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An Extended Solution Procedure for Single Item Inventory System with Uncertain Demand and Shortages.

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Abstract

In this paper, an extended solution procedure for inventory system for single item with uncertain demand is examined. In this model shortage is permitted. A new expression for the inventory total cost and the quantity is derived for the system. We use convex property to develop a simple and direct search method for the inventory system with uncertain demand. Numerical example is presented to illustrate our proposed model.

Keywords: Inventory, uncertain demand, carrying cost, shortage cost, convexity.

1.0 Introduction

The amount of inventory needed should depend on the safety stock as to protect against the demand uncertainty, and to achieve a high service level for satisfying customers' demand Lee [1]. Osagiede and Oriakhi [2] considered a new approach to inventory replenishment with twistling demand and shortages. In their model, they presented two cases and modeled the mathematical expectation for the inventory total cost for the system incorporating shortages. A solution was provided to a level where methods of calculus in solving the equations will be

applied but they left off there as a result of the mathematical complexity involved in the solution procedure. For detailed review see elsewhere [2 - 5]

In this paper, an extension of Osagiede and Oriakhi [2] is proposed. We provide a simplified mathematical procedure for the inventory problem. Further, a direct search method is developed to the optimal inventory level at time *t* with the convex properties for the inventory system with uncertain demand which we believe to be necessary for the validation of our results. Numerical example is presented to illustrate our new proposed model. We organized this paper as follows: In the next section, the adopted notation and assumptions is presented. Next we present the mathematical model in which we shall illustrate the case presented by Osagiede and Oriakhi [2] diagrammatically. This is followed by the main result by way of proving the proposition stated to show that the total cost obtained is convex. The validation of this proposition shall be verified in our next paper to give an expression for the optimal quantity. Following this, numerical results by our proposed model are presented. The last section presents the conclusion and direction for further studies.

2.0 Notations And Assumptions

The notation and assumptions used here are adopted from Osagiede and Oriakhi [2].

Let $f(x)$ be the probability that the total demand is x during a time interval T; k_1 be the storage cost per unit per unit time, plus interest on capital they represent while k_2 be the shortage cost per unit time. q is the inventory level at time *t*; X the ordered quantity for each ordering cycle. λ the shortage cost per unit per unit time; $W(q)$ be total inventory cost; T is the length of each ordering cycle; q_1^* the optimal value of the inventory level q at time *t* and T_1 is the time when

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there is no shortage; T_2 be the time when there is shortage and n be the number of replenishments. The assumption is as follows; demand is discontinuous, but in practice, we can assume that its rate is constant. The demand during a certain time interval *T* is uncertain and the materials of inventory do not lose value during the interval *T*. The storage cost, ordering cost, shortage cost remain constant over time. Shortages in the inventory are allowed.

3.0 Model Formulation

In this inventory situation, two cases shall be considered.

Case I: If the total demand x is less than the inventory q, $(q \geq x)$.

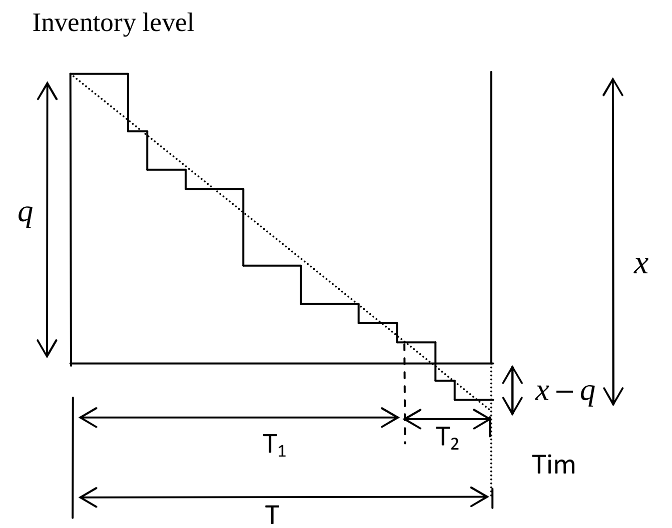

Fig. 1. A diagrammatic representation of the total demand x is less than the inventory q, $(q \geq x)$.

