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Effect Of Forchheimer Term On Pressure-Driven Flow In Composite Channelwithwall Suction/Injection

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Abstract

Solutions for the transient fully developed laminar fluid flow in the parallel-plates partially filled with a uniform porous medium and partially with a clear fluid are presented in the presence of suction and injection using numerical technique. The Brinkman-Darcy-Forchheimer extended equation is utilized to model the flow in the porous region in order to discuss the effect of inertia term on the flow. The dependence of the flow on the adjustable coefficient in the stress jump boundary condition, Darcy number, Forchheimer term and suction/injection are investigated. It is found that there is an excellent agreement with the results earlier presented.

Keywords: Forchheimer term, composite channel, suction, injection, pressure-driven

NOMENCLATURE

$u_f' =$ Dimensional Velocity in clear fluid region

$u_p' =$ Dimensional Velocity in porous medium region

$u_f =$ Non-Dimensional Velocity in clear fluid region

$u_p =$ Non-Dimensional Velocity in porous medium region

$u_i =$ Interfacial non-dimensional velocity

$V_0 =$Dimensional suction/injection velocity

Y= Non-dimensional y-coordinate

y'= Dimensional y Coordinate

$\rho =$ Density of the fluid

$\beta =$ Stress jump Coefficient

Da= Darcy number

P=Dimensionless pressure gradient

$\frac{\delta p'}{\delta z'} =$ Dimensional Pressure Gradient

$\upsilon_f =$ Kinematics Viscosity of the Fluid

$\upsilon_{eff} =$ Effective Kinematics Viscosity of the fluid saturated Porous Domain

K= Permeability of the Porous Material

s= Non dimensional suction/injection velocity

d=Interfacial position

A=Forchheimer (Inertia) term

1.0 Introduction

Fluid flow in a channel which is partially filled with a porous medium and partially with a clear fluid occurs in many practical applications. These applications include thermal insulation, crude oil extraction, nuclear waste storage, storage and dryingof grains and many other applications. The problem of the fluid flow at the porous medium/clear fluid interface was

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first investigated by[1],who utilized the Darcy law to model the flow in the porous medium. There after many results are presented in composite channels. Examples are in [2, 3] which presented an important step towards understanding fluid mechanics and heat transfer in composite region. The non-Darcian effects accounted for using the Brinkman-Forchheimer-extended Darcy equation.For the first time, exact solution for the fully developed steady flow in the interface region was presented in [3], where the fluid layer is sandwiched between a semi-infinite porous body and an external impermeable boundary was presented.Similarly, [4] presented numerical analysis of the natural convection in a squared cavity with isothermal vertical walls. The cavity is filled with a saturated porous medium, and the classical equations for natural convection, mass, momentum, and energy balance, together with Brinkman's and Forchheimer models, are used to study the phenomenon. Also [5] studied numerical solution of a laminar piston-driven flow and heat transfer in a cylinder partially filled with a laterally heated saturated porous medium. The Brinkman-Lapwood-Forchheimer-extended Darcy model, with variable porosity, is used in the compressible momentum equations. [6]Analyzed the steady magnetohydrodynamicflow of an incompressible electrically conducting visco-elasticfluidthrough a porousmedium between two porous parallel plates under the influence of a transverse magnetic fieldusing Brinkman-Forchheimer extension of Darcy's momentum equation for flow. The boundary conditions at the porous medium/clear fluid interface have been recently investigated in [7, 8]. By applying sophisticated volume averaging technique, Ochoa-Tapia and Whitaker have shown that the process of matching the Brinkman-extended Darcy law to the stoke equations requires a discontinuity in the stress jump but retains the continuity of the velocity. [9]Presented results on the studies conducted for the interface velocity in parallel-plate with both variable and constant permeability. [10] Presented the transient flow in parallel plate partially filled with porous material using numerical method. The role of the local macroscopic inertia term in the porous domain was studied. Other results presented in horizontal composite channel are in [11, 12, 13]. While in [14]result in fluid flow invoking vertical interface wasanalyzed and presented. Others are in [15, 16]. None of the results quoted have presented results considering the inertia term in modeling the problems. This paper is an extension of [10] in the presence of wall suction/injection. The condition suggested by [7, 8] and also reported by [17] was utilized to match the flows at the interface. It is also discussed the effects of parameters including the variation of Forchheimer term on the flow.

