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Adaptive Control and Synchronization of a new Uncertain Hyperchaotic Systems with Application to Secure Communication

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Abstract

This work investigates control and synchronization of chaos of a new uncertain hyperchaotic system using the adaptive control method. By Lyapunov stability theory, the adaptive control laws and parameters update laws are derived to ensure stable chaos control and chaos synchronization of the two new identical hyperchaotic systems evolving from different initial conditions. Adaptive synchronization result is computationally demonstrated for secure information transmission. The effectiveness of all the results are verified by numerical simulations.

1.0 Introduction

Chaotic phenomenon occurs naturally in many physical, biological, engineering and social systems. Chaotic phenomenon could be beneficial in some applications, however, it is undesirable in many engineering and other physical applications and should therefore be controlled to improve the system performance. Chaos control is concerned with using some designed control input to modify the characteristics of a parameterized nonlinear system so that the system becomes stable at a chosen position or track a desired trajectory. The control could be static feedback, dynamics feedback or open-loop feedback [1].

Chaos synchronization is an important subject both theoretically and practically for applications requiring; oscillation out of chaos; machine and building structural stability analysis; chaos generators design and so on. Chaos synchronization first describe by Fujisaka and Yamada [2] in 1983, did not received great attention until 1990 [3]. From then on, chaos synchronization has been developed extensively due to its various applications [4-7]. During the last decades, many techniques for handling chaos synchronization have been developed, such as Pecora and Carroll method [3], OGY method [5], feedback approach [8], adaptive methods [9-13], time-delay-feedback approach [14], active control [15], backstepping design technique [16], impulsive control [17] etc.

Most of the works mentioned so far deal mainly with low dimension chaotic systems with one positive Lyapunov exponent. Hyperchaotic systems possessing at least two positive Lyapunov exponents have more complex behaviour and abundant dynamics than chaotic systems and are more suitable for engineering applications such as secure communication. Hence, how to realize hyperchaotic systems synchronization is interesting and challenging work. Fortunately, some existing methods of synchronizing low dimension chaotic systems like adaptive control, active control, active backstepping control, sliding mode control methods can be generalized to synchronize hyperchaotic systems [18-26]. In practical situations, parameters are probably unknown and may change from time to time. Therefore, how to effectively synchronize hyperchaotic systems with fully unknown parameters is an important problem for theoretical research and practical applications. Among different methods of synchronizing two hyperchaotic systems, adaptive control method is an effective one for achieving synchronization of hyperchaotic systems with fully unknown parameters [27].

Despite the numerous advantages of adaptive synchronization of chaotic systems its application to synchronization of hyperchaotic sytems with application to secure communication has not been adequately explored. In this paper, we consider adaptive control and synchronization of a new hyperchaotic system with uncertain parameters with application to secure communication. The organization of the paper is as follows. Section 2 deals with the system description, section 3 and 4 deal with adaptive control and adaptive synchronization respectively. Section 5 deals with secure communication while section 6 concludes the paper.

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2.0 Description of the mathematical model.

The mathematical model of interest in this paper is the new hyperchaotic system described by the following four-dimensional continuous-time autonomous hyperchaotic systems:

$$\begin{aligned} \dot{x} &= a(y-x)+u \\ \dot{y} &= cx - xz \\ \dot{z} &= -bz + xy \\ \dot{u} &= -yz + du \end{aligned} \qquad (1)$$

Where a, b, c, and d are constant parameters, x, y, z, u are the state variables with the overdots representing differentiation with respect to time. The new hyperchaotic system (1) was constructed in [28] from the three-dimensional autonomous chaotic system [29].

As reported in [28], system (1) exhibits hyperchaotic dynamics with two positive Lyapunov exponents: $LE_1 = 0.3331$, $LE_2 = 0.2301$ for parameter values $a = 10, b = \frac{8}{3}, c = 16$ and $d = -0.3$ with the hyperchaotic attractors shown in Fig.1

