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Special Cases of the Gamma Distribution and its Applications

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Abstract

This paper examines the Gamma Distribution as a type of continuous distribution and also a method of measuring uncertainties that occurs within intervals. It is prominent to note that uncertainties are bound to occur as long as man lives. The gamma distribution becomes a vital tool in measurement especially in measurement that involves scale and intervals and also in checking the rate of uncertainties in all human Endeavour's.

1.0 Introduction

The gamma distribution is a topic under the probability distribution which form the basis of this research work. A probability distribution is a correspondence which assigns probabilities to the values of random variable [1].

The gamma distribution is a type of continuous distribution it is a very useful model for measuring continuous data especially data that are obtained by comparison with a scale of some kind, for example, length, time, weight, mass e.t.c. this occurs when we measure the speed of a car, the amount of alcohol in a person's blood, the net weight of a package of frozen food, or the amount of tar in cigarette e.t.c. usually this will be decimal places even if all recorded data are whole numbers, decimals could occur by greater precision of measurement is one of facilitating stories of technology.

The properties of the gamma distribution will be considered with proofs as they are stated in theorem form, yet this work will also reveal two special cases of the gamma distribution and finally areas of application will be highlighted.

2.0 The Gamma Distribution

In discussing the gamma distribution, there is need to talk about the gamma function ,is most important not only in probability theory but in many areas of Mathematics. The function is denoted by the symbol " Γ " and it is defined as Γ (r) =

$$\int_o^\infty x^{r-1} e^{-x} \quad dx \qquad \text{for } r > 0 \qquad (2.1)$$

The gamma function is a generalization of the factorial to non integral values. The factorial is written as (!) with n! defined as the product

(1 x 2 x 3 x………xn). The gamma function can also be defined as the value that is approached by the quotient.

$$\frac{n!n^2}{z(z+1)(z+2)\ldots\ldots(z+n)} \qquad (2.2)$$

As n gets larger and larger it is equivalent to its definition as a type of infinite sum given by the integral (2.1)

With the aid of the gamma function, we can now introduce the gamma probability distribution. Gamma distribution is a very important model for measuring data within intervals, it can be regarded as a second approximation. This distribution depends on two parameters, r and α of which r and α are positive.

Let x be a continuous random variable assuming only non negative values, x is said to have the gamma distribution with parameters r and α if x has probability density function (pdf) given by

$$f(x) = \begin{cases} \frac{\alpha}{\Gamma(r)}[(\alpha x)^{r-1}e^{-\alpha x}] & x > 0 \\ 0, \quad otherwise & \end{cases} \qquad (2.3)$$

Just as the other distributions have mean and variance so does the gamma distribution. The mean is given by $r/_{\alpha}$ while variance is $r/_{\alpha^2}$.

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2.1 Properties of Gamma Distribution

The properties of the gamma distribution are stated in theorem forms [3]

Theorem 1 : $\Gamma(1) = 1$

Theorem 2 : $\Gamma(r+1) = r\,\Gamma(r)$, or > o

Theorem 3: $\Gamma(n) = (n-1)!$

Theorem 4: $\Gamma(^1/_2) = \Gamma(\pi)$

2.2 Moment Generating Function For The Gamma Distribution

The moment generating function for a gamma distribution is derived from the moment generating function (mgf) of a continuous distribution since gamma distribution is a type of continuous distribution.

Suppose x has a gamma distribution with parameter α and r which implies that if x assumed only non negative values, then x has a gamma pdf given in (2.3).

Then the mgf of x is given as

$$M_{x(t)} = \frac{\alpha}{\Gamma(r)} \int_0^\infty e^{tx} (\alpha x)^{r-1} e^{-\alpha x}\, dx = \frac{\alpha^r}{\Gamma(r)} \int_0^\infty x^{r-1} e^{-x(\alpha-t)} dx \quad (2.4)$$

This integral converges provided $\alpha > t$.

If we let $(\alpha - t) = u$, $x = \frac{u}{(\alpha-t)}$, $\frac{du}{dx} = \alpha - t$, $du = (\alpha - t)dx$, $dx = \frac{du}{(\alpha-t)}$ (2.5)

By substituting (2.5) into (2.4) we have

$$M_{x(t)} = \left(\frac{\alpha}{\alpha-t}\right)^r \frac{1}{\Gamma(r)} \int_0^\infty u^{r-1} e^{-u}\, du \quad (2.6)$$

But $\int_0^\infty u^{r-1} e^{-u}\, du = \Gamma(r)$

$$\therefore M_{x(t)} = \left(\frac{\alpha}{\alpha-t}\right)^r \quad (2.7)$$

3.0 Special Cases Of The Gamma Distribution

The gamma distribution has too very important and useful special cases. These are the exponential distribution with parameter α and the chi square distribution with n degrees of freedom.