As presented in diagram I, parallel-plates channel partially filled with constant porous material and partially with clear fluid was considered. The walls assumed to have suction/injection as indicated in the illustration of diagram I. The porous medium is assumed as isotropic and homogeneous. The inertial effects in the porous region are considered. Hence, the Brinkman-

2.0 Mathematical Analysis

extended Darcy-Forchheimerequation is utilized to model the fluid flow in the porous region. While the stoke equation was utilized to model fluid flow in the clear fluid region. At the interface, the condition suggested by [7, 8] and also reported by [17] was utilized.$u_i$is denoted to be interfacial velocity and $u_i = u_f = u_p$ at the interface [18]. Under these assumptions discussed, for the one-dimensional flow process, the fluid motion in the channel is governed by

$$\frac{\partial u'_f}{\partial t'} = -\frac{1}{\rho}\frac{dP_f}{dx} + \upsilon_f \frac{\partial^2 u'_f}{\partial y'^2} + v_0 \frac{\partial u'_f}{\partial y'} \qquad (1)$$

$$\frac{\partial u_p'}{\partial t'} = -\frac{1}{\rho}\frac{dP_p}{dx} + \upsilon_{eff}\frac{\partial^2 u'_p}{\partial y'^2} - \frac{\upsilon_f}{k'}u'_p + v_0\frac{\partial u'_p}{\partial y'} - \frac{A}{\sqrt{k'}}u'_p\sqrt{u'^2_p + v_0^2} \qquad (2)$$

The boundary and initial conditions:

$$t' \le 0, \quad u'_f = 0, \quad u'_p = 0, \quad \text{at} \quad y' = 0$$

$$t' \ge 0 \quad u'_f = 0, \quad \text{at} \quad y' = 0$$

$$u'_f = u'_p \quad \text{at} \quad y' = d' \qquad \upsilon_{eff}\frac{du'_p}{dy'} - \upsilon_f\frac{du'_f}{dy'} = \upsilon_f \beta \frac{u'_p}{\sqrt{k'}} \qquad (3)$$

$$u'_p = 0, \quad \text{at} \quad y' = h$$

The following non-dimensional parameters are defined to make the Eqs. (1)- (3) non-dimensional.

$$u_f = \frac{\upsilon_f u'_f}{h}, \quad u_p = \frac{\upsilon_f u'_p}{h}, \quad y = \frac{y'}{h}, \quad d = \frac{d'}{h}, \quad \gamma = \frac{\upsilon_{eff}}{\upsilon_f}, \quad s = \frac{v_0 h}{\upsilon_f}, \quad Da = \frac{k'}{h^2}, \quad P = -\frac{1}{\rho}\frac{h^3}{\upsilon_f^2}\frac{dp}{dx},$$

$$T = \frac{t'\upsilon_f}{h^2}$$

Therefore the problem in non-dimensional form is recast as:

$$\frac{\partial u_f}{\partial T} = \frac{\partial^2 u_f}{\partial y^2} + s\frac{\partial u_f}{\partial y} + P \quad (4)$$

$$\frac{\partial u_p}{\partial T} = \gamma\frac{\partial^2 u_p}{\partial y^2} + s\frac{\partial u_p}{\partial y} - \frac{u_p}{Da} + P - \frac{A}{\sqrt{Da}}u_p\sqrt{u_p^2 + s^2} \quad (5)$$

The initial, boundary and matching conditions:

$$T \le 0,\ u_f = 0,\quad u_p = 0,\quad \text{at}\quad y = 0$$

$$T \ge 0:\quad u_f = 0\quad \text{at}\quad y = 0$$

$$u_i = u_f = u_p \quad \text{at} \quad y = d \qquad \gamma\frac{du_p}{dy} - \frac{du_f}{dy} = \beta\frac{u_p}{\sqrt{Da}} \quad (6)$$

$$u_p = 0 \quad \text{at} \quad y = 1$$

In solving the non-dimensional equations (4)-(5), the numerical method utilized by [10] and presented in [19] was utilized.

3.0 Numerical Solution Procedure

The momentum equation in the fluid and porous regions given by equations (4) – (5) are solved numerically using implicit difference method as used in [20,21, 22] .