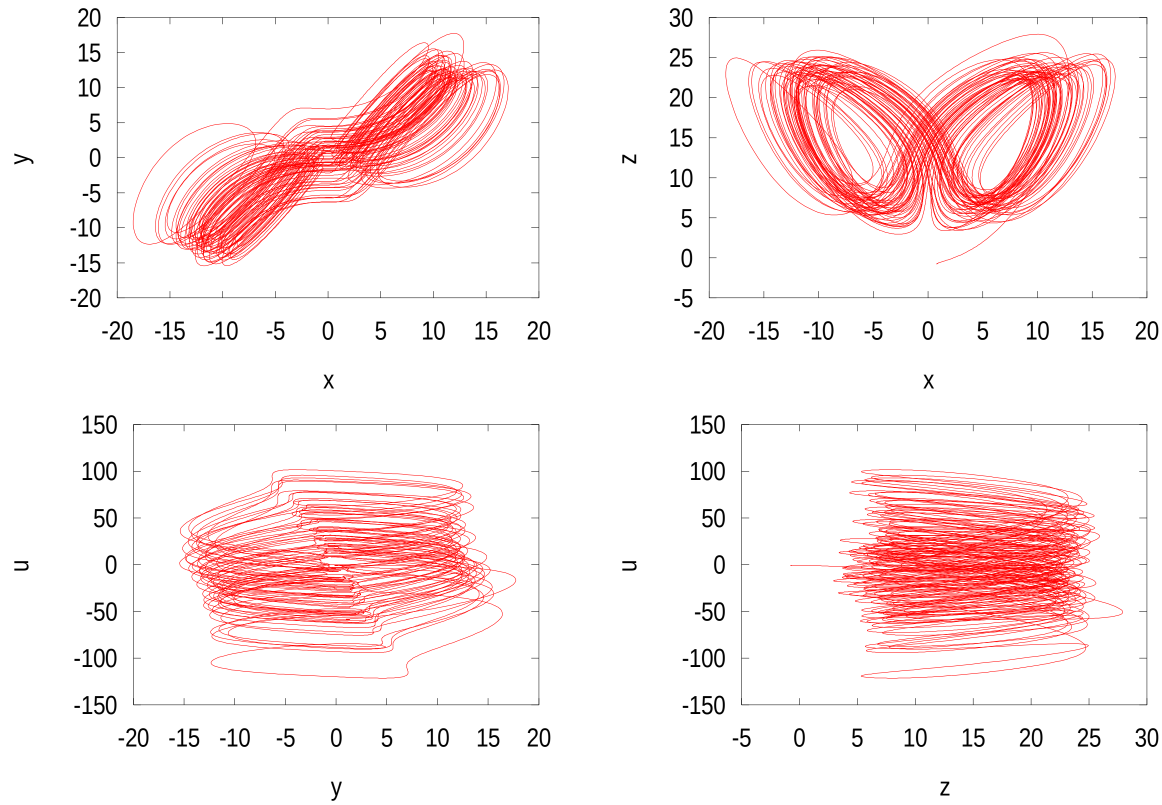

Fig.1. Phase portraits of hyperchaotic attractors of (1) with $a = 10, b = \frac{8}{3}, c = 16, and\ d = -0.3$

3.0 Stabilization of the new hyperchaotic system

3.1 Design of adaptive controllers for stabilization

In this section, the hyperchaotic dynamics of system (1) is suppressed to an unstable equilibrium E_0 in the presence of unknown parameters. The adaptive control theory is applied to achieving this goal. Consider the following controlled system as follow

$$\begin{aligned} \dot{x} &= a(y-x)+u+\mu_1 \\ \dot{y} &= cx - xz + \mu_2 \\ \dot{z} &= -bz + xy + \mu_3 \\ \dot{u} &= -yz + du + \mu_4 \end{aligned} \qquad (2)$$

Where $a, b, c,$ and d are unknown parameters, $\mu_1, \mu_2, \mu_3,$ and μ_4 are the controllers to be designed. According to the Lyapunov stability theory, we choose the following Lyapunov function

$$V = 1/2(x^2 + y^2 + z^2 + u^2 + \tilde{a}^2 + \tilde{b}^2 + \tilde{c}^2 + \tilde{d}^2) \quad (3)$$

In which $\tilde{a} = \bar{a} - a, \tilde{b} = \bar{b} - b, \tilde{c} = \bar{c} - c,$ and $\tilde{d} = \bar{d} - d,$ $\tilde{a}, \tilde{b}, \tilde{c}, \tilde{d}$ are the estimate values of these unknown parameters.

The time derivative of V along the trajectory of system (2) is

$$\dot{V} = x\dot{x} + y\dot{y} + z\dot{z} + u\dot{u} + \tilde{a}\dot{\tilde{a}} + \tilde{b}\dot{\tilde{b}} + \tilde{c}\dot{\tilde{c}} + \tilde{d}\dot{\tilde{d}} \quad (4)$$

Substituting (2) into (4) we have

$$\begin{aligned}
\dot{V} &= x(a(y-x) + u + \mu_1) + y(cx - xz + \mu_2) \\
&\quad + z(-bz + xy + \mu_3) + u(-yz + du + \mu_4) \\
&\quad + \tilde{a}\dot{\tilde{a}} + \tilde{b}\dot{\tilde{b}} + \tilde{c}\dot{\tilde{c}} + \tilde{d}\dot{\tilde{d}} \\
&= x(\bar{a}(y-x) + u + \mu_1) + y(\bar{c}x - xz + \mu_2) \\
&\quad + z(-\bar{b}z + xy + \mu_3) + u(-yz + \bar{d}u + \mu_4) \\
&\quad + \tilde{a}(\dot{\tilde{a}} - xy + x^2) + \tilde{b}\left(\dot{\tilde{b}} + z^2\right) + \tilde{c}(\dot{\tilde{c}} - xy) + \tilde{d}\left(\dot{\tilde{d}} - u^2\right).
\end{aligned}$$