Case II: If demand is greater than inventory q, $(q < x)$

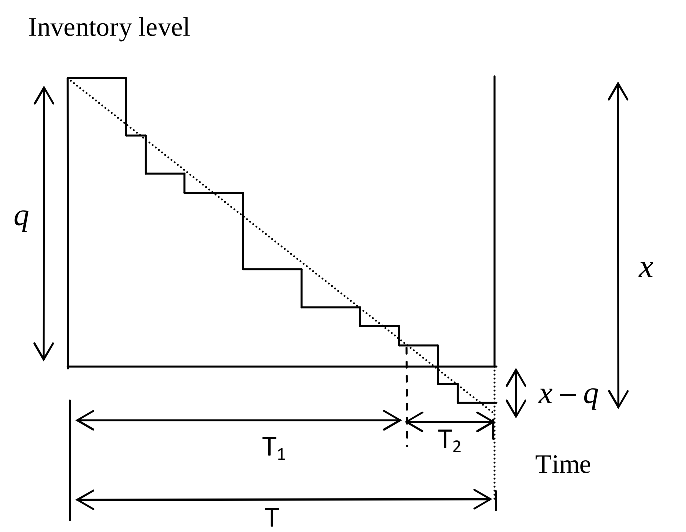

Fig. 2. A diagrammatic representation of the total demand when q, $(q < x)$

We give some preliminary results of Osagiede and Oriakhi [2]

In case 1: Average inventory corresponding to situation in case 1 (see fig. 1) is given by:

$$q_a = \frac{1}{2}[q + (q - x)] = q - \frac{x}{2} \qquad (1)$$

In case II: Average inventory corresponding to situation in case II (see fig 2)is given by

$$q_b = \frac{1}{2} q \frac{T_1}{T} = \frac{1}{2} \frac{q^2}{x} \tag{2}$$

From case II, the Average shortage is given by:

$$\mu_b = \frac{1}{2}(x-q)\frac{T_2}{T} = \frac{1}{2}\frac{(x-q)^2}{x} \tag{3}$$

The mathematical expectation for the total inventory cost will be;

$$W(q) = k_1 \sum_{x=0}^{q}\left(q-\frac{x}{2}\right) f(x) + k_1 \sum_{x=q+1}^{\infty} \frac{1}{2}\frac{q^2}{x} f(x) + k_2 \sum_{x=q+1}^{\infty} \frac{1}{2}\frac{(x-q)^2}{x} f(x) \tag{4}$$

This work is extended as follows:

3.0 The Main Result:

To further describe the structure of the optimal policy for convexity, we need the following proposition. The proposition is an extension of the work of Osagiede and Oriakhi [2].

***Our Proposition*:**

The minimum inventory total cost $W(q)$ as defined by equation (4) occurs for a value of q^* such that:

$G(q^*-1) < \lambda < G(q^*)$, where

$$\lambda = \frac{k_2}{k_1+k_2} \quad and$$

$G(q^*) = f(x \le q^*) + (q^*+1) \sum_{x=q^*+1}^{\infty} \frac{f(x)}{x}$, and q^* is the optimal inventory level.

***Proof*:**

Given that

$$W(q) = k_1 \sum_{x=0}^{q}\left(q-\frac{x}{2}\right) f(x) + k_1 \sum_{x=q+1}^{\infty} \frac{1}{2}\frac{q^2}{x} f(x) + k_2 \sum_{x=q+1}^{\infty} \frac{1}{2}\frac{(x-q)^2}{x} f(x),$$

then

$$W(q+1) = k_1 \sum_{x=0}^{q+1}\left(q+1-\frac{x}{2}\right) f(x) + k_1 \sum_{x=q+2}^{\infty} \frac{1}{2}\frac{(q+1)^2}{x} f(x) + k_2 \sum_{x=q+2}^{\infty} \frac{1}{2}\frac{(x-q-1)^2}{x} f(x) \tag{6}$$

But we can write

$$k_1 \sum_{x=0}^{q+1}\left(q+1-\frac{x}{2}\right) f(x) = k_1 \sum_{x=0}^{q}\left(q-\frac{x}{2}\right) f(x) + k_1 \sum_{x=0}^{q} f(x) + k_2 \frac{q+1}{2} f(q+1) \tag{7}$$

$$k_1 \sum_{x=q+2}^{\infty} \frac{1}{2}\frac{(q+1)^2}{x} f(x) = k_1 \sum_{x=q+1}^{\infty} \frac{q^2 f(x)}{2x} + k_1 \sum_{x=q+1}^{\infty} \frac{q\, f(x)}{x} + \frac{k_1}{2} \sum_{x=q+1}^{\infty} \frac{f(x)}{x} - \frac{k_1}{2}(q+1) f(q+1) \tag{8}$$

and

$$k_2 \sum_{x=q+2}^{\infty} \frac{1}{2}\frac{(x-q-1)^2}{x} f(x) = k_2 \sum_{x=q+1}^{\infty} \left[(x-q)^2 - 2(x-q)+1\right]\frac{f(x)}{2x}$$
$$- k_2 \left[(q+1-q)^2 - 2(q+1-q)+1\right]\frac{f(q+1)}{2(q+1)}$$

and so,

$$k_2 \sum_{x=q+2}^{\infty} \frac{1}{2}\frac{(x-q-1)^2}{x} f(x) = k_2 \sum_{x=q+1}^{\infty} \frac{(x-q)^2}{2x} f(x) - k_2 \sum_{x=q+1}^{\infty} f(x) + q\,k_2 \sum_{x=q+1}^{\infty} \frac{f(x)}{x} + \frac{1}{2}k_2 \sum_{x=q+1}^{\infty} \frac{f(x)}{x} \tag{9}$$

