$

After deterioration sets in, depletion of inventory will depend on both demand and deterioration and the differential equation which describes the situation is given by

$$\frac{dY_d(t)}{dt} + \beta Y_d(t) = -\mu_2 \qquad \text{where } X_1 \le t \le X \tag{8}$$

Equation (8) is a linear differential equation and so using the integrating factor,

$$Y_H = e^{\beta t}$$

we get its solution as

$$Y_d(t) = Fe^{-\beta t} - \frac{\mu_2}{\beta} \qquad \text{(where } F \text{ is a constant)} \tag{9}$$

applying the boundary condition $[Y_d(t) = Y_d \text{ at } t = X_1]$ in (9), we get

$$Y_d = Fe^{-\beta X_1} - \frac{\mu_2}{\beta}$$

$$\Rightarrow \qquad F = \left(Y_d + \frac{\mu_2}{\beta}\right)e^{\beta X_1}$$

substituting F in (9), we get

$$Y_d(t) = \left(Y_d + \frac{\mu_2}{\beta}\right)e^{\beta X_1}e^{-\beta t} - \frac{\mu_2}{\beta}$$

$$\Rightarrow Y_d(t) = \left(Y_d + \frac{\mu_2}{\beta}\right)e^{-\beta(t - X_1)} - \frac{\mu_2}{\beta}$$

$$\Rightarrow Y_d(t) = \frac{\mu_2}{\beta}\left(e^{(X_1 - t)\beta} - 1\right) + Y_d e^{(X_1 - t)\beta} \tag{10}$$

Applying the boundary condition [$Y_d(t) = 0$ at $t = X$] in (10), we get

$$0 = \frac{\mu_2}{\beta}\left(e^{(X_1 - X)\beta} - 1\right) + Y_d e^{(X_1 - X)\beta}$$

$$\Rightarrow \qquad Y_d e^{(X_1 - X)\beta} = -\frac{\mu_2}{\beta}\left(e^{(X_1 - X)\beta} - 1\right)$$

$$\therefore \qquad Y_d = -\frac{\mu_2}{\beta}\left(1 - e^{-(X_1 - X)\beta}\right) \tag{11}$$

Substituting (11) in (7), we obtain

$$Y(t) = -\frac{\mu_2}{\beta}\left(1 - e^{-(X_1 - X)\beta}\right) + (X_1 - t)\mu_1 \tag{12}$$

Substituting (11) in (10), we get

$$Y_d(t) = \frac{\mu_2}{\beta}\left(e^{(X_1 - t)\beta} - 1\right) - \frac{\mu_2}{\beta}\left(1 - e^{-(X_1 - X)\beta}\right)e^{(X_1 - t)\beta}$$

$$\Rightarrow \quad Y_d(t)=\frac{\mu_2}{\beta}e^{(X_1-t)\beta}-\frac{\mu_2}{\beta}-\frac{\mu_2}{\beta}e^{(X_1-t)\beta}+e^{(-X_1+X+X_1-t)\beta}$$

$$\Rightarrow \quad Y_d(t)=-\frac{\mu_2}{\beta}+\frac{\mu_2}{\beta}e^{(X-t)\beta}$$

$$\therefore \quad Y_d(t)=\frac{\mu_2}{\beta}\left(e^{(X-t)\beta}-1\right) \qquad (13)$$

The number of deteriorated items $d(X_2)$ is the difference between the total demand in X_2 i.e. $X_2\mu_2$ and the total inventory at the time deterioration sets in i.e. Y_d ,

$$\therefore \quad d(X_2)=Y_d-X_2\mu_2$$

$$\Rightarrow \quad d(X_2)=-\frac{\mu_2}{\beta}\left(1-e^{-(X_1-X)\beta}\right)-X_2\mu_2$$

and so

$$d(X_2)=-\frac{\mu_2}{\beta}\left(1-e^{-(X_1-X)\beta}+X_2\beta\right) \qquad (14)$$

INVENTORY CARRYING COST (HOLDING COST)