According to the Lyapunov stability theory, the condition $\dot{V} < 0$ can ensure that the controlled system (2) converges to the origin asymptotically. To guarantee the time derivation of the Lyapunov function $\dot{V}$ be negative, choose the controllers $u_i (i = 1,2,3,4)$ as follows

$$\begin{aligned}
\mu_1 &= \bar{a}(x-y) - u - x \\
\mu_2 &= xz - \bar{c}x - y \\
\mu_3 &= \bar{b}z - xy - z \\
\mu_4 &= yz - \bar{d}u - u
\end{aligned} \quad (5)$$

And the following parameter estimation update law

$$\begin{aligned}
\dot{\tilde{a}} &= xy - x^2 - \tilde{a} \\
\dot{\tilde{b}} &= -z^2 - \tilde{b} \\
\dot{\tilde{c}} &= xy - \tilde{c} \\
\dot{\tilde{d}} &= u^2 - \tilde{d}
\end{aligned} \quad (6)$$

With the choice of (5) and (6), the time derivation of the Lyapunov function becomes

$$\dot{V} = -x^2 - y^2 - z^2 - u^2 - \tilde{a}^2 - \tilde{b}^2 - \tilde{c}^2 - \tilde{d}^2 < 0$$

Since the Lyapunov function V is positive definite and its derivative $\dot{V}$ is negative definite in the neighborhood of the zero solution for system (2). Therefore, based on the Lyapunov stability theory, the controlled system (2) can asymptotically converge to the unstable equilibrium $E_0(0,0,0,0)$ with controllers (5) and parameter estimation update law (6). Fig. 2 shows the time responses of the four state variables of the controlled system (2).

3.2 Numerical Results

Using the fourth order Runge-Kutta algorithms with initial conditions $(x, y, z, u) = (0.028, 0.02, 0.03, 0.04)$, a time step of 0.005 and fixing the parameter values of a, b, c, d as in Fig.1 to ensure hyperchaotic dynamics of the state variables, we solve system (2) with the adaptive control law (5) and the parameter estimation update law (6). The results obtained show that the state variables move hyperchaotically with time when the controller is switched off, and when the controller is switched on at $t = 50s$ the state variables are controlled to stabilized at the origin, Fig.2