Substituting equations (7), (8) and (9) into equation (4), we obtain

$$\begin{aligned} W(q) &= k_1 \sum_{x=0}^{q+1}\left(q+1-\frac{x}{2}\right) f(x) - k_1 \sum_{x=0}^{q} f(x) - k_1 \frac{(q+1)}{2} f(q+1) \\ &+ k_1 \sum_{x=q+2}^{\infty} \frac{(q+1)^2}{2x} f(x) - k_1 \sum_{x=q+1}^{\infty} \frac{qf(x)}{x} - k_1 \sum_{x=q+1}^{\infty} \frac{f(x)}{x} \\ &+ \frac{k_1}{2}(q+1)\, f(q+1) + k_2 \sum_{x=q+2}^{\infty} \frac{(x-q+1)^2}{2x} f(x) + k_2 \sum_{x=q+1}^{\infty} f(x) \\ &- q\,k_2 \sum_{x=q+1}^{\infty} \frac{f(x)}{x} - \frac{k_2}{2} \sum_{x=q+1}^{\infty} \frac{f(x)}{x} \\ &= \left[k_1 \sum_{x=0}^{\infty}\left(q+1-\frac{x}{2}\right) f(x) + k_1 \sum_{x=q+2}^{\infty} \frac{(q+1)^2}{2x} f(x) + k_2 \sum_{x=q+2}^{\infty} \frac{(x-q-1)^2}{2x} f(x) \right] \\ &- k_1 \sum_{x=0}^{q} f(x) + k_2 \sum_{x=q+1}^{\infty} f(x) - (qk_1 + qk_2) \sum_{x=q+1}^{\infty} \frac{f(x)}{x} - \left(\frac{k_1}{2} + \frac{k_2}{2}\right) \sum_{x=q+1}^{\infty} \frac{f(x)}{x} \end{aligned}$$

But

$$\begin{aligned} k_2 \sum_{x=q+1}^{\infty} f(x) &= k_2 \sum_{x=0}^{\infty} f(x) - k_2 \sum_{x=0}^{q} f(x) \\ &= k_2 - k_2 \sum_{x=0}^{q} f(x) \end{aligned} \tag{10}$$

Thus

$$W(q) = W(q+1) + k_2 - k_2 \sum_{x=0}^{\infty} f(x) - k_1 \sum_{x=0}^{q} f(x) - k_1\left(q+\frac{1}{2}\right) \sum_{x=q+1}^{\infty} \frac{f(x)}{x} - k_2\left(q+\frac{1}{2}\right) \sum_{x=q+1}^{\infty} \frac{f(x)}{x}$$

This can be re-written as

$$W(q) = W(q+1) + k_2 - (k_1 + k_2) \sum_{x=0}^{q} f(x) - (k_1 + k_2)\left(q+\frac{1}{2}\right) \sum_{x=q+1}^{\infty} \frac{f(x)}{x}$$

Hence $W(q+1) = W(q) + (k_1 + k_2)\, f(x \le q) + (k_1 + k_2)\left(q+\frac{1}{2}\right) \sum_{x=q+1}^{\infty} \frac{f(x)}{x} - k_2$

$$= W(q) + (k_1 + k_2)\left[f(x \le q) + \left(q+\frac{1}{2}\right) \sum_{x=q+1}^{\infty} \frac{f(x)}{x} \right] - k_2 \tag{11}$$

If we set

$$G(q) = f(x \le q) + \left(q+\frac{1}{2}\right) \sum_{x=q+1}^{\infty} \frac{f(x)}{x} \tag{12}$$

Then (11) becomes.

$$W(q+1) = W(q) + (k_1 + k_2)\, G(q) - k_2 \tag{13}$$

In a similar manner;

$$W(q-1) = W(q) - (k_1 + k_2)\ G(q-1) + k_2 \qquad (14)$$

4.0 Numerical Illustrations

In this section, we present numerical results to illustrate the proposed model developed in this paper.