The cost associated with storage of the whole inventory until it is sold or used, i.e. inventory carrying cost H, and is given by:

$$H=iC\int_0^{X_1}Y(t)dt+iC\int_{X_1}^{X}Y_d(t)\,dt \qquad (15)$$

Substituting (1), (12) and (13) in (15), we obtain

$$H=ia(1-r)^{-k}\left(\mu_1\frac{X_1}{X}+\mu_2\frac{X_2}{X}\right)^{-b}\int_0^{X_1}\left(-\frac{\mu_2}{\beta}\left(1-e^{-(X_1-X)\beta}\right)+(X_1-t)\mu_1\right)dt$$

$$+ia(1-r)^{-k}\left(\mu_1\frac{X_1}{X}+\mu_2\frac{X_2}{X}\right)^{-b}\int_{X_1}^{X}\frac{\mu_2}{\beta}\left(e^{(X-t)\beta}-1\right)dt$$

$$=ia(1-r)^{-k}\left(\mu_1\frac{X_1}{X}+\mu_2\frac{X_2}{X}\right)^{-b}\left\{\int_0^{X_1}\left(-\frac{\mu_2}{\beta}\left(1-e^{-(X_1-X)\beta}\right)+(X_1-t)\mu_1\right)dt+\frac{\mu_2}{\beta}\int_{X_1}^{X}\left(e^{(X-t)\beta}-1\right)dt\right\}$$

$$=ia(1-r)^{-k}\left(\mu_1\frac{X_1}{X}+\mu_2\frac{X_2}{X}\right)^{-b}\left\{\left[-\frac{\mu_2}{\beta}\left(t-te^{-(X_1-X)\beta}\right)+\left(X_1t-\frac{t^2}{2}\right)\mu_1\right]_0^{X_1}+\frac{\mu_2}{\beta}\left[-\frac{e^{(X-t)\beta}}{\beta}-t\right]_{X_1}^{X}\right\}$$

$$=ia(1-r)^{-k}\left(\mu_1\frac{X_1}{X}+\mu_2\frac{X_2}{X}\right)^{-b}\left\{-\frac{\mu_2}{\beta}\left(X_1-X_1e^{-(X_1-X)\beta}\right)+\left(X_1^{\,2}-\frac{X_1^{\,2}}{2}\right)\mu_1+\frac{\mu_2}{\beta}\left(-\frac{1}{\beta}-X+\frac{e^{(X-X_1)\beta}}{\beta}+X_1\right)\right\}$$

so that after further simplification, we obtain

$$H=\left(\left(1+\frac{1}{\beta X_1}\right)e^{-(X_1-X)\beta}+\frac{X_1\mu_1\beta}{2\mu_2}-\frac{1}{\beta X_1}-\frac{X}{X_1}\right)\frac{ia(1-r)^{-k}\left(\mu_1X_1+\mu_2\;X_2\right)^{-b}\mu_2X_1}{\beta X^{-b}} \qquad (16$$

TOTAL VARIABLE COST

The total variable cost is the sum of set-up cost, cost of deteriorated items and inventory carrying cost, i.e.

Total variable cost (TVC)

$$=A+Cd(X_2)+H \qquad (17)$$

$\therefore \quad (TVC) =$

$$A+a(1-r)^{-k}\left(\mu_1\frac{X_1}{X}+\mu_2\frac{X_2}{X}\right)^{-b}\left(-\frac{\mu_2}{\beta}\left(1-e^{-(X_1-X)\beta}+X_2\beta\right)\right)+\left\{\left(1+\frac{1}{\beta X_1}\right)e^{-(X_1-X)\beta}+\frac{X_1\mu_1\beta}{2\mu_2}-\frac{1}{\beta X_1}-\frac{X}{X_1}\right\}\frac{ia(1-r)^{-k}\left(\mu_1X_1+\mu_2X_2\right)^{-b}\mu_2X_1}{\beta X^{-b}} \tag{18}$$

The total variable cost per unit time is given as

$$Z(X) = \frac{Total\ variable\ cost\ per\ production\ run}{The\ production\ cycle\ length}$$

$$\Rightarrow \qquad Z(X) = \frac{A}{X} - \frac{a(1-r)^{-k}\left(\mu_1X_1+\mu_2X_2\right)^{-b}\mu_2}{\beta X^{-b}}\left(\frac{1}{X}\right)+\frac{a(1-r)^{-k}\left(\mu_1X_1+\mu_2X_2\right)^{-b}\mu_2}{\beta X^{-b}}\left(\frac{e^{-(X_1-X)\beta}}{X}\right)$$