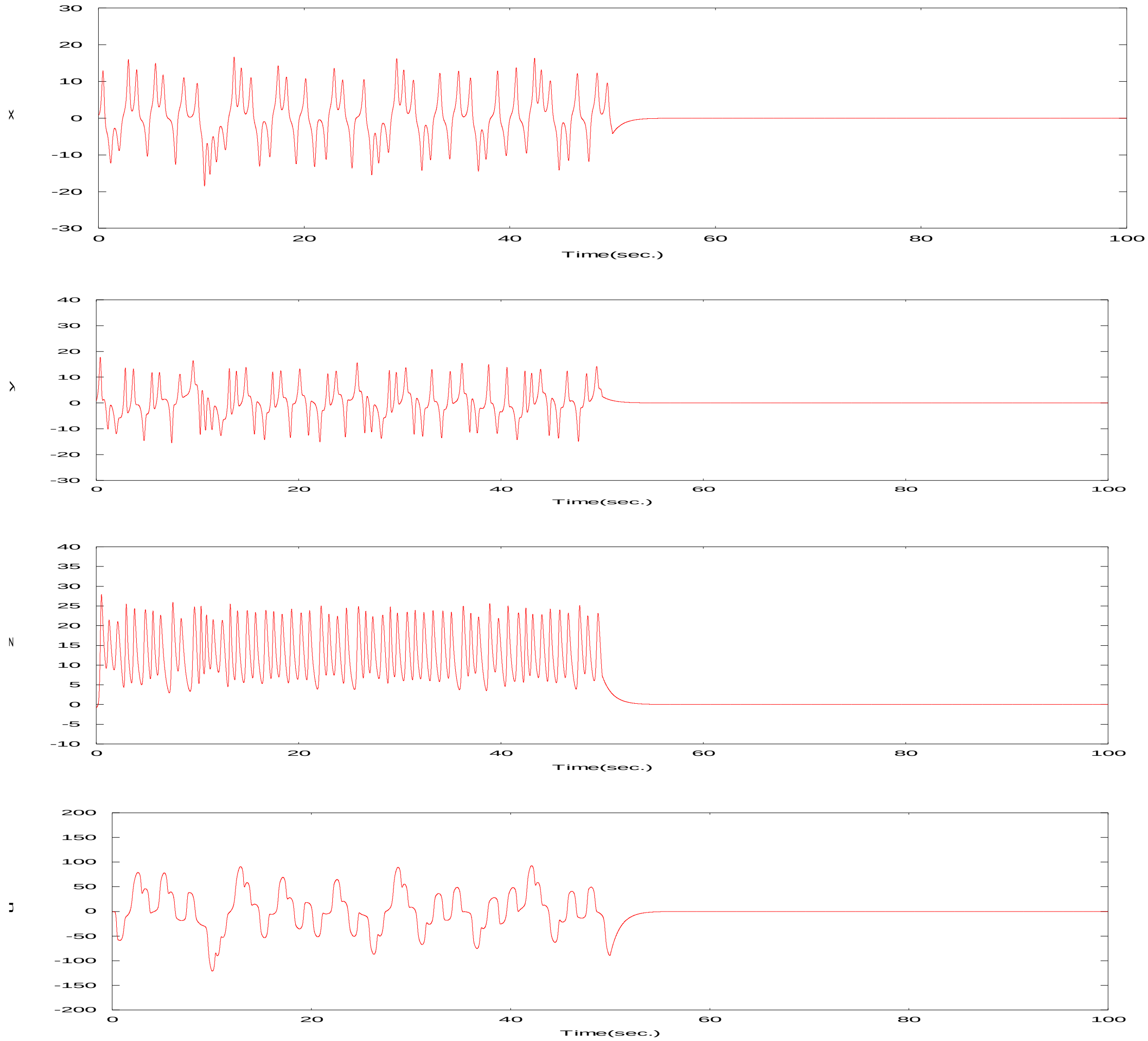

Fig. 2: Time responses of state variables (x, y, z, u) stabilizing at the origin when the control activated at $t = 50$

4.0 Adaptive Synchronization of Identical New Hyperchaotic Systems

In this section, adaptive synchronization between two identical new hyperchaotic systems [28] with unknown parameters is achieved based on the Lyapunov stability theory and adaptive control.

4.1 Design of the adaptive controller

We assume that there are two new hyperchaotic systems and that the drive system with the subscript 1 is to control the response system with subscript 2. The drive and the response systems are:

$$\begin{aligned} \dot{x}_1 &= a(y_1 - x_1) + u_1 \\ \dot{y}_1 &= cx_1 - x_1 z_1 \\ \dot{z}_1 &= -bz_1 + x_1 y_1 \\ \dot{u}_1 &= -y_1 z_1 + du_1 \end{aligned} \tag{7}$$

and

$$\begin{aligned} \dot{x}_2 &= a(y_2 - x_2) + u_2 + \mu_1 \\ \dot{y}_2 &= cx_2 - x_2 z_2 + \mu_2 \\ \dot{z}_2 &= -bz_2 + x_2 y_2 + \mu_3 \\ u_2 &= -y_2 z_2 + du_2 + \mu_4 \end{aligned} \tag{8}$$

Where a, b, c, d are unknown parameters, which need to be estimated, $\mu_1, \mu_2, \mu_3, and\ \mu_4$ are the controllers which are to be designed such that the two hyperchaotic systems can synchronized with each other.

Subtracting drive system (7) from response system (8) yields the following error dynamical system:

$$\begin{aligned}
\dot{e}_1 &= a(e_2 - e_1) + e_4 + \mu_1 \\
\dot{e}_2 &= ce_1 - e_1e_2 - z_1e_1 - x_1e_3 + \mu_2 \\
\dot{e}_3 &= -be_3 + e_1e_2 + x_1e_2 + y_1e_1 + \mu_3 \\
\dot{e}_4 &= de_4 - e_2e_3 - z_1e_2 - y_1e_3 + \mu_4
\end{aligned} \tag{9}$$

Where $e_1 = x_2 - x_1,\ e_2 = y_2 - y_1,\ e_3 = z_2 - z_1,\ e_4 = u_2 - u_1$

Choose the following Lyapunov function

$$V = 1/2(e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_a^2 + e_b^2 + e_c^2 + e_d^2) \tag{10}$$

In which n

$$e_a = a - \bar{a},\ e_b = b - \bar{b},\ e_c = c - \bar{c},\ e_d = d - \bar{d}. \tag{11}$$

$\bar{a}, \bar{b}, \bar{c}, \bar{d}$ are the estimate values of these unknown parameters $a, b, c, and\ d$, respectively.

Note that

$$\dot{e}_a = -\dot{\bar{a}},\ \dot{e}_b = -\dot{\bar{b}},\ \dot{e}_c = -\dot{\bar{c}},\ \dot{e}_d = -\dot{\bar{d}} \tag{12}$$

The time derivation of the Lyapunov function along the trajectory is

$$\dot{V} = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4 + e_a\dot{e}_a + e_b\dot{e}_b + e_c\dot{e}_c + e_d\dot{e}_d \tag{13}$$

Substituting Eqs.(9), (11) and (12) into Eq.(13), we have

$$\begin{aligned}
\dot{V} &= e_1(\bar{a}(e_2 - e_1) + e_4 + \mu_1) + e_2(\bar{c}e_1 - e_1e_3 - z_1e_1 - x_1e_3 + \mu_2) \\
&+ e_3(-\bar{b}e_3 + e_1e_2 + x_1e_2 + y_1e_1 + \mu_3) + e_4(\bar{d}e_4 - e_2e_3 - z_1e_2 - y_1e_3 + \mu_4) \\
&+ e_a(e_1(e_2 - e_1) - \dot{\bar{a}}) + e_b\left(-e_3^2 - \dot{\bar{b}}\right) + e_c(e_1e_2 - \dot{\bar{c}}) + e_d(e_4^2 - \dot{\bar{d}})
\end{aligned}$$