For the sake of comparison, the same problem was solved numerically withoutwall Suction or injection i.e. S=0 as in [10]. Similarly, the same problem was again solved analytically withoutboth inertia termi.e. A=0 and Suction or injectioni.e. S=0 as in[18].Therefore, the analytical solutions when S=0 and A=0.0 areas follows:

$$u_f = -\frac{py^2}{2} + C_1 y$$

$$u_p = C_2\exp\left(\sqrt{a}\,y\right) - C_3\exp\left(-\sqrt{a}\,y\right) + a_3$$

$$u_i = \frac{a_{17} + a_{16}}{a_{15}}$$

Where

$$C_1 = \frac{a_{17} + a_{16}}{a_{15}d} + \frac{pd}{2},\quad C_2 = \frac{-\left(a_{17} + a_{16}\right)}{a_5}a_9 - a_8a_9 - a_{10},\quad C_3 = \frac{a_{17} + a_{16}}{a_5} + a_8$$

$$a = \frac{1}{\gamma Da},\ b = \frac{p}{\gamma},\ a_2 = (d-1)\sqrt{a},\ a_1 = (d-2)\sqrt{a},\ a_3 = \frac{b}{a},\ a_4 = \sqrt{a}\,d,\ a_5 = \exp(-a_4) - \exp(a_1),$$

$$a_6 = \frac{a_3\exp(a_2)}{a_5},\ a_7 = \frac{a_3}{a_5}$$

$$,a_9 = \exp\left(-2\sqrt{a}\right), a_{10} = a_3\exp\left(-\sqrt{a}\right), a_{11} = a_8a_9 + a_{10},\ a_{12} = \frac{a_9}{a_5}$$

$$a_{13} = \gamma\sqrt{a}\exp\left(\sqrt{a}\,d\right) - \frac{\beta}{\sqrt{Da}}\exp\left(\sqrt{a}\,d\right),\ a_{14} = -\gamma\sqrt{a}\exp\left(-\sqrt{a}\,d\right) - \frac{\beta}{\sqrt{Da}}\exp\left(-\sqrt{a}\,d\right)$$

$$a_{15} = -a_{13}a_{12} + \frac{a_{14}}{a_5} - \frac{1}{d},\ a_{16} = a_{11}a_{13} + a_{14}a_8,\ a_{17} = -\frac{pd}{2} + \frac{b}{a}\frac{\beta}{\sqrt{Da}}$$

4.0 Results And Discussions

The results are presented in graphs as figures in the appendix. Figure 1 (a) & (b) depict transient velocity at t=0.04 and steady velocity at large time against y for different values of s and b and fixed values of d=0.5, A=10.0 and Da=0.1 respectively. It is observed that almost the velocity is constant in the porous region for s=1.5 and b=-0.6, s=-1.5 and b=-0.6. This shows that negative value of beta, b, has impact on suction and injection of fluid in the porous region because of the effect of inertia term. It is also observed that in both figures for a fixed value of beta, b, the fluid flow is high when s=0.0. Generally, velocity is constant in the porous region for the transient state. By comparison of the figures, the velocity is high in its steady state (t) than in its transient state. This shows that inertia term affects transient velocity more than steady state velocity.

Figure 2(a) & (b) depict transient velocity at t=0.04 and steady velocity at large time t both against y for different A and Da=0.1, s=1.5, b=0.6 respectively. It is noticed in the both figures, velocity in the porous region is constant for big values of A (A=100, A=1000). This shows that inertia term affect flow in the porous region. That is why when Da is decreases for