3.1 Exponetial Distribution With Parameter α

The exponential distribution is widely used for length of life of equipments or parts, in fact it is the standard distribution in area of reliability. A continuous random variable x assuming all non negative value is said to have an exponential distribution with parameter α where $\alpha > 0$ if its pdf is given by

$$f(x) = \begin{cases} \alpha e^{-\alpha x} & , x > 0 \\ 0, \text{ otherwise} & \end{cases} \quad (3.1)$$

The gamma distribution has an interesting connection with the exponential distribution is regarded as a special case of the gamma distribution. This is obtained form the gamma distribution by letting r= 1. This will be illustrated from three cases, the definition, pgf and mgf of the gamma distribution.

Case 1

From (2.3), if we put r = 1, we have

$$f(x) = \alpha\, e^{-\alpha x} \quad (3.2)$$

Case II

From (2.1), and putting r = 1, we have (3.3)

$$\Gamma(1) = \int_0^\infty e^{-x}\, dx$$

Since x^0 and $\Gamma(1) = 1$, (3.3) becomes

$$\Gamma(1) = 1 = \int_0^\infty e^{-x}\, dx \quad (3.4)$$

Case III

From the moment of generating function, the mgf of the exponential distribution is given as

$$M_{x(t)} = \left(\frac{\alpha}{\alpha-t}\right)^r \quad (3.5)$$

Comparing (3.5) and (2.6), putting r=1, we have

$$M_{x(t)} = \left(\frac{\alpha}{\alpha - t}\right) \qquad (3.6)$$

From the above illustrations, it is obvious that the exponential distribution is obtained from the gamma distribution by letting r = 1 hence it is a special case of the gamma distribution. This special connection between the gamma and the exponential distribution is very useful in total length of life after replacing each failed unit by a new unit in life testing. [4]

3.2 Chi - Square Distributions with n - degrees of Freedom

The chi square distribution has many important applications in statistical reference hence it is tabulated for various values of the parameter n. The chi square distribution is symbolized as χ^2. The probability density function for a chi square random variable is given as

$$f(x) = \frac{1}{2^{½}\Gamma(½f)} x^{½}(f-2)\, e^{½x} \qquad (3.7)$$

This is also a probability density function when $f > 0$. The chi square distribution is a special and very important case of the gamma distribution with n degrees of freedom. It is obtained from the gamma distribution by letting α = ½ and f= n where n is positive integer. A random variable Z having a pdf given by

$$f(Z) = \frac{1}{2^{n/2}\Gamma(n/2)} Z^{n/2-1} e^{-z/2} \qquad Z > 0 \qquad (3.8)$$

is said to have a chi square distribution with n degrees of freedom (df). This special case of gamma distribution can also be verified in a similar manner to that of the exponential we shall use the pdf and mgf of the gamma distribution by letting α = ½ and r = $n/2$.

Case 1 (using the pdf).

If we let α = ½ and r = $^n/_2$ in (2.3), we discovered that the gamma distribution is a generalization of the chi-square

$$f(x) = \frac{(½)^{n/2}}{\Gamma(n/2)} x^{n/2-1} e^{½x} = \frac{1}{2^{n/2}\Gamma(n/2)} x^{n/2-1} e^{-x/2} \qquad (3.9)$$

If x = z, (3.9) will be same as (3.8)

Case II (using mgf)

From (2.6), letting α = ½ and r = $^n/_2$, (2.6) becomes

$$M_{x(t)} = (1-2t)^{-n/2} \qquad (3.10)$$

If x = z, we have $M_{z(t)} = (1-2t)^{-n/2}$ (3.11)

Therefore the mgf of a random variable with chi square distribution is given, as it is derived from the mgf of the gamma distribution.

(3.11) is the mgf of a random variable x with a chi square with n degrees of freedom. From the above verification, it is obvious that the gamma distribution is a generalization of the chi square distribution, hence it is an important special case of the gamma distribution.

The mean and variance of the chi square distribution can also be evaluated from its mgf by differentiating. The first derivative evaluated at t = 0 is the mean while the second derivative evaluated at t = 0 is the variance of the distribution. We now differentiate (3.10), to have

$$M'_{x(0)} = n\,(1)^{-(n/2+1)} = n \qquad (3.12)$$

Therefore, mean of chi square distribution = n.