According to the Lyapunov stability theory, to ensure the error dynamical system converges to the origin asymptotically, the condition $\dot{V} < 0$ should be satisfied. So we choose the following controllers:

$$\begin{aligned}
\mu_1 &= \bar{a}(e_1 - e_2) - e_4 - e_1 \\
\mu_2 &= -\bar{c}e_1 + e_1e_3 + z_1e_1 + x_1e_2 - e_2 \\
\mu_3 &= \bar{b}e_3 - e_1e_2 - x_1e_2 - y_1e_1 - e_3 \\
\mu_4 &= -\bar{d}e_4 + e_2e_3 + z_1e_2 + y_1e_3 - e_4
\end{aligned} \tag{14}$$

And the parameter estimation updates laws

$$\begin{aligned}
\dot{\bar{a}} &= e_1(e_2 - e_1) + e_a \\
\dot{\bar{b}} &= -e_3^2 + e_b \\
\dot{\bar{c}} &= e_1e_2 + e_c \\
\dot{\bar{d}} &= e_4^2 + e_d
\end{aligned} \tag{15}$$

With the choices of (14) and (15), the time derivation of the Lyapunov function $\dot{V}$ becomes

$$\dot{V} = -e_1^2 - e_2^2 - e_3^2 - e_4^2 - e_a^2 - e_b^2 - e_c^2 - e_d^2 < 0$$

In the light of the Lyapunov stability theory, the error dynamical system can converge to the origin asymptotically. Consequently, the drive system (7) is synchronous asymptotically with the response system (8) with the adaptive controllers (14) and the parameter estimation update laws (15).

4.2 Numerical Results

To verify the effectiveness of the proposed synchronization scheme, we simulate the dynamics of the drive system and the response system. In the simulation, the fourth order Runge-Kutta algorithm is empolyed with time step of 0.005 and fixing the parameter values $a = 10, b = \frac{8}{3}, c = 16, and\ d = -0.3$ to ensure hyperchaotic dynamics of the state variables without control, we solved (7) and (8) with the control functions as defined in (14). The initial conditions of the drive system and the response are $(-1.0, -1.0, -1.0, -1.0) and\ (1.0, 1.0, 1.0, 1.0)$, respectively. The initial condition of the parameter update law is $(5, 5, 10, 5)$.

The results shows that the error state variables move hyperchaotically with time when the controller is switched off and when the controller is switched on at $t = 20\ Fig$ state variables converge to zero, thereby guaranteeing the synchronization of systems (7) and (8). The estimated values of the unknown parameters converges to $a = 10, b = \frac{8}{3}, c = 16, and\ d = -0.3$ as $t \to \infty$, respectively.