$$-\frac{a(1-r)^{-k}\left(\mu_1X_1+\mu_2X_2\right)^{-b}\mu_2}{X^{-b}}\left(\frac{X-X_1}{X}\right)$$

$$+\left\{\left(1+\frac{1}{\beta X_1}\right)\left(\frac{e^{-(X_1-X)\beta}}{X}\right)+\frac{X_1\mu_1\beta}{2\mu_2}\left(\frac{1}{X}\right)-\frac{1}{\beta X_1}\left(\frac{1}{X}\right)-\left(\frac{1}{X}\right)\frac{X}{X_1}\right\}\frac{ia(1-r)^{-k}\left(\mu_1X_1+\mu_2X_2\right)^{-b}\mu_2X_1}{\beta X^{-b}}$$

since $\quad X_2 = X - X_1,$

$$\Rightarrow \qquad Z(X) = \frac{A}{X} - \frac{a(1-r)^{-k}\left(\mu_1X_1+\mu_2(X-X_1)\right)^{-b}\mu_2}{\beta}\left(\frac{1}{X^{-b+1}}\right)$$

$$+\frac{a(1-r)^{-k}\left(\mu_1X_1+\mu_2(X-X_1)\right)^{-b}\mu_2}{\beta}\left(\frac{e^{-(X_1-X)\beta}}{X^{-b+1}}\right)-\frac{a(1-r)^{-k}\left(\mu_1X_1+\mu_2(X-X_1)\right)^{-b}\mu_2}{1}\left(\frac{X-X_1}{X^{-b+1}}\right)$$

$$+\left\{\left(1+\frac{1}{\beta X_1}\right)\left(\frac{e^{-(X_1-X)\beta}}{X}\right)+\frac{X_1\mu_1\beta}{2\mu_2}\left(\frac{1}{X}\right)-\frac{1}{\beta X_1}\left(\frac{1}{X}\right)-\left(\frac{1}{X}\right)\frac{X}{X_1}\right\}\frac{ia(1-r)^{-k}\left(\mu_1X_1+\mu_2(X-X_1)\right)^{-b}\mu_2X_1}{\beta X^{-b}} \tag{19}$$

To obtain the value of X which minimizes the total variable cost per unit time, we take the derivative of $Z(X)$ with respect to X and then set the result to zero. The value of X satisfying that equation gives the minimum provided $\frac{d^2(Z)}{dX^2} > 0$. The derivative is given as:

$$\frac{dZ(X)}{dX} =$$

$$-\frac{A}{X^2}-\frac{a(b-1)(1-r)^{-k}\left(\mu_1X_1+\mu_2(X-X_1)\right)^{-b}\mu_2}{\beta}\left(\frac{1}{X^{-b+2}}\right)+\frac{ab\mu_2^2(1-r)^{-k}\left(\mu_1X_1+\mu_2(X-X_1)\right)^{-b-1}}{\beta}\left(\frac{1}{X^{-b+1}}\right)$$

$$+\frac{a(1-r)^{-k}\left(\mu_1X_1+\mu_2(X-X_1)\right)^{-b}\mu_2}{\beta}\left(\frac{\beta X^{-b+1}+(b-1)X^{-b}}{X^{2(-b+1)}}\right)e^{-(X_1-X)\beta}-\frac{ab\mu_2^2(1-r)^{-k}\left(\mu_1X_1+\mu_2(X-X_1)\right)^{-b-1}}{\beta}\left(\frac{e^{-(X_1-X)\beta}}{X^{-b+1}}\right)$$

$$-\frac{a(1-r)^{-k}\left(\mu_1X_1+\mu_2(X-X_1)\right)^{-b}\mu_2}{1}\left(\frac{X^{-b+1}+(b-1)(X-X_1)X^{-b}}{X^{2(-b+1)}}\right)+\frac{ab\mu_2^2(1-r)^{-k}\left(\mu_1X_1+\mu_2(X-X_1)\right)^{-b-1}}{1}\left(\frac{X-X_1}{X^{-b+1}}\right)$$

$$+\left\{\left(1+\frac{1}{\beta X_1}\right)\left(\frac{\beta X-1}{X^2}\right)e^{-(X_1-X)\beta}-\frac{X_1\mu_1\beta}{2\mu_2}\left(\frac{1}{X^2}\right)+\frac{1}{\beta X_1}\left(\frac{1}{X^2}\right)\right\}\frac{ia(1-r)^{-k}\left(\mu_1X_1+\mu_2(X-X_1)\right)^{-b}\mu_2X_1}{\beta X^{-b}}$$