If we take the second derivatives of (3.10), we have,

$$M''_{x(t)} = -\,n(2+n)(1-2t)^{-\frac{4+n}{2}} \qquad (3.13)$$

at t = 0

$$M''_{x(0)} = n\,(2+n) \qquad (3.14)$$

But variance

$$\sigma = \sum(x)^2 - (\sum(x)^2) = M''_{x(0)} - \left(M'_{x(0)^2}\right) = n(n+2) - n^2 = 2n \qquad (3.15)$$

Therefore the variance $\sigma^2{}_x$ of the chi square distribution is given by $\sigma^2{}_x$ = 2n.

4.0 Applications of the Gamma Distribution

The gamma distribution as an important technique for measuring uncertainties has a wide range of applications which are useful in life and all ramification of human endeavour. Some of these applications are highlighted as follows

4.1 Relationship between the CDF of the Gamma and the Poisson Distribution.

There is an interesting relationship between the cumulative density function (cdf) of the gamma distribution and the Poisson distribution. For the purpose of application, consider the integral

$$I = \int_0^{\infty} (e^{-y} y^r / r!)\, dy \qquad (4.1)$$

Where r is a positive integer and a >0.
Multiplying (4.1) by r! and integrating by part, we have

$$r!I = e^{-a}\, a^{r} + r\int_{0}^{\infty} e^{-y}\, y^{r-1}\ dy \qquad (4.2)$$

The integral in this expression is exactly of the same form as the original integral with r replaced by (r-1). Thus if we continue to integrate by part, we have

$$r!I = e^{-a}[a^{r} + ra^{r-1} + r(r-1)a^{r-2} + \ \dots\dots + r!] \qquad (4.3)$$

Since r >0,

$$I = \quad e^{-a}\left[1 + a + \frac{a^{2}}{2!} + \ \dots . + \frac{a^{r}}{r!}\right] \ = \sum p(y = k) \qquad (4.4)$$

Where y has a Poisson distribution with parameter a.
We now consider the cdf of a random variable x whose pdf is given in (2.3)
Since r>o, (2.3) can be written as

$$f(x) = \frac{\alpha}{(r-1)!}(\alpha x)^{r-1}e^{-\alpha x} \qquad x > 0 \qquad (4.5)$$

and consequently the cdf of x becomes

$$f(x) = \ 1 - \ p(X\ > x) = 1 - \int_{x}^{\infty}\frac{\alpha}{(r-1)!}(\alpha s)^{r-1}e^{-\alpha s}\, ds \qquad (4.6)$$

Note that s is used because of the x in the integrating limit
Putting (αs) = u, (4.6) becomes

$$f(x) = 1 - \int_{\alpha x}^{\infty}\frac{u^{r-1}e^{-u}}{(r-1)!}\ du\ , > 0 \qquad (4.7)$$

This integral is precisely of the form considered above namely I(with a = αx) and thus

$$f(x)\ =\ 1 -\ \sum_{k=0}^{r-1}\frac{e^{-\alpha x}(\alpha x)^{k}}{k!} \qquad , x > 0 \qquad (4.8)$$

Hence the cdf of the gamma distribution may be expressed in terms of the tabulated cdf of the Poisson distribution if only r >0. (4.8) establishes the relationship between the cdf of the gamma and the Poisson distributions.

When we deal with Poisson distribution, we are essentially concerned about the numbers of occurrence of some event during a fixed time period, and the gamma distribution arises when we ask for the distribution of the time required to obtained a specific number of occurrences of the event.[5]

Specifically,

Suppose x is the number of occurrence of the event A during (0, t). Then under suitable conditions, x has Poisson distribution with parameter α where α is the expected number of occurrence of A during a unit time interval.
Let T = time required to observed r occurrence of A, we have

$$H(t)\ =\ p(T \le t) = 1 - p(T > t) = 1 - p(x < r)\ =\ 1 - \sum_{k=0}^{r-1}\ \frac{e^{-\alpha t}}{k!} \qquad (4.9)$$

Comparing (4.9) with (4.8), it obvious that there is a relationship between the cdf of the gamma distribution and the Poisson distribution which is established by

$$\mathbf{H}(\boldsymbol{t}) = 1 - \sum_{k=0}^{r-1}\frac{e^{-\alpha t}(\alpha t)^{k}}{k!} \qquad , \quad t > 0 \qquad (4.10)$$