higher values of A the flow in the porous region is zero. Generally, the intensity of the velocity is higher in its steady state than in its transient state. This shows that velocity increase with increase in time and attains steady at large time($t \to \infty$).
Figure 3(a) & (b) show transient velocity at a fixed time (t=0.04) and steady state velocity at large time ($t \to \infty$) both against y for different Da and A=10, s=1.5, b=0.6 and d=0.5 respectively. It is clearly observed that velocity is decreasing to constant in the porous region as Darcy number, Da decreases. Similarly, velocity is decreasing to zero in the porous region for a small value of Darcy number, Da. These physically show that small value of Da means that the pores of the porous medium are very small which will not allow fluid to flow fast.
Figure 4 shows Interfacial velocity, ui, against time, t, for different s and b and d=0.5, Da=0.1 and A=10.0. It is observed that, ui, increase and becomes steady with increase in time. It is also observed ui is constant for the value of b=-0.6 and s=1.5 for all time. Similarly, ui, increase with increase in s, the value of suction/injection in the system.
Figure 5 depicts interfacial velocity, ui, against $\log Da$ for different value of s and b for d=-0.5, A=10.0. It is observed that, for all values of b and s, ui decrease with decrease in $\log Da$. This shows that, as $\log Da$ decrease means the porous medium has small pores which will not allow fluid to flow easily. Similarly, ui is high for s=-1.5 and s=0.0 than for s=1.5. This shows that injection of fluid affects flow in the system than suction of fluid.
Figure 6 depicts interfacial velocity, ui against beta, b for different values of Da and A and s=1.5, d=0.5 and t=0.04. It is observed that, ui decrease with increase in b and become constant after zero (i.e. positive values of beta, b) for Da=0.0001, A=100 and Da=0.0001 and A=10. Similarly, ui, increase and become constant after zero for Da=0.001, Da=0.01 and Da=0.1 and A=1000, A=100, A=10. These clearly show that inertia term affect fluid flow in the porous medium for small values of Da. Figure 7 shows interfacial velocity, ui against $\log A$ for different values of Da and d=0.5, s=1.5 and b=0.6. It is noticed that ui decrease with increase in $\log A$ for all values of Da. It is observed that ui is high for small value of Da (0.0001) than the other values of Da. It is similarly, observed that the interfacial velocities intersect for a big value of $\log A$ and all values of Da excluding Da=0.1. These mean inertia term affect porosity of porous medium.
Figure 8(a)-(c) show transient velocity against y for different time, t and A=10, A=100, A=1000 and d=0.5, s=1.5 and b=0.6 respectively. It is clearly observed that in all the figures transient velocities are constant in the porous medium for A=10 and A=100 and it is almost zero for A=1000. These physically show that inertia term affect the transient fluid flow in the porous medium. Figure 9 depicts skin friction against time t for different A and Da in both clear fluid wall (SKINO) and porous wall (SKIN1). It is observed that both skin frictions increase and become constant as time increases. It is similarly observed that skin friction at porous wall is higher than skin friction at clear wall for all values of A. This physically shows that inertia term affects skin friction in the porous region than clear fluid region.

Conclusion

Fluid flow in parallel-plates in horizontal channels partially filled with porous material and partially with clear fluid and wall suction/injection is modeled considering the inertia term in the porous region is presented. Numerical solutions are obtained for the transient fluid flow problem under the effect of sudden change in the imposed pressure gradient. The effects of the parameters involved are investigated. In particular, it is found:

- That inertia term has effect on the flow in the porous medium with big values of Darcy number, Da and small values of inertia term.
- That there are excellent agreements with the results found in [18] in the absence of the inertia term and suction/injection
- That similarly there are excellent agreements with the results found in [9] in the absence of suction/injection.