$$+\left\{\left(1+\frac{1}{\beta X_1}\right)\left(\frac{e^{-(X_1-X)\beta}}{X}\right)+\frac{X_1\mu_1\beta}{2\mu_2}\left(\frac{1}{X}\right)-\frac{1}{\beta X_1}\left(\frac{1}{X}\right)-\left(\frac{1}{X}\right)\frac{X}{X_1}\right\}\frac{iab(1-r)^{-k}\left\{-\mu_2\left(\mu_1X_1+\mu_2(X-X_1)\right)^{-1}+X^{-1}\right\}\mu_2X_1}{X^{-b}\left(\mu_1X_1+\mu_2(X-X_1)\right)^b\beta}=0$$

$$\Rightarrow\quad -\frac{A}{X^2}-\frac{a(b-1)(1-r)^{-k}\left(\mu_1X_1+\mu_2(X-X_1)\right)^{-b}\mu_2}{\beta}\left(\frac{1}{X^{-b+2}}\right)+\frac{ab\mu_2^2(1-r)^{-k}\left(\mu_1X_1+\mu_2(X-X_1)\right)^{-b-1}}{\beta}\left(\frac{1}{X^{-b+1}}\right)$$

$$+\frac{a(1-r)^{-k}\left(\mu_1X_1+\mu_2(X-X_1)\right)^{-b}\mu_2}{\beta}\left(\frac{X^{-b}\left(X\beta+(b-1)\right)}{X^{2(-b+1)}}\right)e^{-(X_1-X)\beta}-\frac{ab\mu_2^2(1-r)^{-k}\left(\mu_1X_1+\mu_2(X-X_1)\right)^{-b-1}}{\beta}\left(\frac{e^{-(X_1-X)\beta}}{X^{-b+1}}\right)$$

$$-\frac{a(1-r)^{-k}\left(\mu_1X_1+\mu_2(X-X_1)\right)^{-b}\mu_2}{1}\left(\frac{X^{-b}\left(bX-X_1(\mathrm{b}-1)\right)}{X^{2(-b+1)}}\right)+\frac{ab\mu_2^2(1-r)^{-k}\left(\mu_1X_1+\mu_2(X-X_1)\right)^{-b-1}}{1}\left(\frac{X-X_1}{X^{-b+1}}\right)$$

$$+\left\{\left(1+\frac{1}{\beta X_1}\right)\left(\frac{\beta X-1}{1}\right)e^{-(X_1-X)\beta}-\frac{X_1\mu_1\beta}{2\mu_2}+\frac{1}{\beta X_1}\right\}\frac{ia(1-r)^{-k}\left(\mu_1X_1+\mu_2(X-X_1)\right)^{-b}\mu_2X_1}{\beta X^{-b+2}}$$

$$+\left\{\left(1+\frac{1}{\beta X_1}\right)\left(\frac{e^{-(X_1-X)\beta}}{1}\right)+\frac{X_1\mu_1\beta}{2\mu_2}-\frac{1}{\beta X_1}-\frac{X}{X_1}\right\}\frac{iab(1-r)^{-k}\left\{-\mu_2\left(\mu_1X_1+\mu_2(X-X_1)\right)^{-1}+X^{-1}\right\}\mu_2X_1}{X^{-b+1}\left(\mu_1X_1+\mu_2(X-X_1)\right)^b\beta}=0$$

$$\Rightarrow\quad \frac{A}{X^2}+\frac{a(b-1)(1-r)^{-k}\left(\mu_1X_1+\mu_2(X-X_1)\right)^{-b}\mu_2}{\beta}\left(\frac{1}{X^{-b+2}}\right)-\frac{ab\mu_2^2(1-r)^{-k}\left(\mu_1X_1+\mu_2(X-X_1)\right)^{-b-1}}{\beta}\left(\frac{1}{X^{-b+1}}\right)$$

$$-\frac{a(1-r)^{-k}\left(\mu_1X_1+\mu_2(X-X_1)\right)^{-b}\mu_2}{\beta}\left(\frac{\left(X\beta+(b-1)\right)}{X^{-b+2}}\right)e^{-(X_1-X)\beta}+\frac{ab\mu_2^2(1-r)^{-k}\left(\mu_1X_1+\mu_2(X-X_1)\right)^{-b-1}}{\beta}\left(\frac{e^{-(X_1-X)\beta}}{X^{-b+1}}\right)$$