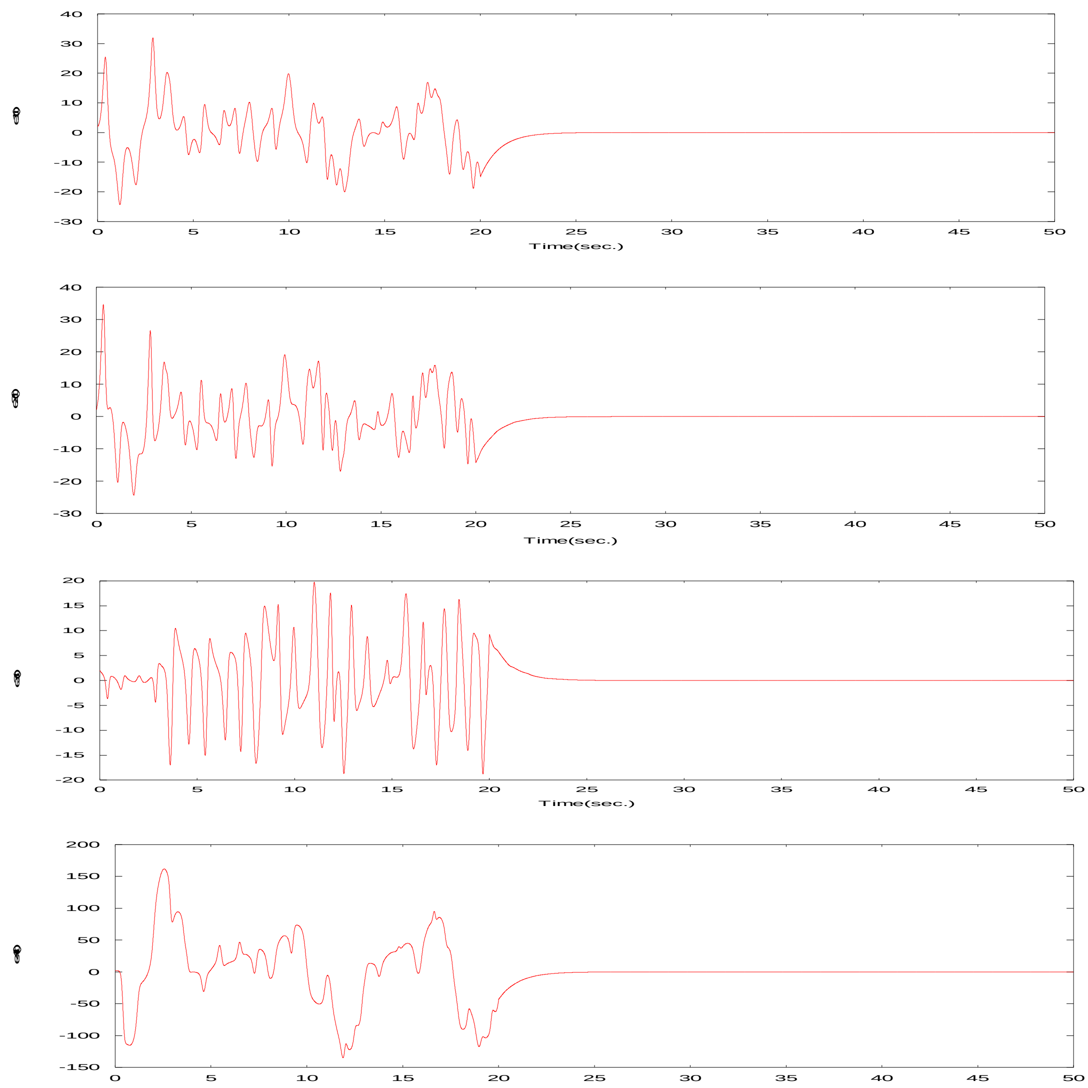

Fig. 3. Error dynamics between the two new hyperchaotic systems with the controller deactivated for $0 < t < 20$ and activated for $t \geq 20$

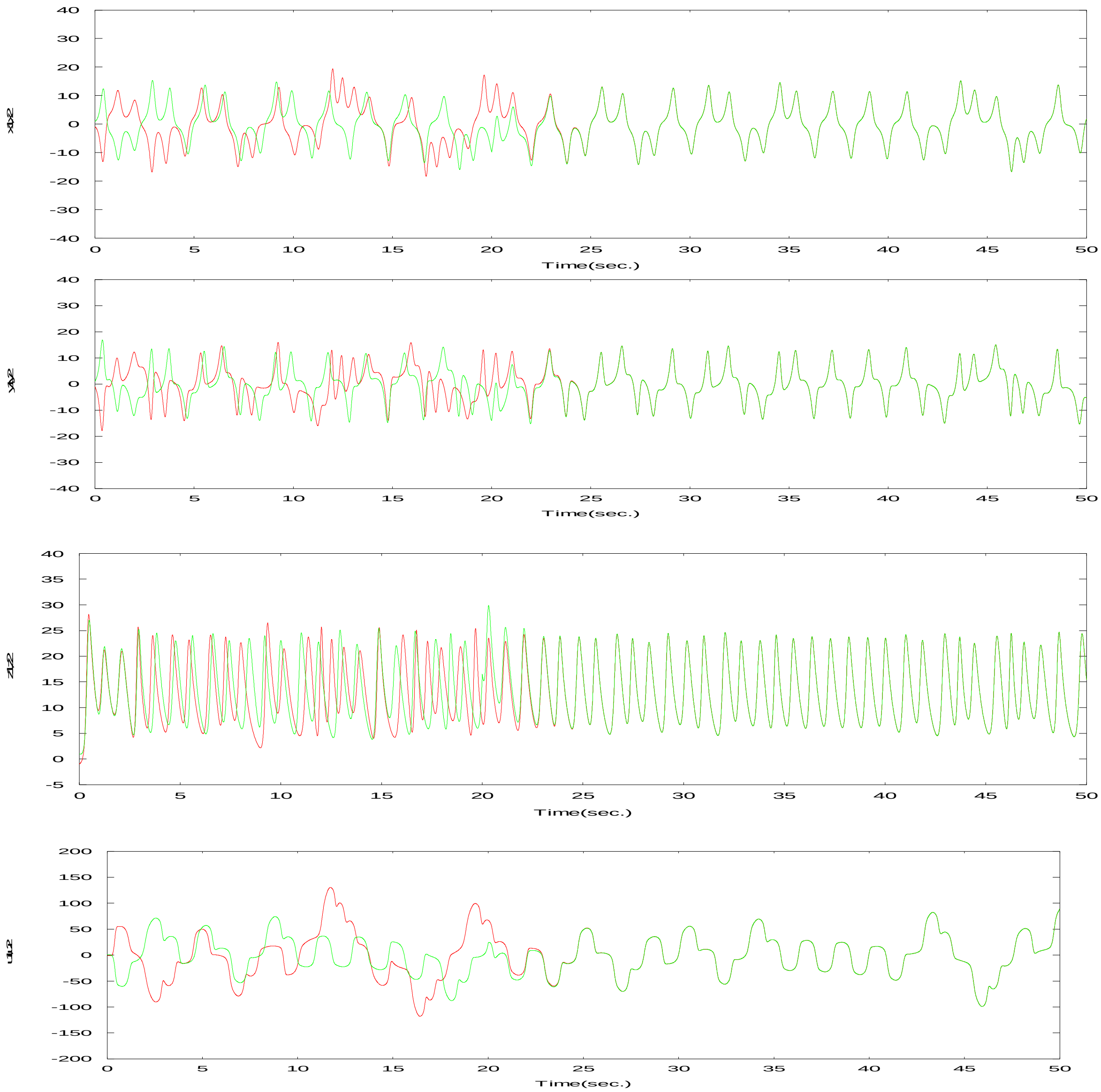

Fig. 4 Time evolution of the drive system (red colour) and the response system (green colour)

