# Erratum: Development of Methods For Determining The Lateral Surface of Tank 

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#### Abstract

The names of the Authors in the inside title header in this paper were wrongly typesetted in the vol. 18 issue of the Journal of NAMP. The entire article is therefore reproduced below as it ought to appear in pages 165-174 (Vol. 18).


## Abstract


#### Abstract

Ensuring the security and life of civil engineering structures is paramount to the designer, the users and the environment. To this end, it is necessary to carry out periodic monitoring of structures. The monitoring of Civil engineering structures is not limited to Geomatics Engineers only. Other professional are also involved in structural monitoring with a view of ensuring its safety and integrity. In the paper we develop a Geomatics technique of structural deformation study. The approach divides the storage tank with diameter of 76.2 m and 22 m high into circular cross section with points distributed to cover the perimeter of the cross section. These monitoring points (studs) were situated at equal distances on the outer surface of the tanks and located around the tank base. Geodetic Total Station instruments were setup at these monitoring stations (occupied stations) and observations carried out to determine the coordinates of monitoring points on the tank surface. In the past traditional geodetic was used which was time consuming, and problematic data acquisition and analysis. The new technique is very fast, cost effective, continuous data acquisition, compactable with computer Aided design (CAD), and the easy of developing digital terrain model (DTM) with the data. We also presented the mathematical model and least squares technique necessary for the deformation measurement and analysis.


Key words: Monitoring, deformation, diameter, oil volume, intersection, accuracy, oil tanks.

### 1.0 Introduction:

In the past, conventional classical methods of deformation measurement has been used which was time consuming, not economic and observation is associated with errors. During the last decade, the world of engineering surveying has seen enormous developments in the techniques for spatial data acquisition. One of these developments has been the appearance of geodetic Total station, global positioning system (GPS) and laser scanner device [4].

As a result of tanks age, geological of the area, non uniform settlement of tanks foundations, loading and off loading of oil and temperature of the crude will cause stress and strain of tanks membrane and settlement of sediments. The tanks tend to undergo radial deformation or out of roundness. Therefore, monitoring the structural deformation of the circular oil storage tanks must be carried out using accurate geodetic observations and analysis methods.

Historically, different methods have been used to monitor the deformations of large structures. New monitoring techniques and methodologies emerge as new technology is developed and enhanced, for example, the combination of a total station with image based measurement systems or laser scanners [1]. Each monitoring scheme has unique advantages, disadvantages, and limitations whether it is based on traditional geodetic surveying techniques, geotechnical measurements, the global positioning system (GPS), or remote sensing principles [6, 7, 8]. The cost, effectiveness, and reliability of a monitoring scheme are important factors in the decision to implement a certain monitoring system over another. Among geodetic techniques, the Total Station provides a reliable tool for automated and continuous (if required) monitoring of large structures at a relatively low cost.

Most deformation monitoring schemes consist of measurements made to the monitored object that are referred to several reference points (assumed to be stable) [3].

To obtain correct object point displacements (and thus deformations), the stability of the reference points must be ensured [1]. The main conclusion from the many papers written on this topic states that every measurement made to a monitored object must be connected to stable control points. This is accomplished by creating a reference network of control points surrounding a particular structure (Figure 1).
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### 2.0 MONITORING OF VERTICAL STORAGE TANKS

The tanks under study are designed with floating roof plate of thickness 6.0 mm were constructed in the 70 s with the following properties (tables 1 and 2)

Table 1: Tank property

| Nominal <br> Diameter | Temperature | Norminal <br> Volume | Height | Liquid <br> Gravity | Hydrostatic <br> pressure |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 76.2 m | $58^{0} \mathrm{f}$ | $100,000 \mathrm{~m}^{3}$ | 22 m | 0.85 to 0.9 | 2 bars |

Table 2: Tank Thickness

| Tank Segment | Plate Thickness |
| :--- | :--- |
| Bottom Plate | 6.0 mm |
| $1^{\text {st }}$ plate | 34.5 mm |
| $2^{\text {nd }}$ plate | 30.6 mm |
| $3^{\text {rd }}$ plate | 26.7 mm |
| $4^{\text {th }}$ plate | 22.7 mm |
| $5^{\text {th }}$ plate | 19.0 mm |
| $6^{\text {th }}$ plate | 15.0 mm |
| $7^{\text {th }}$ plate | 11.3 mm |
| $8^{\text {th }}$ plate | 10.0 mm |
| $9^{\text {th }}$ plate | 10.0 mm |

It is necessary to model the structure of oil storage tank by using well-chosen discrete monitoring points located on the surface of the structure at different levels which, when situated correctly, accurately depict the characteristics of the structure.