4.2 Gamma Failure Law

It has earlier been stated that the exponential distribution is a special case of the gamma distribution which is obtained by letting r=1 and so the application of the exponential failure law many as well be used as the gamma failure law [5]

To derive the gamma failure law, we will apply the relationship that exists between the exponential distributions which is characterized in many ways. For the purpose of this research, we will assumed that the failure rate of the exponential is constant which implies that $Z(t) = \alpha$. An immediate consequence of this assumption is stated in one of the theorem of reliability which states that “if T, the time to failure is a continuous random variable with pdf, and if $f(0)$ = 0 where f is the cdf of T, then f may be expressed in terms of the failure rate Z. [5]

$$f(t) = Z(t)e^{-}\int_{0}^{t} Z(s)\ ds \qquad (4.11)$$

From the above, the pdf associated with the time to failure T is given by

$$f(t)\ =\ \alpha e^{-\alpha\, t} \qquad , t > 0 \qquad (4.12)$$

The assumption of constant failure rate may be interpreted to mean that after the item has been used, its probability of failing has not changed. This also means that there is no “wearing” out effect when the exponential model is stipulated.

For many types of components, the assumption leading to the exponential failure law is not only intuitively appealing but, it is in fact coned by empirical evidence. For instance, it is quite reasonable to suppose that a fuse is “as good as new” while it is still functioning i.e. if the fuse has not melted it is in practically new condition.

From the above discussion of the exponential failure law, we now examine its connection with a Poisson process so as to derived the gamma failure law.

Suppose that failure occurs because of the appearance of certain 'random' disturbance. These may be caused by either external forces such as sudden gusts of wind or a rise in voltage or by mechanical malfunctioning. Let x_t be equal to the number of such disturbance occurring during a time interval of length t, and suppose that $x_t$constitute a Poisson process where t $\geq$ 0. That is for any fixed t the random variable x has a Poisson distribution with parameter α.

Suppose that failure during (0, t) is caused if and only if (iff) at least one of such disturbance occurs,

Let T be the time to failure which we shall assume to be a continuous random variable. Then

$$f(t) = p(T \leq t) = 1 - p(T > t) \tag{4.13}$$

T > t iff no disturbance occurs during [0, t],this happen iff $x_t = 0$.

Hence $$f(t) = 1 - p(x_t = 0) = 1 - e^{-\alpha t} \tag{4.14}$$

is the cdf of an exponential failure law. Thus it is clear that the above cause of failure implies an exponential failure law. This may also be generalized as

Suppose again that disturbances appear according to a Poisson process. In this case we shall assume that failure occur whenever r or more disturbance (r ≥ 1) occur during an interval of length t. Therefore if T is time to failure, we have as stated in (4.13). In this case T > t if (r-1) or fewer disturbances occur. Therefore

$$f(t) = 1 - \sum_{k=0}^{r-1} \frac{e^{-\alpha t}}{k!} \tag{4.15}$$

$$\boldsymbol{f(t)} = 1 - \sum_{k=0}^{r-1} \frac{e^{-\alpha t}}{k!} = \int_0^t \frac{\alpha}{(r-1)!} (\alpha s)^{r-1} e^{-\alpha s} \, ds \tag{4.16}$$

(4.16) represent the cdf of a gamma distribution.

Thus the above caused of exponential failure law leads to the conclusion that the time to failure follows a gamma failure law.

4.3 Application of The Chi-Square Distribution

Since the chi-square distribution is a special case of the gamma distribution with n degrees of freedom obtained by letting a = ½ and r = $^{n}/_{2}$, its applications may as well be regarded as those of the gamma distribution. [6]

4.3.1 Test of Goodness of Fit

In testing the goodness of fit, we make use of the chi-square distribution. In this case, the chi- square distribution is denoted by

$$\boldsymbol{\chi}^2 = \frac{(O - E)^2}{E} \tag{4.17}$$

Where

$\boldsymbol{\chi}^2$ = test statistics

O = observed frequency and

E = expected frequency

This test usually involves two types of hypothesis

H_o = null hypothesis

H_a = alternative hypothesis

If the computed value of $\boldsymbol{\chi}^2$ is lesser than the table value, we accept H_o and reject H_a. On the other hand, if computed value $\boldsymbol{\chi}^2$ is greater than the table value we reject H_o and accept H_a. Test of goodness of fit helps to determine whether the sample data are in conformity with the hypothesized distribution. In other words, it determines the closeness between the observed number (frequency) and the number that one would expect from the hypothesized distribution.

