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Differential Transform Method for the Solution of Convective Heat Transfer over a Vertical Plate with Internal Heat Generation Embedded in Saturated Porous Media

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Abstract

In this research work, we provide a differential transform solution of the convective heat transfer over vertical plate with exponential heat generation. The coupled partial differential equations are transformed into nonlinear system of ordinary differential equation using similarity transform. This nonlinear system is solved using the differential transform method. The values of the skin friction for various thermo physical parameters are also computed and compared with that obtained from the Runge Kutta shooting method, the results obtained shows a good agreement between the two methods.

Keywords: boundary layer, differential transform, porous media, suction.

1.0 Introduction

Many problems in applied mathematics, theoretical physics and engineering are often nonlinear ordinary or partial differential equations. Most of which are difficult to solve and defy analytical solutions. In recent years, differential transform method which was first introduced by Zhou[1] in 1986 to solve initial value problems arising in electric circuit analysis has been used to solve several forms of linear and nonlinear differential equations. The differential transform method is used to construct analytic solutions in the form of Taylor series polynomial, unlike the traditional higher order Taylor method, the differential transform method reduces the the size of the computational algorithm and can easily be applied to a wide range of linear and nonlinear differential equations. Several applications of the differential transform method to convective heat transfer include the work of Rashidi et'al[2] who applied the multi-step differential transform method to the flow of second grade fluid over a stretching or shrinking sheet, Islam et'al[3] solve the axisymmetrical squeezing fluid flow between two parallel infinite plates in a porous channel, Rashidi and Keimanesh[4] presented a differential transform method –Pade approximant to the MHD flow in laminar Liquid film from a horizontal strctching surface and the paper of Peker et'al[5] who also solved the Blasius equation using a combined differential transform method-Pade approximant.

Several results have been published on heat transfer through porous surfaces and channels due to its importance in many geophysical, scientific and industrial applications. Examples of such studies includes the work of Igham and Pop[6], Cheng and Minkowycz[7] who studied the steady free convection about a vertical plate embedded in a porous media using the boundary layer assumptions and Darcy model by the similarity method. Islam etal[3] studied the axisymmetrical squeezing fluid flow between the two infinite parallel plates in a porous medium channel using the differential transform method. Ali[8] studied the effect of lateral mass flux on the natural convection boundary layers induced by a heated vertical plate embedded in a saturated porous medium with internal heat generation using the fourth order Runge kutta techniques.

Also Okedayo etal[9] carried out an analysis of viscous dissipation effect on the mixed convection MHD Flow towards a stagnation point with convective Boundary condition in a porous media using the similarity transformation together with the Runge-Kutta shooting method. In this paper we apply the differential transform method to obtain a semi analytic result for heat transfer over a vertical plate embedded in a porous medium with exponential internal heat generation. The results obtained for the skin friction coefficient are compared with numerical result.

The advantage of the method is that it reduces computational complexities and is easy to implement on a computer program.

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2.0 Mathematical formulations

Consider the laminar two-dimensional flow of a viscous incompressible fluid over a vertical plate embedded in a homogeneous porous media. The basic governing boundary layer equations using the Darcy and Boussinesq approximations are Ali [3]

continuity

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

momentum

$$\frac{\partial u}{\partial y}=\frac{gK\beta}{\gamma}\frac{\partial T}{\partial y} \tag{2}$$

energy

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{q}{\rho C} \tag{3}$$

Subject to the following boundary conditions

$$\left.\begin{aligned} &v(x,0)=V(x),\ T(x,0)=T_W(x)\\ &u(x,\infty)=0,\ \ T(x,\infty)=T_\infty \end{aligned}\right\} \tag{4}$$

The u and v are the velocity components in x and y directions respectively, T is the temperature, g is the acceleration due to gravity, β is the thermal expansion coefficient, α is the thermal diffusivity, γ is the kinematic viscosity, and q_r is the internal heat rate, $T_\omega = T_\infty + Ax^m$ is wall surface temperature, T_∞ is the free stream temperature, while, u_w and u_e are the surface and free stream velocity respectively , σ electrical conductivity, ρ is the density of the fluid, k is the permeability of the porous media and (x, y) are the Cartesian coordinates along the surface normal to it.