Any movements of the monitoring point locations (and thus deformations of the structure) can be detected by maintaining the same point locations over time and by performing measurements on them at specified time intervals. This enables direct point displacement comparisons to be made. A common approach for this method is to place physical targets on each chosen discrete point on which measurements can be made. However, there are certain situations in which monitoring the deformations of a large structure using direct displacement measurements of targeted points is uneconomical, unsafe, inefficient, or simply impossible. The reasons for this limitation vary, but it may simple due to the difficulty or cost of placing permanent prisms on the structure [3].

To obtain the correct object point displacements (and thus its deformation), the stability of the reference stations and control points must be ensured. The main conclusion from the many papers written on this topic states that every measurement made to a monitored object must be connected to stable control points [1]. This is accomplished by creating a reference network of control points surrounding a particular structure (Figure 1).

To develop a reliable and cost effective monitoring system of any of the storage oil tanks, the deformation monitoring scheme consisted of measurements made on the tanks from several monitoring stations (occupied stations), which are chosen in the area around the tank, and that are referred to several reference control points [4]. The geodetic instruments are setup at these monitoring stations (occupied stations) and observations are carried out to determine the coordinates of monitoring points on the tank surface.

The circular cross section of the oil storage tank is divided into several monitoring points distributed to cover the perimeter of this cross section, as shown in Figure 1. These monitoring points are situated at equal distances on the outer surface of the tank. The (stud) points are fixed, with each stud carrying an identification number and made permanent throughout the life of the tank. The purpose is to maintain the same monitoring point during each epoch of observation.
From Figure 1, (A to I) are monitoring stations, B.M.1, B.M. 2 and B.M. 3 are control net work while number ( 1 to 20) are studs permanently attached to the tank surface for monitoring. To determine the coordinates of occupied stations around the monitored oil storage tank, traverse network was run from the control points around the vicinity of the tank to connect the monitoring stations.

The easiest way of visualizing the traversing process around the tank is to consider it as the formation of a polygon on the ground using standard survey procedures. The traverse was being measured using total station. The slope distances and horizontal angles were measured to survey stations on both faces for a given number of rounds, and recorded accordingly. Appropriate corrections were applied, and the distances reduced to horizontal distance. There are a total number of 18 tanks monitored, in this work; traverse network around tank № 8 is presented in fig 1.

To determine the coordinates of the eight occupied stations, a closed loop traverse was designed around tank № 8 as shown in Fig 1.


Figure 1 - Structural deformation monitoring system
In this closed traverse there are 9 interior angles and 9 side lengths. The observed interior angles and sides of the traverse loop together with computed accuracy using Calson2011 software are presented in table 3 to table 8 .

### 3.0 COMPUTATION AND ADJUSTMENT OF OBSERVATIONS

By using least square theory, method of condition equation adjustment technique was used and is presented thus:
The number of total observations $(\mathrm{n})=10$ angles +10 distances
This gives total number of (20) observations
The number of conditions ( r$)=3$ and these include:

1. Angular misclosure condition:
$\Delta_{1}=($ sum interior angle of loop traverse $)-\left(n_{\text {angles }}-2\right)\left(180^{\circ}\right)$
2. Sum of the departures is equal to zero:

$$
\begin{equation*}
\Delta_{2}=\sum_{i=1}^{n} D * \operatorname{Sin} \theta_{i}=0 \tag{1}
\end{equation*}
$$

Where $D_{i}$ - the length of traverse side, $\theta_{i}$ - bearing of traverse side
3. Sum of the latitude is equal to zero:

$$
\begin{equation*}
\Delta_{3}=\sum_{i=1} D_{i} * \cos \theta_{i}=0 \tag{3}
\end{equation*}
$$

Hence, the number of necessary observations ( $\mathrm{n}_{0}$ )

$$
\begin{equation*}
\mathrm{n}_{0}=\mathrm{n}-\mathrm{r}=17 \tag{4}
\end{equation*}
$$

The first step in solving traverse using conditional least square is finding the adjusted values of observations (9 interior angles and 9 lengths) and its accuracy. Secondly, from these values and accuracies, the adjusted coordinates of the traverse stations (eight occupied points) and its accuracy can be determined depending on the geometry of the traverse figure. All of these steps were carried out using Carlson2011 program. The adjusted coordinates of the traverse stations are presented in table 4.