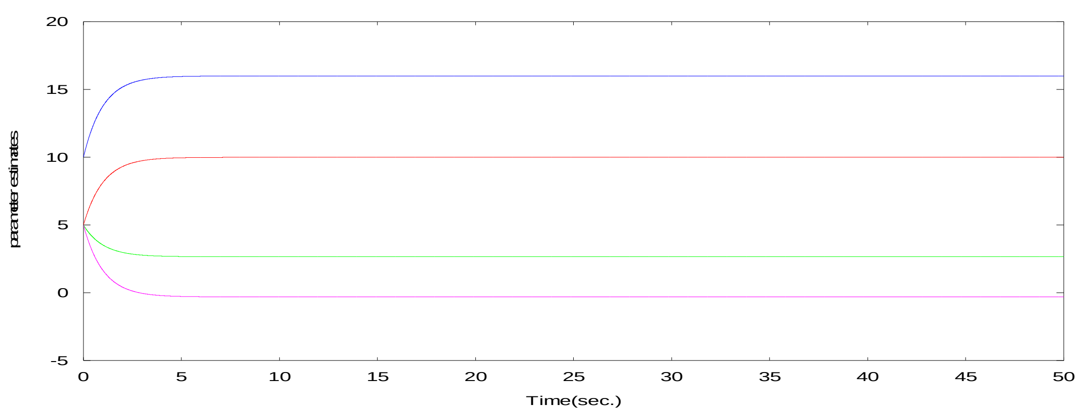

Fig. 5. Time series of the response system parameters

5.0 Secure information transmission via adaptive synchronization

The basic idea of chaos secure communication is based on using chaotic nonlinear oscillator as a broadband signal generation. The signal is combined with message to produce unpredictable signal which is transmitted from the transmitter to the receiver. At the receiver the pseudo-random is generated through the inverse operation, original message is recovered. In order for this scheme to properly work, the receiver must synchronize robustly enough so as to admit the small perturbation in the drive signal due to the addition of the message. The power of information signal must be lower than that of chaotic signal to effectively bury the information signal. The signal from the master serves two purposes: to control the slave system so as to synchronize it with the master and to carry the information signal, just like any other communication scheme, the purpose of chaos secure communication is to hide message during transmission. The suitability of chaotic systems for application in secure communication is based on the feature of chaotic carriers such as: broadband or wide spectrum (which reduces the fading of the signal and increase the transmission capacity); orthogonality (which reduces signal distortion); sensitivity to slight changes in the initial conditions and system parameter as well complexity and noise-likeness dynamics which lead to unpredictability, thereby making extraction of hidden message difficult [30]. The secret keys are the set of value of the system parameters and since the system parameter are real number, the number of possible keys is infinite, thereby, enhancing confidentiality.

In this chaotic masking scheme, encryption is achieved by mixing by information signal with the chaotic carrier signal using mixing algorithm which is simply a function of information and chaotic carrier signals. So far many mixing algorithm have been proposed to achieve chaotic masking: some of which are additive masking; multiplicative masking etc [28]. Here we demonstrate our secure communication scheme using the additive encryption masking scheme. The information signal is chosen to be a periodic function $sm = 5\sin 2t$, with this choice the chaotic carrier x_2 remain chaotic. The encrypted information is given by the masking algorithm $se = sm - 2x_2$. Consequently, the decrypted information rm is given by the inverse function $rm = se + 2y_2$. The chaotic signal x_2 of the master is transmitted to the slave via a coupling channel for synchronization between the master and the slave, the information signal $sm = 5\sin 2t$ is masked in the encrypted signal em and transmitted to the receiver. The decrypted information rm is extracted by inverse function. The block diagram for the communication scheme is shown in Fig. 6 while, the numerical simulation results for secure information transmission is shown in Fig. 7.