Appendix

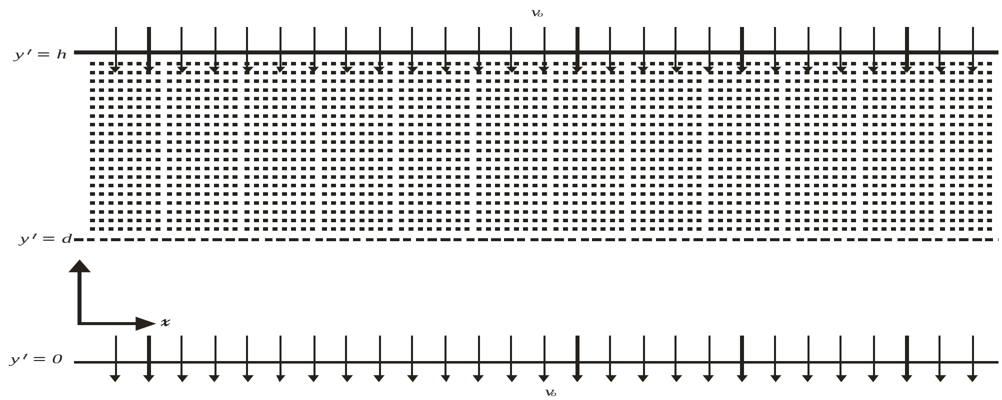

Diagram I: Illustration of the problem

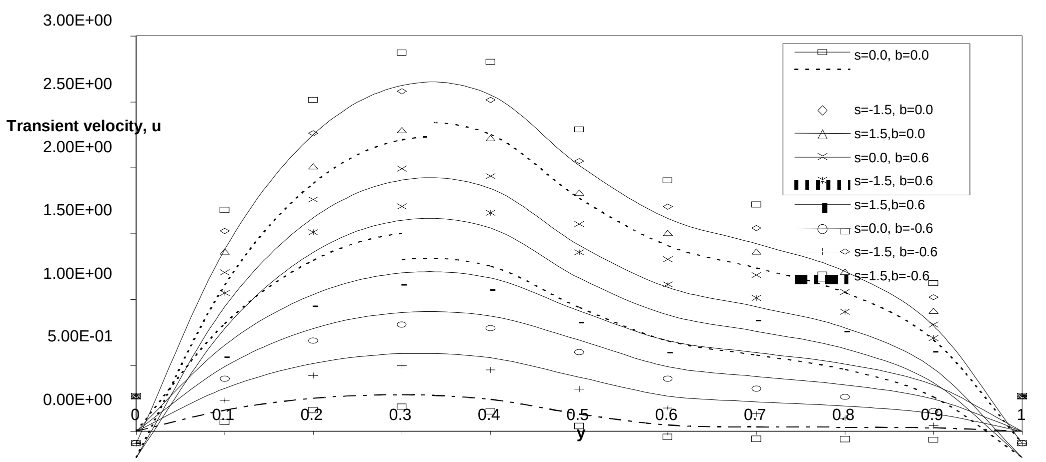

Figure 1 (a) Transient velocity against y for different s and b and fixed d=0.5, A=10, Da=0.1

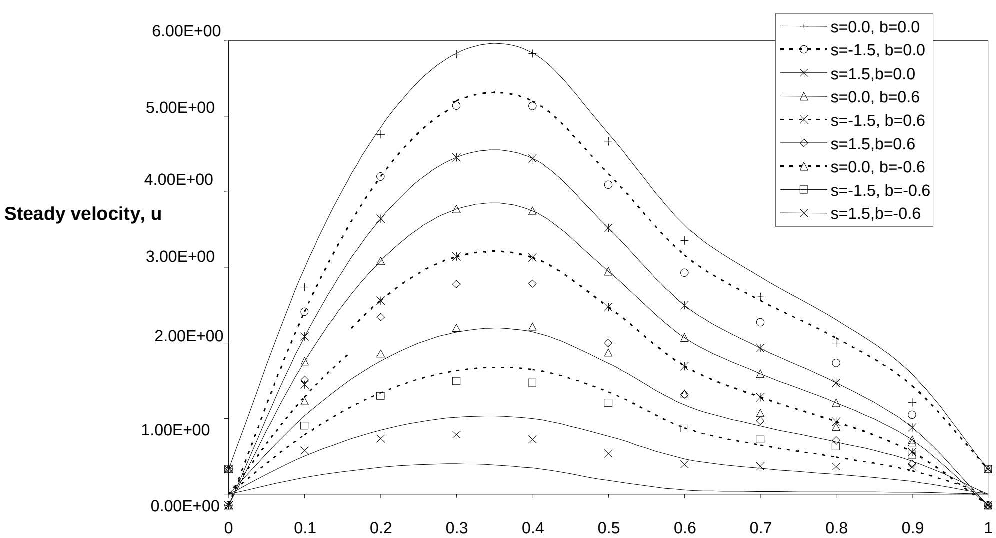

Figure 1 (b) Velocity at steady state against y for different s and b and d=0.5, A=10.0 and Da=0.1