Table 3 - Least - square solution of Tank 8 observations
Process Transit Results
Raw file: C:/Carlson Projects/TANKS/TANK8/TANK8.rw5
Coordinate file: C:/Carlson Projects/TANKS/TANK8/TANK8.crd
Scale Factor: 1.00000000
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Correct for Earth Curvature: OFF
Closure Results (Before Angle Balance)
Starting Point 2: E 324951.639 N 148189.825 Z 0.000
Closing Reference Point BM-8A: E 325174.013 N 148157.213 Z 0.000
Ending Point 11: E 325174.098 N 148157.340 Z -0.000
Azimuth Of Error: $33^{\circ} 51^{\prime} 59{ }^{\prime \prime}$
North Error : 0.02717
East Error : 0.08535
Vertical Error : -0.00000
Hz Dist Error : 0.15316
Sl Dist Error : 0.15316
Traverse Lines : 9
SideShots : 0
Store Points : 0
Horiz Dist Traversed: 708.171
Slope Dist Traversed: 708.171
Closure Precision: 1 in 46240
Starting Point 2: E 324951.639 N 148189.825 Z 0.000
BackSight Point 1: E 325174.013 N 148157.213 Z 0.000

| Point | Horizontal | Zenith | Slope | Inst | Rod | Easting | Northing | Elev |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. | Angle | Angle | Dist | HT | HT |  |  |  |
| Description |  |  |  |  |  |  |  |  |
| 3 | AR268.3816 | 90.0000 | 65.966 | 0.000 | 0.000 | 324959.656 | 148255.302 | -0.000 |
| PEG1 |  |  |  |  |  |  |  |  |
| 4 | AR196.4010 | 90.0000 | 36.402 | 0.000 | 0.000 | 324974.259 | 148288.647 | -0.000 |
| PEG2 |  |  |  |  |  |  |  |  |
| 5 | AR220.2333 | 90.0000 | 58.134 | 0.000 | 0.000 | 325026.529 | 148314.092 | -0.000 |
| PEG3 |  |  |  |  |  |  |  |  |
| 6 | AR231.3346 | 90.0000 | 58.504 | 0.000 | 0.000 | 325079.287 | 148288.808 | -0.000 |
| PEG4 |  |  |  |  |  |  |  |  |
| 7 | AR210.5030 | 90.0000 | 64.962 | 0.000 | 0.000 | 325115.192 | 148234.670 | -0.000 |
| PEG5 |  |  |  |  |  |  |  |  |
| 8 | AR2 44.0404 | 90.0000 | 70.803 | 0.000 | 0.000 | 325079.241 | 148173.673 | -0.000 |
| PEG6 |  |  |  |  |  |  |  |  |
| 9 | AR2 48.3307 | 90.0000 | 75.559 | 0.000 | 0.000 | 325004.626 | 148185.581 | -0.000 |
| PEG7 |  |  |  |  |  |  |  |  |
| 10 | AR175.3321 | 90.0000 | 53.088 | 0.000 | 0.000 | 324951.710 | 148189.860 | -0.000 |
| BM-8B |  |  |  |  |  |  |  |  |
| 11 | AR3. 4147 | 90.0000 | 224.753 | 0.000 | 0.000 | 325174.098 | 148157.340 | -0.000 |
| BM-8A |  |  |  |  |  |  |  |  |

Table 4-Adjusted Point Comparison

| Original |  | Adjusted |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Point\# | Easting | Northing | Easting | Northing | Dist | Bearing |
| 3 | 324959.656 | 148255.302 | 324959.655 | 148255.275 | 0.027 | S 02* ${ }^{\circ} 1^{\prime \prime} 01^{\prime \prime}$ |
| 4 | 324974.259 | 148288.647 | 324974.256 | 148288.606 | 0.040 | S 05*00'28" |
| 5 | 325026.529 | 148314.092 | 325026.517 | 148314.041 | 0.052 | S 12059'27" |
| 6 | 325079.287 | 148288.808 | 325079.267 | 148288.747 | 0.064 | S $18^{\circ} 05^{\prime} 40^{\prime \prime}$ |
| 7 | 325115.192 | 148234.670 | 325115.166 | 148234.587 | 0.086 | S $17^{\circ} 05^{\prime} 11^{\prime \prime}$ |
| 8 | 325079.241 | 148173.673 | 325079.210 | 148173.566 | 0.112 | S 16605'42" |
| 9 | 325004.626 | 148185.581 | 325004.583 | 148185.469 | 0.120 | S 2046'34" |
| 10 | 324951.710 | 148189.860 | 324951.660 | 148189.746 | 0.125 | S $24^{\circ} 01^{\prime \prime} 25^{\prime \prime}$ |
| 11 | 325174.098 | 148157.340 | 325174.013 | 148157.213 | 0.153 | S 33051'59" |