TRANSMITTER **RECEIVER**

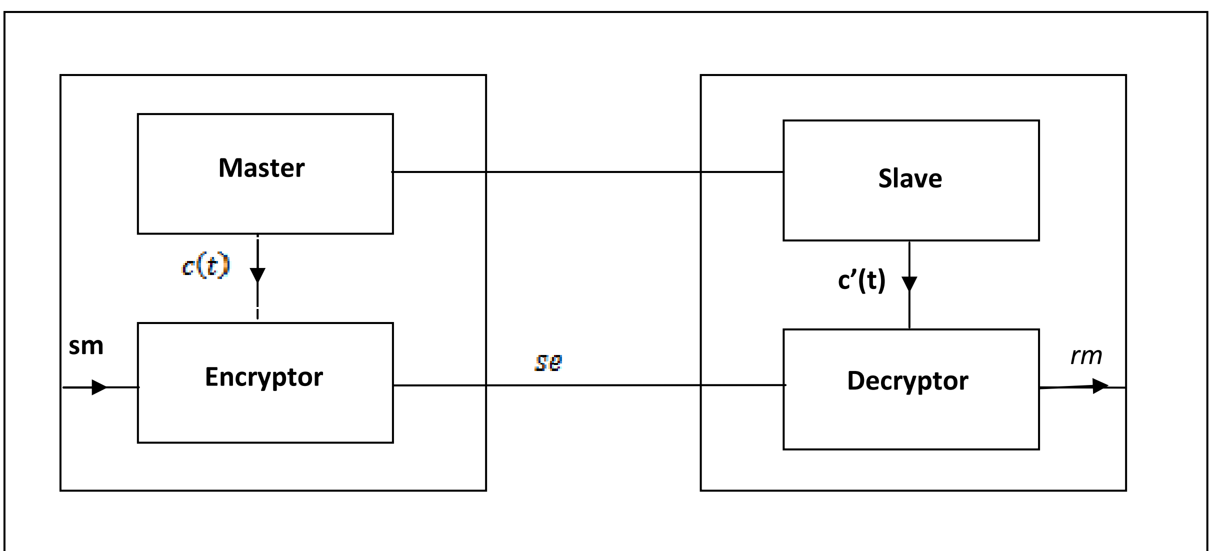

Fig. 6: Block diagram of a typical chaotic communication system**.**

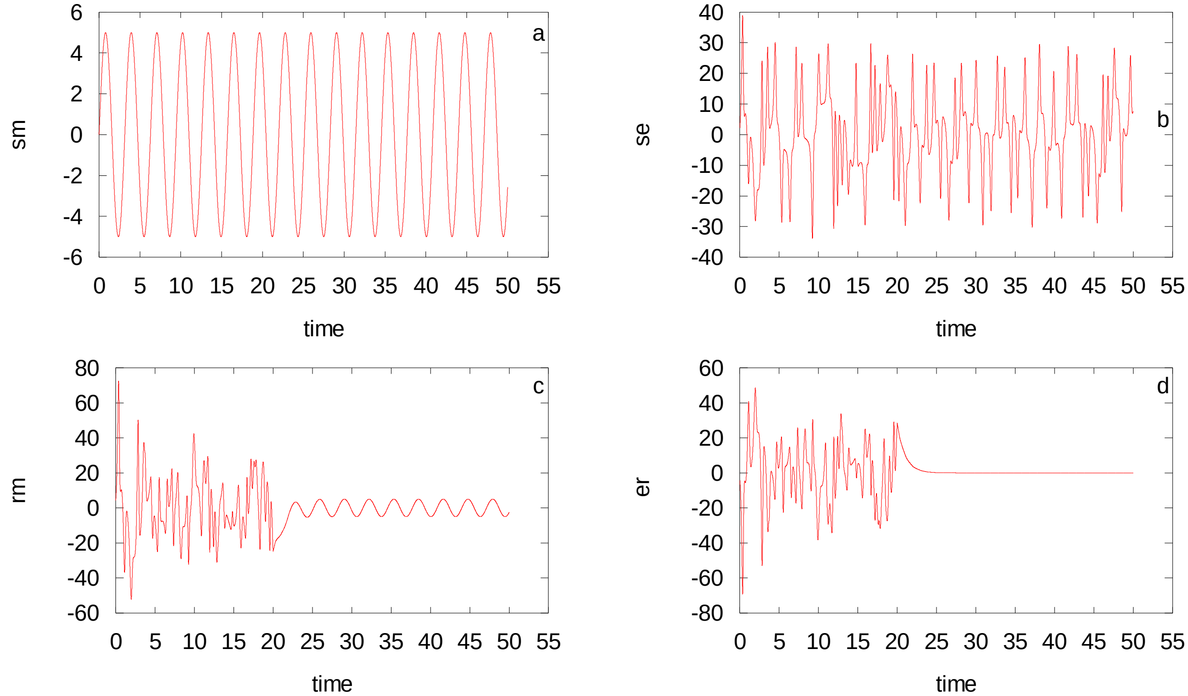

Fig.7: (a) original information sm; (b) encrypted signal se; (c) decrypted signal rm; (d) decrypted error er-rm

6.0 Conclusion

We investigate control and synchronization of chaos of a new uncertain hyperchaotic system using the adaptive control method. By Lyapunov stability theory, the adaptive control laws and parameters update laws are derived to ensure stable chaos control and chaos synchronization of the two new identical hyperchaotic systems evolving from different initial conditions. Adaptive synchronization result is computational demonstrated for secure information transmission. The effectiveness of all the results are validated by numerical simulations.

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