- **Solution by the Proposed Model:**

In our proposed model, we shall determine the optimum q^* and compare results with the solution values obtained in Osagiede and Oriakhi [2].

Here, we shall calculate $G(q)$ for different values of q without going through the rigorous of calculating various values of q as obtained in Osagiede and Oriakhi [2] solution

Table I: Various Values for the Proposed Model. *Indicates Minimum Cost.

q	x	$f(x)$	$\frac{f(x)}{x}$	$\sum_{x=q+1}^{\infty}\frac{f(x)}{x}$	$\left(q+\frac{1}{2}\right)\sum_{x=q+1}^{\infty}\frac{f(x)}{x}$	$f(x \le q)$	$G(q) = f(x \le q) + \left(q+\frac{1}{2}\right)\sum_{x=q+1}^{\infty}\frac{f(x)}{x}$
0	0	0.1	∞	0.445	0.2225	0.1	0.3225
1	1	0.2	0.2	0.245	0.3675	0.3	0.6675
2	2	0.2	0.1	0.145	0.3625	0.5	0.8625
3 *	**3**	**0.3**	**0.1**	**0.145**	**0.1575**	**0.8**	**0.9575**
4	4	0.3	0.025	0.020	0.0900	0.9	0.9900
5	5	0.1	0.020	0.000	0.0000	1	1
>5	>5	0.1	0.000	0.000	0.0000	1	1

We shall out of curiosity briefly give the procedure of obtaining our various values in the column shown above.
The entries in Table I were obtained as follows when $q = 0$

$$x = 0, \quad f(x) = 0\cdot 1, \quad \frac{f(x)}{x} = \infty$$

$$\sum_{\infty=q+1}^{\infty}\frac{f(x)}{x} = \sum_{x=1}^{\infty}\frac{f(x)}{x} = 0\cdot 2 + 0\cdot 1 + 0\cdot 1 + 0\cdot 025 + 0\cdot 020 + 0\cdot 000 = 0.445$$

$$\left(q+\frac{1}{2}\right)\sum_{x=1}^{\infty}\frac{f(x)}{x} = \left(0+\frac{1}{2}\right)(0\cdot 445) = 0\cdot 2225$$

$$f(x \le q) = f(x \le 0) = f(x = 0) = 0.1$$

$$G(q) = f(x \le q) + \left(q+\frac{1}{2}\right)\sum_{x=q+1}^{\infty}\frac{f(x)}{x} = 0\cdot 1 + 0\cdot 2225 = 0\cdot 3225$$

Again, when $q = 3$

$$x = 3, \quad f(x) = 0\cdot 3, \quad \frac{f(x)}{x} = \frac{0\cdot 3}{3} = 0\cdot 1$$

$$\sum_{x=q+1}^{\infty}\frac{f(x)}{x} = \sum_{x=3+1}^{\infty}\frac{f(x)}{x} = \sum_{x=4}^{\infty}\frac{f(x)}{x} = 0\cdot 025 + 0\cdot 020 = 0\cdot 045$$

$$\left(q+\frac{1}{2}\right)\sum_{x=3+1}^{\infty}\frac{f(x)}{x} = \left(3+\frac{1}{2}\right)(0\cdot 045) = 0\cdot 1575$$

$$f(x \le q) = f(x \le 3) = 0\cdot 1 + 0\cdot 2 + 0\cdot 2 + 0\cdot 3 = 0\cdot 8$$

$$= f(0) + f(1) + f(2) + f(3).$$

$$G(q) = f(x \le q) + \left(q + \frac{1}{2}\right) \sum_{x=q+1}^{\infty} \frac{f(x)}{x}$$

$$= 0 \cdot 8 + 0 \cdot 1575 = 0 \cdot 9575 .$$

Continuing in this manner, we obtain other values in the table I.

Notice that in Table I, for $q = 3$, we have

$$[G(2) = 0 \cdot 8625] < \left(\lambda = \frac{20}{21} = 0 \cdot 9524\right) < [G(3) = 0 \cdot 9575].$$

From this illustration, we can absolutely say that, the optimum inventory level q is therefore

3 units. This follows that the corresponding cost and the inventory level q as obtained by Osagiede and Oriakhi [2] is N 0.29million.

5.0 Conclusion

An extended solution procedure for inventory system for single item with uncertain demand has been examined. In this model shortages are permitted. We indeed derived a new expression with a proposition stated and a proof to the proposition for the inventory total cost. Further, we use the convex property established to develop a simple and direct search method for the inventory system with uncertain demand. Work is currently going on using the computer program written in MATLAB for solving more problems. Secondly, we shall give a proposition to also show the convex nature of the properties of the quantity function for the system.

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