In order to reduce the set of equations (1)-(4) to a system of ordinary differential equations we introduce the following dimensionless variables.

$$\eta = Ra^{\frac{1}{2}} f(\eta),\ u=\frac{\alpha}{x}Raf'(\eta),\ \psi=\alpha Ra^{\frac{1}{2}} f(\eta), T=T_\infty + Ax^m\theta(\eta) \tag{5}$$

$$\theta(\eta)=\frac{T-T_\infty}{T_W-T_\infty},\ q=\frac{k(T_W-T_\infty)}{x^2}Rae^{-\eta}, v=-\frac{\alpha}{2x}Ra^{\frac{1}{2}}[(m+1)f+(m-1)\eta f']$$

Substituting the above into the governing equations (1) - (4) we have

$$f''=\theta' \tag{6}$$

$$\theta''+\frac{m+1}{2}f\theta'-mf'\theta+e^{-\eta}=0 \tag{7}$$

subject to

$$\theta(0)=1,\ f(0)=f_W,\quad f'(\infty)=0 \tag{8}$$

Applying (6) in (7)-(8) we have

$$f'''+\frac{m+1}{2}ff''-mf'^2+e^{-\eta}=0 \tag{9}$$

subject to

$$f'(0)=1,\ \ f(0)=f_W,\ \ f'(\infty)=0 \tag{10}$$

3.0 The Differential Transform Method

If f(x) is a given function, it can be expanded in a Taylor series form about a point x_0 as

$$f(x)=\sum_{K=0}^{\infty}\frac{(x-x_0)^K}{K!}\left(\frac{d^K f}{dx^K}\right)_{x=x_0} \tag{11}$$

If $f(K)$ is defined as $\frac{1}{K!}\left(\frac{d^K f}{dx^K}\right)_{x=x_0}$ then

$$f(x)=\sum_{k=0}^{\infty} f(k)(x-x_0)^K \tag{12}$$

Table 1: differential transform of functions

Function	Transformed function
$f(x)=u(x)\pm v(x)$	$f(k)=u(k)\pm v(k)$
$f(x)=au(x)$	$f(k)=au(k)$
$f(x)=\frac{d^n u(x)}{dx^n}$	$f(k)=\frac{(k+n)!}{k!}u(k+n)$
$f(x)=\frac{du(x)}{dx}.\frac{du(x)}{dx}$	$f(k)=\sum_{r=0}^{k}(k-r+1)(r+1)u(r+1)u(k-r+1$
$f(x)=u(x)\frac{d^2u}{dx^2}$	$f(k)=\sum_{r=0}^{k}(k-r+2)(k-r+1)u(r)u(k-r+$
$f(x)=\exp(ax)$	$f(k)=\frac{a^k}{k!}$

Differential Transform Method of our problem

Recall equation (9) and (10)

$$f'''+\frac{m+1}{2}ff''-mf'^2+e^{-\eta}=0$$

$$subject\ to$$

$$f'(0)=1,\ \ f(0)=f_W,\ \ f'(\infty)=0$$

The differential transform is given by

$$\frac{(k+3)!}{k!}F(k+3)+\frac{m+1}{2}\sum_{r=0}^{k}(k-r+1)(k-r+2)F(r)F(k-r+2)$$

$$-m\sum_{r=0}^{k}(r+1)(k-r+1)F(r+1)F(k-r+1)+\frac{(-1)^k}{k!}=0 \tag{13}$$

$$F(1)=1,\ F(0)=F_W,\ \sum_{k=0}^{n}k(\beta)^{k-1}F(k)=0, F(2)=a \tag{14}$$

Where $a=\frac{f''(0)}{2!}$

4.0 Results and Discussion

The nonlinear equation (13) and (14) were solved using a computer program written in maple. We have the first seven terms for $F_W = -1/2$ as follows because of space