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Table 5 - Control Points

| Point\# | Easting | Northing |
| :--- | :---: | :--- |
| 1 | 325174.013 | 148157.213 |
| 2 | 324951.639 | 148189.825 |
| 11 | 325174.013 | 148157.213 |
|  |  |  |
| Adjust | Points |  |
| Point\# | Easting | Northing |
| 3 | 324959.656 | 148255.302 |
| 4 | 324974.259 | 148288.647 |
| 5 | 325026.529 | 148314.092 |
| 6 | 325079.287 | 148288.808 |
| 7 | 325115.192 | 148234.670 |
| 8 | 325079.241 | 148173.673 |
| 9 | 325004.626 | 148185.581 |
| 10 | 324951.710 | 148189.860 |

Table 6 - Distance Observations and standard errors

| Occupy | FSight | Distance | StdErr |
| :--- | :--- | :--- | :--- |
| 2 | 3 | 65.966 | 0.011 |
| 3 | 4 | 36.402 | 0.011 |
| 4 | 5 | 58.134 | 0.011 |
| 5 | 6 | 58.504 | 0.011 |
| 6 | 7 | 64.962 | 0.011 |
| 7 | 8 | 70.803 | 0.011 |
| 8 | 9 | 75.559 | 0.011 |
| 9 | 10 | 53.088 | 0.011 |
| 10 | 11 | 224.753 | 0.011 |

Table 7 - Angle Observations and standard Errors

| BSight | Occupy | FSight | Angle | StdErr |
| :--- | :--- | :--- | :--- | :--- |
| 1 | 2 | 3 | $268^{\circ} 38^{\prime} 16^{\prime \prime}$ | $24.571^{\prime \prime}$ |
| 2 | 3 | 4 | $196^{\circ} 40^{\prime} 10^{\prime \prime}$ | $54.569^{\prime \prime}$ |
| 3 | 4 | 5 | $20^{\circ} 23^{\prime} 33^{\prime \prime}$ | $55.434^{\prime \prime}$ |
| 4 | 5 | 6 | $231^{\circ} 33^{\prime} 46^{\prime \prime}$ | $41.162^{\prime \prime}$ |
| 5 | 6 | 7 | $210^{\circ} 50^{\prime} 30^{\prime \prime}$ | $40.282^{\prime \prime}$ |
| 6 | 7 | 8 | $244^{\circ} 04^{\prime} 04^{\prime \prime}$ | $34.575^{\prime \prime}$ |
| 7 | 8 | 9 | $248^{\circ} 33^{\prime} 07^{\prime \prime}$ | $31.723^{\prime \prime}$ |
| 8 | 9 | 10 | $175^{\circ} 33^{\prime} 21^{\prime \prime}$ | $40.749^{\prime \prime}$ |

Table 8- Least-Squares Closure

| Control Points |  |  |  |
| :---: | :---: | :---: | :---: |
| Point\# | Easting | Northing |  |
| 1 | 325174.013 | 148157.213 |  |
| 2 | 324951.639 | 148189.825 |  |
| 11 | 325174.013 | 148157.213 |  |
| Distance Observations |  |  |  |
| Occupy | FSight | Distance | StdErr |
| 2 | 3 | 65.966 | 0.011 |
| 3 | 4 | 36.402 | 0.011 |
| 4 | 5 | 58.134 | 0.011 |
| 5 | 6 | 58.504 | 0.011 |
| 6 | 7 | 64.962 | 0.011 |
| 7 | 8 | 70.803 | 0.011 |
| 8 | 9 | 75.559 | 0.011 |
| 9 | 10 | 53.088 | 0.011 |
| 10 | 11 | 224.753 | 0.011 |


| Angle Observations |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| BSight | Occupy | FSight | Angle | StdErr |
| 1 | 2 | 3 | 268*38'16" | 24.571" |
| 2 | 3 | 4 | 196** $40^{\prime \prime} 1^{\prime \prime}$ | $54.569^{\prime \prime}$ |
| 3 | 4 | 5 | 220으'33" | 55.434 " |
| 4 | 5 | 6 | 231*33'46" | $41.162^{\prime \prime}$ |
| 5 | 6 | 7 | 210ㅇํ ${ }^{\prime \prime} 30^{\prime \prime}$ | 40.282" |
| 6 | 7 | 8 | 244* $04^{\prime \prime} 04^{\prime \prime}$ | $34.575^{\prime \prime}$ |
| 7 | 8 | 9 | 248*33'07" | $31.723^{\prime \prime}$ |
| 8 | 9 | 10 | 175*3 $33^{\prime \prime} 1^{\prime \prime}$ | $40.749^{\prime \prime}$ |
| 9 | 10 | 11 | $3^{\circ} 41^{\prime \prime} 47^{\prime \prime}$ | 27.671" |