For the purpose of indicating the procedure, let us consider the following examples.

Example 4.1

It is believed that the proportion of people with A, B, O and AB blood types in a population are respectively 0.4, 0.2, 0.3 and 0.1. When 400 randomly picked people were examined, the observed numbers of each were 148, 96, 106, and 50

At 5% level of significance, we test the hypothesis that this data bear out the stated belief.

Solution:

Let P_A = probability that a person has type A blood.

P_B= probability that a person has type B blood.

P_O = probability that a person has type O blood.

P_{AB} = probability that a person has type AB blood.

Then H_o: P_A= 0.4, P_B = 0.2, P_O = 0.3, P_{AB} = 0.1

The above information is displayed in table 4.1

Table 4.1: blood type and observed frequency

Blood type	Observed frequency (O)
A	148
B	96
O	106
AB	50
TOTAL	400

We now compute the expected frequency using E= np.

Table 4.2: observed and expected frequencies of blood type and computation of χ^2 value.

Blood type	O	E	O-E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
A	148	160	-12	144	0.90
B	96	80	16	256	3.20
O	106	120	-14	196	1.63
AB	50	40	10	100	2.50
TOTAL	400	400	0	696	8.23

At 5%, α = 0.05, $\chi^2{}_{3,0.05}$ = 7.815

Since the computed value is larger than the table value, we reject H_o. Thus, there is strong evidence that the above beliefs regarding the distribution of blood is not correct?

4.3.2 Tests of Homogeneity

This has to do with the comparison between proportions of a characteristic in more than two populations. For instance one may compare or rather consider three states, say New York, California and Indiana, and wanted to test whether, in this three states, the proportion of republicans are the same. In short, the most important thing is to test whether all these states are homogeneous with respect to the affiliation of their residents. Once again the measure of departure from homogeneity is provided by a test statistic χ^2 whose value for any sample is given by (4.17).

The computational procedure for the test of homogeneity is the same as for the test of independence variables although the conclusion to be drawn are of a slightly different nature. For the purpose of application, let consider the following example.

Example 4.2

In order to investigate whether the distribution of the blood types in Europe is the same as in the united state information was collected on 200 randomly picked people in Europe and 300 people in the united states, from the data summarized in table 4.3 below, is it true that the distribution of blood types in Europe and the united states are significantly different? At 5% of significant level, we test the hypothesis that the distribution of blood types in Europe is the same as in the United States.

Table 4.3 Observed frequencies of the distribution of blood types in Europe and the United States.

Blood Type	**Europe**	**United States**	**Total.**
A	95	125	220
B	50	70	120
O	45	90	135
AB	10	15	25
Total	200	300	500

Solution

Following similarly from section 4.3.1

H_o: The distribution of blood types in Europe is the same as in the united state.

H_a: The distribution of blood types differ in Europe and in the united state.

Table 4.4: The Expected frequencies And The Computation Of x^2 Value.

Location	**Blood Type**	**O**	**E**	**(O – E)**	**$(O – E)^2$**	**$(O – E)^2/E$**
Europe	A	95	88	7	49	0.55
	B	50	48	2	4	0.08
	O	45	54	-9	81	1.50
	AB	10	10	0	0	0
	A	125	132	-7	49	0.37
United State	B	70	72	-2	4	0.06
	O	90	81	9	81	1.00
	AB	15	15	0	0	0
	Total	500	500	0	268	3.56

The chi–square distinction has (4-1) (2-1) = 3df at 5%, α = 0.05, $\chi^2{}_{3,0.05}$=7.81.

Since the computed value 3.56 < 7.81, we accept H_o and reject H_a. Therefore there is no reason to believe that the distinction of blood types in Europe is different from the distribution in the United State.

4.4 Gamma Fitting

In gamma fitting, we apply the procedure of fitting any theoretical distribution. Suppose we have a distribution and we want to fit the distribution by a gamma curve, the following are the necessary procedures: -

(i) Evaluate the mean and variance of the distribution.

(ii) Estimate the parameters associated with gamma distribution i.e. α and r using point estimate by moment.

(iii) Recall the mean and variance of the gamma distribution and equate it with the mean and variance earlier obtained from the distribution i.e. E(x) = $\frac{r}{\alpha}$ and $\sigma_x^2 = \frac{r}{\alpha^2}$ (4.18)

(iv) Solve the above equations simultaneously to get values of r and α respectively.