$$F(0) = -1/2, F(1) = 1, \ F(2) = \frac{160659}{7951446},$$

$$F(3) = \frac{ma}{12} + \frac{a}{12} + \frac{m}{6} - \frac{1}{6}, \qquad F(4) = \frac{m^2 a}{192} + \frac{13ma}{96} + \frac{m^2}{96} - \frac{7a}{192} + \frac{1}{32}$$

$$F(5) = \frac{m^3 a}{3840} + \frac{43m^2 a}{1920} + \frac{m^3}{1920} + \frac{19ma}{3840} - \frac{29m}{1920} + \frac{17m^2}{1920} + \frac{ma^2}{20} - \frac{23a}{3840} + \frac{1}{640} - \frac{a^2}{60}$$

$$F(6) = \frac{m^4 a}{92160} + \frac{13m^3 a}{23040} + \frac{m^4}{46080} + \frac{127m^2 a}{9216} - \frac{m^2}{1280} + \frac{13m^3}{23040} + \frac{11m^2 a^2}{1440} - \frac{37ma}{1536} - \frac{m}{23040}$$
$$+ \frac{ma^2}{240} + \frac{73a}{10240} - \frac{1}{9216} - \frac{a^2}{288},$$

In order to determine the value of $\frac{f''(0)}{2!}$ we substitute the first 12 terms 0f the series into the boundary condition at infinity, the resulting expression is then evaluated for a at various values of m.

For

$$m = -1/3 \ \ we \ have \ F(0) = -1/2, F(1) = 1, \ F(2) = \frac{160659}{7951446}, F(3) = -\frac{283409}{1325241},$$

$$F(4) = \frac{160657}{7951446}, F(5) = \frac{45592361719}{7805616480360}, F(6) = \frac{207762556219}{9366797764320}, F(7) = -\frac{2794659654037}{1686013159757760}$$

$$F(8) = \frac{127379267580316643}{308954466361231672320}, \ F(9) = \frac{429074505157934442 49}{50049972355051953091 5840},$$

$$F(10) = \frac{880720876832799953 71}{428999763043302455070 7200}, \ F(11) = -\frac{14005478781528257299572737}{926490196255994233698390589440},$$

$$F(12) = -\frac{9097783563613203190306 39}{2830006417654673295660538527744}$$

The computed values of $\frac{f''(0)}{2!}$ for various values of the exponent m and suction parameter F_W are tabulated in Table.2. Since exact solutions are not available in literature, we compare our results with numerical solution obtained from the Runge Kutta shooting method.

Table 2: Computed Values of $\frac{f''(0)}{2!}$

F_W	M	Runge-Kutta	Differential Transform method
-1/2	-1/3	0.135445399	0.1506156239
5	1/3	-1.543331892	-1.534125716
-1/2	-1	Not converging	0.5470885884
½	1	-0.4226027161	-0.4225141461

4.1 Results and Discussion

Table.2 shows a numerical comparison of the differential transform method with the classical fourth order Runge-Kutta shooting method. Various values of the skin friction coefficient were computed for different suction parameter and the exponent m. It observed that the two values are closely related, it is also observed that the differential transform method converges for all values tested while the Runge-Kutta shooting methods does not. In Fig.1-4 the dimensionless velocity profile for various values of the suction parameter is also depicted. It is discovered that the profile follows the usual boundary layer profile, which shows that the differential transform method is an efficient method of solving the boundary layer equations**.**