Adjusted Point Comparison

| Original |  | Adjusted |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Point\# | Easting | Northing | Easting | Northing | Dist | Bearing |
| 3 | 324959.656 | 148255.302 | 324959.658 | 148255.301 | 0.002 | S 56¹1'32" |
| 4 | 324974.259 | 148288.647 | 324974.267 | 148288.642 | 0.009 | S 57 ${ }^{\circ} 00^{\prime} 45^{\prime \prime}$ |
| 5 | 325026.529 | 148314.092 | 325026.545 | 148314.066 | 0.031 | S 31*35'45" |
| 6 | 325079.287 | 148288.808 | 325079.290 | 148288.755 | 0.052 | S 02033'11" |
| 7 | 325115.192 | 148234.670 | 325115.162 | 148234.597 | 0.079 | S $22^{\circ} 05^{\prime} 09^{\prime \prime}$ |
| 8 | 325079.241 | 148173.673 | 325079.172 | 148173.622 | 0.086 | S 52\%54'12" |
| 9 | 325004.626 | 148185.581 | 325004.564 | 148185.577 | 0.062 | S 86\% ${ }^{\circ} 0^{\prime \prime} 07^{\prime \prime}$ |
| 10 | 324951.710 | 148189.860 | 324951.650 | 148189.893 | 0.069 | N 61¹6'24" |


| Adjusted | Points |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| Point\# | Easting | Northing | N-StdErr | E-StdErr |
| 3 | 324959.658 | 148255.301 | 0.010 | 0.007 |
| 4 | 324974.267 | 148288.642 | 0.013 | 0.012 |
| 5 | 325026.545 | 148314.066 | 0.018 | 0.018 |
| 6 | 325079.290 | 148288.755 | 0.025 | 0.017 |
| 7 | 325115.162 | 148234.597 | 0.029 | 0.014 |
| 8 | 325079.172 | 148173.622 | 0.027 | 0.016 |
| 9 | 325004.564 | 148185.577 | 0.031 | 0.013 |
| 10 | 324951.650 | 148189.893 | 0.040 | 0.011 |

Solution Converged in 3 Iterations
Reference Standard Deviation: 0.945
Chi-Square statistic: 1.786, Range for 95\%: 0.103 to 5.990
Adjustment Passes Chi-Square test at $95 \%$ confidence level
Max adjustment: 0.086
Starting Point 2: E 324951.639 N 148189.825 Z 0.000
Backsight Point 1: E 325174.013 N 148157.213 Z 0.000

| Point <br> No. <br> Descrip | Horizontal <br> Angle <br> n | Zenith <br> Angle | Slope <br> Dist | Inst HT | Rod <br> HT | Easting | Northing | Elev |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | AR2 68. 3821 | 90.0000 | 65.965 | 0.000 | 0.000 | 324959.658 | 148255.301 | -0.000 |
| PEG1 |  |  |  |  |  |  |  |  |
| 4 | AR196.4045 | 90.0000 | 36.401 | 0.000 | 0.000 | 324974.267 | 148288.642 | -0.000 |
| PEG2 |  |  |  |  |  |  |  |  |
| 5 | AR220.2414 | 90.0000 | 58.132 | 0.000 | 0.000 | 325026.545 | 148314.066 | -0.000 |
| PEG3 |  |  |  |  |  |  |  |  |
| 6 | AR231.3409 | 90.0000 | 58.503 | 0.000 | 0.000 | 325079.290 | 148288.755 | -0.000 |
| PEG4 |  |  |  |  |  |  |  |  |
| 7 | AR210.5047 | 90.0000 | 64.962 | 0.000 | 0.000 | 325115.162 | 148234.597 | -0.000 |
| PEG5 |  |  |  |  |  |  |  |  |
| 8 | AR2 44.0412 | 90.0000 | 70.804 | 0.000 | 0.000 | 325079.172 | 148173.622 | -0.000 |
| PEG6 |  |  |  |  |  |  |  |  |
| 9 | AR2 48.3311 | 90.0000 | 75.560 | 0.000 | 0.000 | 325004.564 | 148185.577 | -0.000 |
| PEG7 |  |  |  |  |  |  |  |  |
| 10 | AR175.3331 | 90.0000 | 53.089 | 0.000 | 0.000 | 324951.650 | 148189.893 | -0.000 |
| BM-8B |  |  |  |  |  |  |  |  |
| 11 | AR3.4153 | 90.0000 | 224.752 | 0.000 | 0.000 | 325174.013 | 148157.213 | -0.000 |

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By the same way, the coordinates of occupied stations around each oil storage tank of ten studied tanks in the studied area in Forcados Terminal Nigeria were determined. It is important to note that the number of monitoring points on the tank surface and the number of occupied stations around each tank differ from one tank to another depending on the topography and visibility around [3].