$$+\frac{a(1-r)^{-k}\left(\mu_1X_1+\mu_2(X-X_1)\right)^{-b}\mu_2}{1}\left(\frac{\left(bX-X_1(\mathrm{b}-1)\right)}{X^{-b+2}}\right)-\frac{ab\mu_2^2(1-r)^{-k}\left(\mu_1X_1+\mu_2(X-X_1)\right)^{-b-1}}{1}\left(\frac{X-X_1}{X^{-b+1}}\right)$$

$$-\left\{\left(1+\frac{1}{\beta X_1}\right)\left(\frac{\beta X-1}{1}\right)e^{-(X_1-X)\beta}-\frac{X_1\mu_1\beta}{2\mu_2}+\frac{1}{\beta X_1}\right\}\frac{ia(1-r)^{-k}\left(\mu_1X_1+\mu_2(X-X_1)\right)^{-b}\mu_2X_1}{\beta X^{-b+2}}$$

$$-\left\{\left(1+\frac{1}{\beta X_1}\right)\left(\frac{e^{-(X_1-X)\beta}}{1}\right)+\frac{X_1\mu_1\beta}{2\mu_2}-\frac{1}{\beta X_1}-\frac{X}{X_1}\right\}\frac{iab(1-r)^{-k}\left\{-\mu_2\left(\mu_1X_1+\mu_2(X-X_1)\right)^{-1}+X^{-1}\right\}\mu_2X_1}{\beta X^{-b+1}\left(\mu_1X_1+\mu_2(X-X_1)\right)^b}=0$$

Multiplying through by $X^{(-b+2)}$, we get

$$\frac{A}{X^b}+\frac{a(b-1)(1-r)^{-k}\left(\mu_1X_1+\mu_2(X-X_1)\right)^{-b}\mu_2}{\beta}-\frac{ab\mu_2^2(1-r)^{-k}\left(\mu_1X_1+\mu_2(X-X_1)\right)^{-b-1}}{\beta X^{-1}}$$

$$-\frac{a(1-r)^{-k}(\mu_1X_1+\mu_2(X-X_1))^{-b}\mu_2}{\beta}\left(\frac{(X\beta+(b-1))}{1}\right)e^{-(X_1-X)\beta}+\frac{ab\mu_2^{\,2}(1-r)^{-k}(\mu_1X_1+\mu_2(X-X_1))^{-b-1}}{\beta X^{-1}}\left(\frac{e^{-(X_1-X)\beta}}{1}\right)$$

$$+\frac{a(1-r)^{-k}(\mu_1X_1+\mu_2(X-X_1))^{-b}\mu_2}{1}\left(\frac{(bX-X_1(\mathrm{b}-1))}{1}\right)-\frac{ab\mu_2^{\,2}(1-r)^{-k}(\mu_1X_1+\mu_2(X-X_1))^{-b-1}}{1}\left(\frac{X-X_1}{X^{-1}}\right)$$

$$-\left\{\left(1+\frac{1}{\beta X_1}\right)\left(\frac{\beta X-1}{1}\right)e^{-(X_1-X)\beta}-\frac{X_1\mu_1\beta}{2\mu_2}+\frac{1}{\beta X_1}\right\}\frac{ia(1-r)^{-k}(\mu_1X_1+\mu_2(X-X_1))^{-b}\mu_2X_1}{\beta}$$

$$-\left\{\left(1+\frac{1}{\beta X_1}\right)\left(\frac{e^{-(X_1-X)\beta}}{1}\right)+\frac{X_1\mu_1\beta}{2\mu_2}-\frac{1}{\beta X_1}-\frac{X}{X_1}\right\}\frac{iab(1-r)^{-k}\left\{-\mu_2(\mu_1X_1+\mu_2(X-X_1))^{-1}+X^{-1}\right\}\mu_2X_1}{\beta X^{-1}(\mu_1X_1+\mu_2(X-X_1))^{b}}=0 \quad (20)$$