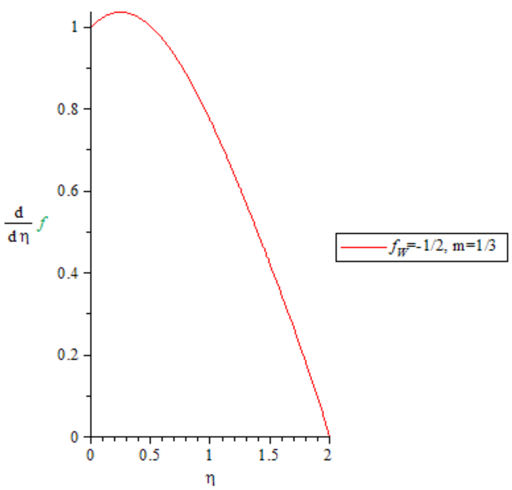

Fig 1: Dimensionless velocity profile

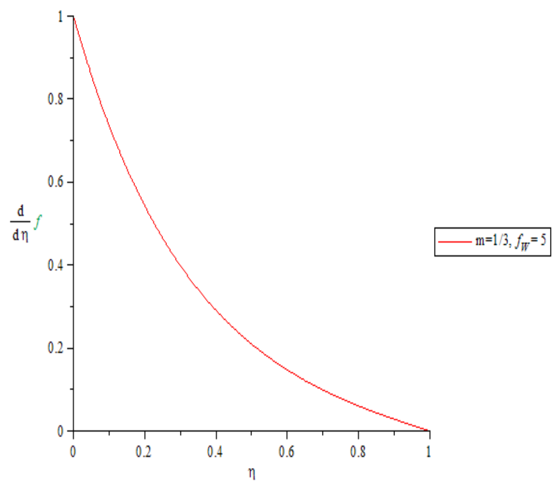

Fig 2.dimensionless velocity profile

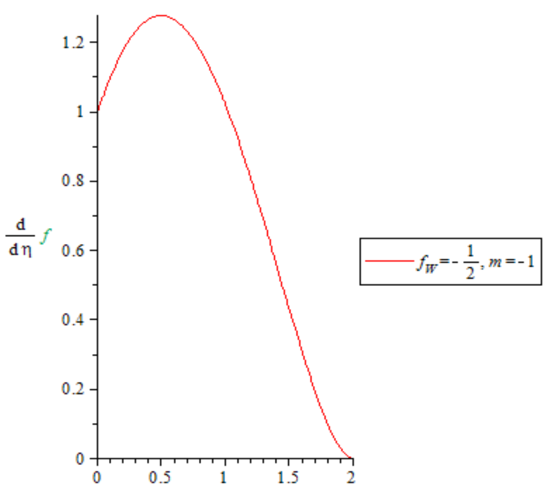

Fig 3 Dimensionless velocity profile

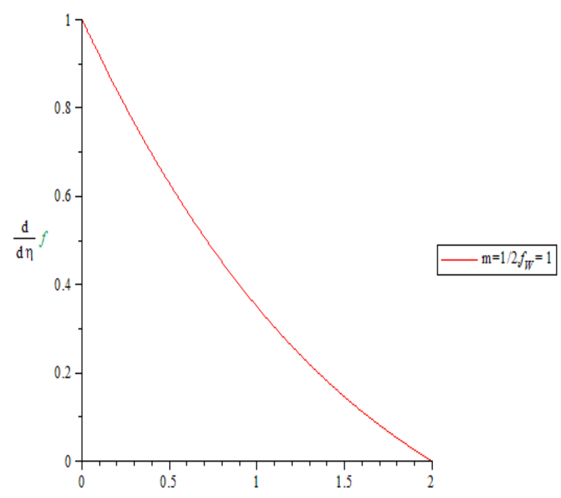

Fig 4: Dimensionless velocity profile

5.0 Conclusion

In this paper, we applied the differential transform method to solve the boundary layer heat transfer over a vertical plate with internal heat generation, The results obtained by the application of differential transform method is very reliable. The method is simple and easy to implement. It requires no restrictions of large and small parameters. It avoids massive computational complexity encountered in other numerical techniques such linearization, discretization and perturbation.

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