### 3.1 Coordinates of tank surface points by linear-angular 2D intersection

To achieve accurate determination of coordinates of monitoring points on the outer surface of oil tank at Forcados terminal and its accuracy during the process of structural deformation monitoring, linear-angular intersection was used. This is because it has the advantages of least squares application. In this case, four observations were carried out from the two occupied stations (two distances and two angles). In angular intersection or linear intersection, the number of observations (two angles or two distances) equals the number of unknowns (coordinates of point P ) but in case of linear-angular intersection the number of observations is more than the number of unknowns, and consequently least square method must be used to determine the coordinates of point P (Figure 2). Figure (2) illustrates the geometry of the linear-angular intersection. There are two known coordinates points $\left(\mathrm{X}_{\mathrm{A}}, \mathrm{Y}_{\mathrm{A}}\right)$ and $\left(\mathrm{X}_{\mathrm{B}}, \mathrm{Y}_{\mathrm{B}}\right)$. From these two known points (A and B), we can determine the coordinates of unknown point $P$; $\left(X_{P}, Y_{P}\right)$ by measuring horizontal angles $\alpha_{1}$ and $\alpha_{2}$ and horizontal distances $S_{1}$ and $S_{2}$. Adjustment will be carried out in this case by using observation equation method.


Figure 2a, 2b and $\mathbf{2 c}$ - Geometry of linear - angular intersection for determining point coordinates
It is important to note that the horizontal distances $S_{1}, S_{2}$ was measured by using reflectorless total station. Modern total station has reflectorless ability, so it can measure the inclined distance and horizontal distance without prisms.
In this model of adjustment (observational least square), the number of equations equals the number of observations ( $n=4$ ), every equation contains one observation and one or more than one unknowns. In this case, the observations are $S_{1}, S_{2}, \alpha_{1}, \alpha_{2}$ and the unknowns are $\mathrm{X}_{\mathrm{P}}, \mathrm{Y}_{\mathrm{P}}$.
The two lengths of the lines $\left(S_{1}, S_{2}\right)$ in horizontal projection can be written in coordinates form as:

$$
\left.\begin{array}{rl}
S_{1} & =\sqrt{\left(X_{P}-X_{A}\right)^{2}+\left(Y_{P}-Y_{A}\right)^{2}}  \tag{5}\\
S_{2} & =\sqrt{\left(X_{P}-X_{B}\right)^{2}+\left(Y_{P}-Y_{B}\right)^{2}}
\end{array}\right\}
$$

From figure (2), the horizontal angles ( $\alpha_{1}$ and $\alpha_{2}$ ) can be calculated as follows:

$$
\begin{align*}
& \alpha_{1}=\cos ^{-1}\left(\frac{A P^{2}+A B^{2}-P B^{2}}{2 \cdot A P \cdot A B}\right) \\
& \alpha_{2}=\cos ^{-1}\left(\frac{B A^{2}+B P^{2}-A P^{2}}{2 \cdot B A \cdot B P}\right) \tag{6}
\end{align*}
$$

By using the coordinates of points, we can write equation (2) as:

$$
\left.\begin{array}{l}
\alpha_{1}=\cos ^{-1}\left[\frac{\left(X_{P}-X_{A}\right)^{2}+\left(Y_{P}-Y_{A}\right)^{2}+A B^{2}-\left(X_{P}-X_{B}\right)^{2}-\left(Y_{P}-Y_{B}\right)^{2}}{2 A B \sqrt{\left(X_{P}-X_{A}\right)^{2}+\left(Y_{P}-Y_{A}\right)^{2}}}\right]  \tag{7}\\
\alpha_{2}=\cos ^{-1}\left[\frac{\left(X_{P}-X_{B}\right)^{2}+\left(Y_{P}-Y_{B}\right)^{2}+A B^{2}-\left(X_{P}-X_{A}\right)^{2}-\left(Y_{P}-Y_{A}\right)^{2}}{2 A B \sqrt{\left(X_{P}-X_{B}\right)^{2}+\left(Y_{P}-Y_{B}\right)^{2}}}\right]
\end{array}\right\}
$$

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The equations (5) and (7) are the four observational equations, these equations are nonlinear function of both parameters and observations; they can be treated by least squares adjustment technique. The first step in the solution is finding the approximate values of unknowns. The approximated values (input data) of coordinates of point $\mathbf{P}$ (Vector $X^{0}$ ) can be assumed by using angular intersection according to the following formulae [3]:

$$
\begin{align*}
& X_{P}^{0}=\frac{X_{A} \cot \alpha_{2}+X_{B} \cot \alpha_{1}-Y_{A}+Y_{B}}{\operatorname{Cot} \alpha_{1}+\operatorname{Cot} \alpha_{2}} \\
& Y_{P}^{0}=\frac{Y_{A} \cot \alpha_{2}+Y_{B} \cot \alpha_{1}-X_{A}+X_{B}}{\operatorname{Cot} \alpha_{1}+\operatorname{Cot} \alpha_{2}} \tag{8}
\end{align*}
$$