(v) Fix these values into the pdf of the gamma distribution to get the probability of occurrence of each observation.

(vi) To test whether the fix is good enough for the distribution or not, we apply test of goodness of fit using the test statistics χ^2 (Chi – square) distribution.

(vii) If the computed value of χ^2 < table value, then the fit is good enough but if χ^2 > table value the fit is not good enough. Practically, if given any theoretical distribution, one can fit the distribution by the gamma curve. An example will help to illustrate this.

Example 4.3

Of the cars observed on a free way, 160 were GM 100 ford, 80 Chrysler, 25 American and 135 foreign imports, if the expected frequencies of the observed cars are in the ratio of 30 : 15 : 10 : 5 : 20 respectively fit the distribution by a gamma curve at 5% level of significance.

Solution

$$E(x) = \frac{160+100+80+25+135}{5} = 100$$

$$\sigma_x^2 = \frac{(160)^2+(100)^2+(80)^2+(25)^2+(135)^2}{5} = 12170$$

Now we estimate the parameter associated with gamma distribution using point estimate by moment. Let us make two assumptions

$$\mu_1' = \mu \text{ and } \mu_2' = \sigma^2 + \mu^2 \qquad (4.19)$$

Now x the distribution has a pdf given in (2.3). Also applying (2.6), the cumulate function.

$$k_{x(t)} = \ln M_{x(t)} = r\log\alpha - r\log(\alpha - t) \qquad (4.20)$$

On differentiating (4.20) with respect to t, and evaluating at t=0 we have

$$k'_{x(t)} = \frac{r}{\alpha-t} = \frac{r}{\alpha} = \mu \qquad (4.21)$$

Similarly, second derivative gives

$$k''_{x(t)} = \frac{r}{(\alpha-t)^2} = \frac{r}{\alpha^2} \qquad (4.22)$$

From (4.19) $\mu = \frac{r}{\alpha} = 100$ and $\sigma^2 + \mu^2 = \frac{r}{\sigma^2} + \left(\frac{r}{\alpha}\right)^2 = 12170$ (4.23)

This imply that α =0.04 and r = 4 are the two parameters associated with gamma distribution

Substituting α and r into the pdf of the gamma distribution given in (2.3) we have

$$\text{Pdf} = f(x) = \int \frac{0.04}{\Gamma(4)}(0.04x)e^{-0.04x}dx \qquad (4.24)$$

Obtaining the probability of each observation from the pdf of the gamma distribution directly is cumbersome and so we obtained the probability from the ratio of the expected frequencies since they are given in the question.

Let R_t / T_r = P (4.25)

Where R_t = ratio of expected frequency,

T_r = Total ratio and

P = Probability

Table 4.5 Observed and Expected frequencies, probability and the computational value of χ^2 of the cars

	Probability	**O**	**E**	**O-E**	**(O – E)^2**	**(O – E)^2/E**
GM	0.375	160	187.5	-27.5	756.25	4.033
Ford	0.1875	100	93.75	6.25	39.06	0.416
Chrylser	0.125	80	62.5	17.5	306.25	4.9
America	0.0625	25	31.25	-6.25	39.625	1,268
Foreign Import	0.25	135	125	10	100	0.8
Total		500	500			11.417

df = n – k where k is the number of parameters estimated. The chi – square distribution has 5-2 = 3df. Since we estimated 2 parameters α and r at 5% α = 0.05

$\chi^2_{3,0.05} = 7.81$

Since the computed value of $\chi^2 = 11.42$ > table value 7.82, we concluded that the fit is not satisfactory. Therefore the above distribution could be any other such as normal, binomial or Poisson.

5.1 Summary and Conclusion

We have examined the Gamma distribution as a type of continuous distribution and also as a method of measuring uncertainties that occur within intervals,. This work also reveals two special cases of the Gamma distribution; the exponential distribution with parameter α which is widely used for length of life of equipment or parts and the chi-square distribution with n degrees of freedom.

The chi - square distribution is a very important case of the Gamma distribution which is obtained by letting $\alpha = \frac{1}{2}$ and

f = n. The Gamma distribution being an important and second approximation to data have a wide range of applications, and some of the areas of applications were highlighted in this paper.

It is glaring from the above that the use of the Gamma distribution cannot be over emphasized. It is useful in all areas of human endeavors', hence the knowledge is of great importance. Managers need it for decision making, it helps students to measure *skewers* of a distribution accurately, business men needs to measure uncertainties in their business life especially in payment of salaries and even in recruitment.

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