Now, if other parameters are given, equation (20) can be used to determine the best X which minimizes the total variable cost.The EPQ of the corresponding X will be computed from:

$$EPQ = \text{demand before deterioration starts} + \text{demand after deterioration starts} + \text{number of deteriorated items}$$

$$=\mu_1X_1+\mu_2X_2+d(X_2)$$

Substituting (14), we have

$$EPQ=\mu_1X_1+\mu_2(X-X_1)-\frac{\mu_2}{\beta}\left(1-e^{-(X_1-X)\beta}+X_2\beta\right)$$

$$=\mu_1X_1+\mu_2(X-X_1)-\frac{\mu_2}{\beta}\left(1-e^{-(X_1-X)\beta}+(X-X_1)\beta\right)$$

$$=\mu_1X_1+\mu_2(X-X_1)-\frac{\mu_2}{\beta}\left(1-e^{-(X_1-X)\beta}\right)-\mu_2(X-X_1)$$

$$\therefore \quad EPQ=\mu_1X_1-\frac{\mu_2}{\beta}\left(1-e^{-(X_1-X)\beta}\right) \quad (21)$$

NUMERICAL EXAMPLES

Solution of ten different numerical examples having different parameters as indicated with $a = 10, b = 1$ and $k = 6$.

Table1: Solutions of ten different numerical examples

Examples	A $(naira)$	μ_1 $(units)$	μ_2 $(units)$	r	i	X_1 $(years)$	β	X $(years)$	Z $(naira)$	EPQ $(units)$
Example1	2500	2000	1200	0.90	0.13	0.019178 (7days)	0.40	0.038356 (14days)	101462.1	61
Example2	2700	2500	1000	0.91	0.15	0.038356 (14days)	0.50	0.046575 (17days)	120365.7	104
Example3	2650	3000	1600	0.88	0.14	0.057534 (21days)	0.30	0.084932 (31days)	51542.96	217
Example4	2400	3500	2000	0.87	0.12	0.076712 (28days)	0.45	0.10411 (38days)	36977.83	324
Example5	3000	3200	800	0.89	0.11	0.095890 (35days)	0.35	0.112329 (41days)	58586.68	320
Example6	2300	2400	1100	0.86	0.10	0.115068 (42days)	0.51	0.145205 (53days)	25627.45	310
Example7	1900	3300	1400	0.85	0.16	0.134247 (49days)	0.55	0.161644 (59days)	22629.26	482
Example8	2600	2000	1300	0.84	0.17	0.153425 (56days)	0.60	0.183562 (67days)	23621.13	346
Example9	3500	2300	500	0.92	0.01	0.172603 (63days)	0.20	0.180822 (66days)	52952.8	401
Example10	1500	1800	700	0.83	0.18	0.191781 (70days)	0.32	0.232877 (85days)	14478.15	374

SENSITIVITY ANALYSIS

We have carried out a sensitivity analysis on the effect of changes in the system parameters ($a, b,$ and k) on the total variable cost per unit time $Z(X)$, using the following example which is example 1 in Table 1.

Table 2: Example 1 in Table 1

a	b	k	A	μ_1	μ_2	r	i	X_1	β	X	Z	EPQ
10	1	6	2500	2000	1200	0.90	0.13	0.019178	0.40	0.038356	101462.1	61

Table 3: Sensitivity analysis

System parameter	% change in the system parameter	% change in Z
a	60	21
	40	14
	20	7.2
	-20	-7.2
	-40	-14
	-60	-21
b	60	-35
	40	-34
	20	-26
	-20	121
	-40	649
	-60	2956
k	60	142329
	40	8949
	20	531
	-20	-34
	-40	-36
	-60	-36

From the table 3, it can be observed that:

- Z(x) is sensitive to changes in the parameter.
- Z(x) is more sensitive to changes in k and is less sensitive to a.

Conclusion

In this paper, an EPQ model for deteriorating items which do not start deteriorating until after some time is presented. Deterioration also depends on product reliability (quality assurance) of the product. Before deterioration begins, depletion of the inventory is purely dependent on demand but when deterioration begins, depletion is dependent on both demand and deterioration. The manufacturer is paid for the items immediately they are received.

There are items that have the property especially in tropical countries such as those in the West-African Sub-region like Nigeria. These items include firsthand vegetables, fruits, bread, cakes, etc. The quality of those products depends largely on process reliability (quality assurance) of the production line. The proposed model can be used in inventory control of non-instantaneous deteriorating items such as food items, like bread, cakes, sandwitches and so on.

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