By substituting these approximate values in the four observation equations, the approximate values of observations $\left(\mathrm{L}^{0}\right)$ can be computed, and then we can compute the misclosure vector ( L ) as follows:

$$
\begin{equation*}
L=L^{0}-L_{o b s} \tag{9}
\end{equation*}
$$

The linearised model may be expressed in the matrix form as follows:

$$
\begin{equation*}
\underset{(4,1)}{V}=\underset{(4,2)}{A} \cdot \underset{(2,1)}{X}+\underset{(4,1)}{L} \tag{10}
\end{equation*}
$$

Where A - the coefficient matrix of parameters with dimension (4, 2); L - The misclosure vector with dimension (4, 1); V The residuals vector with dimension $(4,1) ; \mathrm{X}$ is the unknown parameter with 2 x 1 matrix

Matrix A can be computed by differentiation of the four equations with respect to the two unknowns and can be written in the form:

$$
A_{(4,2)}=\left[\begin{array}{cc}
\frac{\partial S_{1}}{\partial X_{P}} & \frac{\partial S_{1}}{\partial Y_{P}}  \tag{11}\\
\frac{\partial S_{2}}{\partial X_{P}} & \frac{\partial S_{2}}{\partial Y_{P}} \\
\frac{\partial \alpha_{1}}{\partial X_{P}} & \frac{\partial \alpha_{1}}{\partial Y_{P}} \\
\frac{\partial \alpha_{2}}{\partial X_{P}} & \frac{\partial \alpha_{2}}{\partial Y_{P}}
\end{array}\right]
$$

By using MathCAD program, the elements $\left(\mathrm{a}_{\mathrm{ij}}\right)$ of the matrix $(\mathbf{A})$ can be found by differentiating the four observation equations.
Then, the normal equation system is written thus:

Where,

$$
\begin{align*}
& \underset{(2,2)}{N} \cdot \stackrel{\stackrel{A}{X}}{(2,1)}+\underset{(2,1)}{U}=0  \tag{12}\\
& \underset{(2,2)}{N}=\underset{(2,4)}{A} \cdot \underset{(4,4)}{W} \cdot{\underset{(4,2)}{A}}_{A}^{A} \tag{13}
\end{align*}
$$

And

$$
\begin{equation*}
\underset{(2,1)}{U}=\underset{(2,4)}{A^{T}} \cdot \underset{(4,4)}{W} \cdot \underset{(4,1)}{L} \tag{14}
\end{equation*}
$$

Subscript $(2,1)$ are the dimension of matrix $U$
The solution for normal equation is

$$
\begin{equation*}
\stackrel{\Lambda}{X}_{(2,1)}^{\Lambda}=-\underset{(2,2)}{N^{-1}} \cdot \underset{(2,1)}{U} \tag{15}
\end{equation*}
$$

Where $\stackrel{\hat{X}}{ }$ is the solution for normal equation
Then, the adjusted unknown parameters can be estimated as:

$$
\begin{equation*}
\underset{(2,1)}{\bar{X}}=\underset{(2,1)}{\hat{X}}+\underset{(2,1)}{X}{ }^{0} \tag{16}
\end{equation*}
$$

Where $\bar{X}$ is adjusted unknown parameter
The vector of adjusted observations can be estimated as:

$$
\begin{equation*}
\underset{(4,1)}{\bar{L}}=\underset{(4,17}{L}+\underset{(4,1)}{V} \tag{17}
\end{equation*}
$$

Where $\stackrel{\Lambda}{V}$ is the adjusted residual, $\stackrel{(4,1)}{L}$ is the adjusted vector observation
The estimated variance factor is:

$$
\begin{equation*}
\sigma_{0}^{2}=\frac{V^{T} \cdot W \cdot V}{r}=\frac{V^{T} \cdot W \cdot V}{2} \tag{18}
\end{equation*}
$$

The estimated variance covariance matrix of parameters is:

$$
\begin{equation*}
C_{X}=\sigma_{0 \cdot N}^{2} \cdot N^{-1} \tag{19}
\end{equation*}
$$

Finally, the variance covariance matrix of the adjusted observations can be computed as:

$$
\begin{equation*}
C_{L}=A \cdot C_{X} \cdot A^{T} \tag{20}
\end{equation*}
$$

By using MathCAD program, the above normal equation can be solved.
The error in point position $M_{p}$ can then be determined by using the following formula:

$$
\begin{equation*}
M_{P}=\frac{b m_{\alpha}^{\prime \prime}}{\rho^{\prime \prime} \sin \gamma_{1}} \sqrt{\sin { }_{\alpha_{1}}^{2}+\sin { }_{\beta_{1}}^{2}}, \tag{21}
\end{equation*}
$$

Where b - base line (the distance between occupied stations). For example $\mathrm{b}=\mathrm{AB}$ in Fig. 2; $m_{\alpha}^{\prime \prime}-$ mean square error of measuring horizontal angles (taken from specifications of applied instrument); $\rho^{\prime \prime}=206265^{\prime \prime}, \gamma_{1}$ - the horizontal angle at p . In order to accept the observations and adjusted coordinates of point $P$ from the two triangles ABP and BCP, it is necessary that the coordinates must satisfy the following condition [1].

$$
\begin{equation*}
r_{P}=\sqrt{\Delta_{X}^{2}+\Delta_{Y}^{2}} \leq 3 M_{t} \tag{22}
\end{equation*}
$$

Where $\Delta_{X}=X_{1}{ }^{P}-X_{2}^{P} ; \Delta_{Y}=Y_{1}{ }^{P}-Y_{2}{ }^{P}$ and $M_{t}=\sqrt{M_{1}{ }^{2}+M_{2}^{2}}$ and
$\left(X_{1}^{P}, Y_{1}{ }^{P}\right)$ - Coordinates of point P from first triangle (ABP); while $\left(X_{2}^{P}, Y_{2}^{P}\right)$ - Coordinates from second triangle (BCP); $\mathrm{M}_{1}, \mathrm{M}_{2}-$ Error in point position for the first and second triangles respectively $[5,6,7,8]$.
If the coordinates satisfy condition (22), the corrected coordinates of point P can be determined by the arithmetic mean of two triangles.

$$
\begin{equation*}
X_{P}=\frac{X_{1}{ }^{P}+X_{2}^{P}}{2}, \quad Y_{P}=\frac{Y_{1}{ }^{P}+Y_{2}{ }^{P}}{2} \tag{23}
\end{equation*}
$$

The accuracy of coordinates of monitoring point P can then be determined using least square method, consider the following procedure:

## CONCLUSION

Monitoring of tanks and tanks wall helps in identifying and quantifying deteriorations which may lead to tank failure. The history of tank disaster throughout the world reveals that problems often arise undetected due to inaccurate evaluation of the tank defects.

For an effective tank monitoring programme, the equipment used for the monitoring must be precise and of the highest quality. The monitoring personnel must be experienced in not only data capture but also the analysis of the acquired data. The period of observation should be every year and consistent throughout the life of the tank.

Further studies should be carried out on the tank to ascertain the character of the tank over the years. The use of the mathematical model and associated designed MATLAB program to determine the radius and coordinates of center of circular oil tanks from geodetic data especially during the process of monitoring the structural deformation was found to be very correct and economical. The period of observation should be every year and consistent throughout the life of the tank. The results obtained in this study may however be acceptable to the structural Engineer depending on the tank specifications and its properties at the design stage.

## REFERENCES

[1] Ashraf, A. Beshr. Accurate surveying measurements for smart structural members. M.Sc. Thesis. - Mansoura university. - Mansoura. - Egypt, 2004.
[2] Calson Survey 2011software with IntelliCAD 6.6
[3] Ehigiator-Irughe, R. Environmental safety and monitoring of crude oil storage tanks at the Forcados terminal. M. Eng Thesis.- Department of civil engineering, University of Benin, Benin City. Nigeria. - 2005.
[4] Gairns, C. Development of semi-automated system for structural deformation monitoring using a reflectorless total station. M.Sc. Thesis. - Department of Geodesy and Geomatics Engineering - University of New Brunswick, 2008.
[5] Ehigiator - Irughe, R. Ashraf A. A. Beshr, and Ehigiator M. O.(2010) "Structural deformation analysis of cylindrical oil storage tank using geodetic observations" (Paper Presented at Geo -Siberia 2010, International Exhibition and scientific conference VI page 34-37, Novosibrisk Russia Federation)
[6] Ehigiator - Irughe, R. Ehiorobo O. J and Ehigiator M. O.(2010) "Methods of Verticality Measurement of Crude Oil Storage Tanks" publication in International Journal of pure and Applied sciences. A pan - African Journal Series IV pp 20-23)
[7] Ehigiator - Irughe, R. and Ehigiator M. O.(2010) "Estimation of the centre coordinates and radius of Forcados Oil Tank from Total Station data using least square Analysis" International Journal of pure and Applied sciences. A pan - African Journal Series IV 24-27
[8] Ehigiator - Irughe, J.O. Ehiorobo and M.O. Ehigiator (2010) "Distortion Of Oil And Gas Infrastructure From Geomatics Support View. publication in Journal of Emerging trends in Engineering and applied sciences (Jeteas) United Kingdom.vol 1 pp 14 - 23