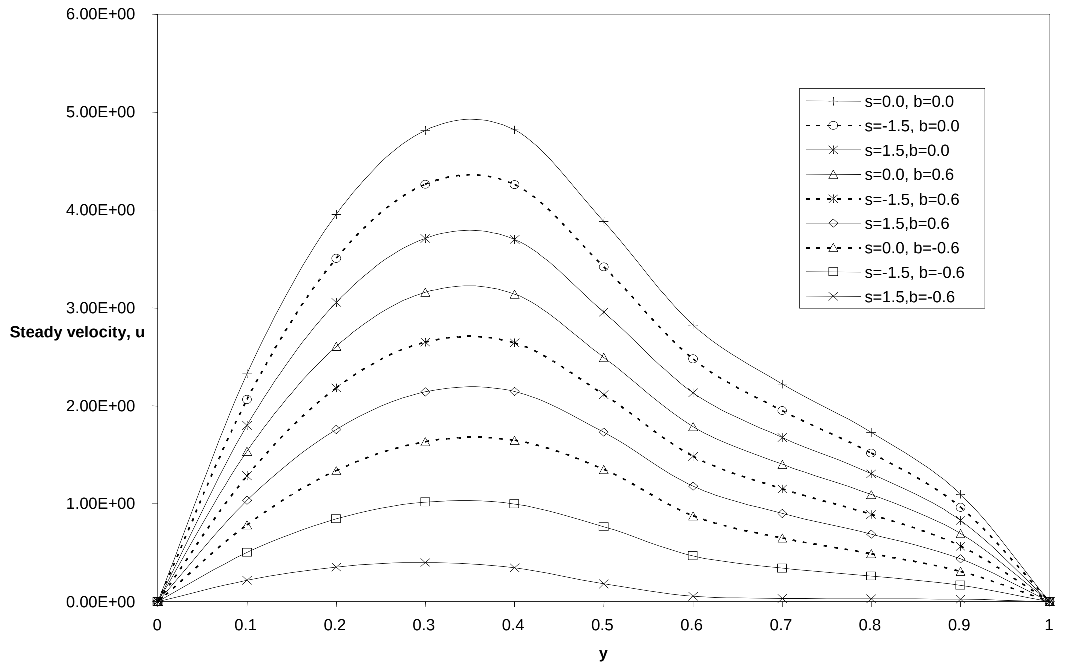

Figure 2(a) Transient velocity against y for different A and s=1.5, Da=0.1, b=-.6 and t=0.04

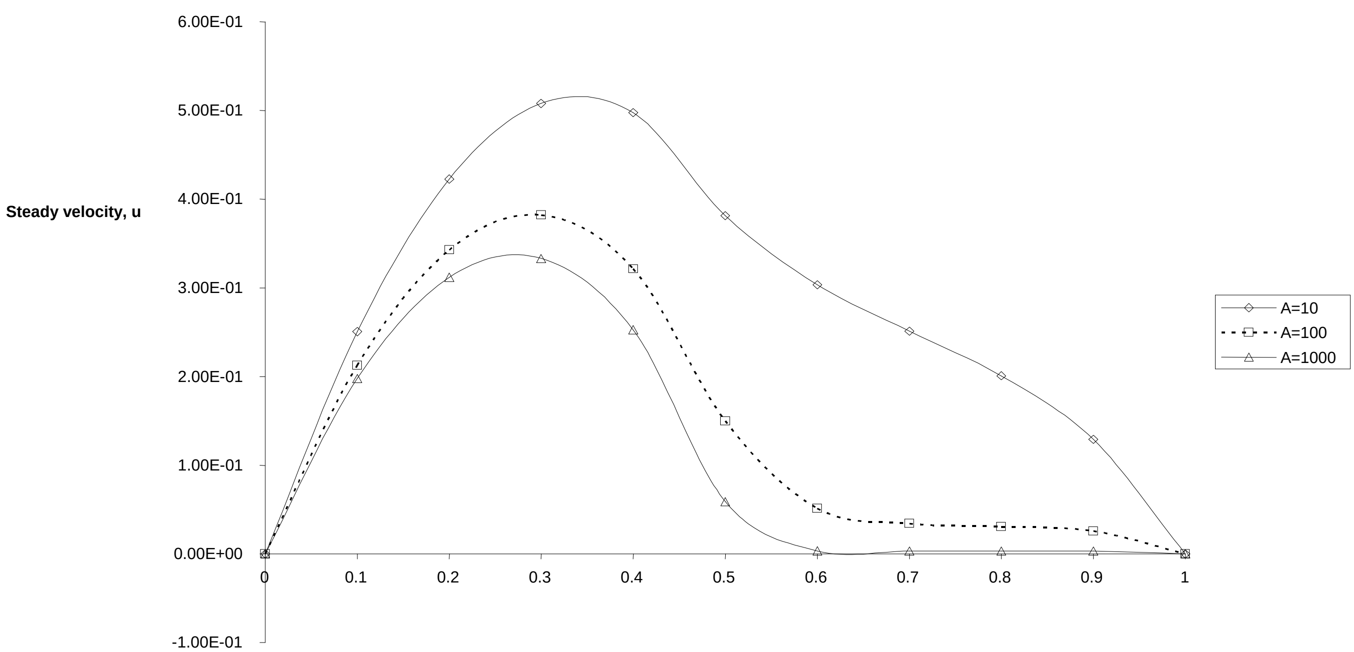

Figure 2 (b) Velocity at steady-state against y for different A and s=1.5, Da=0.1, b=0.6

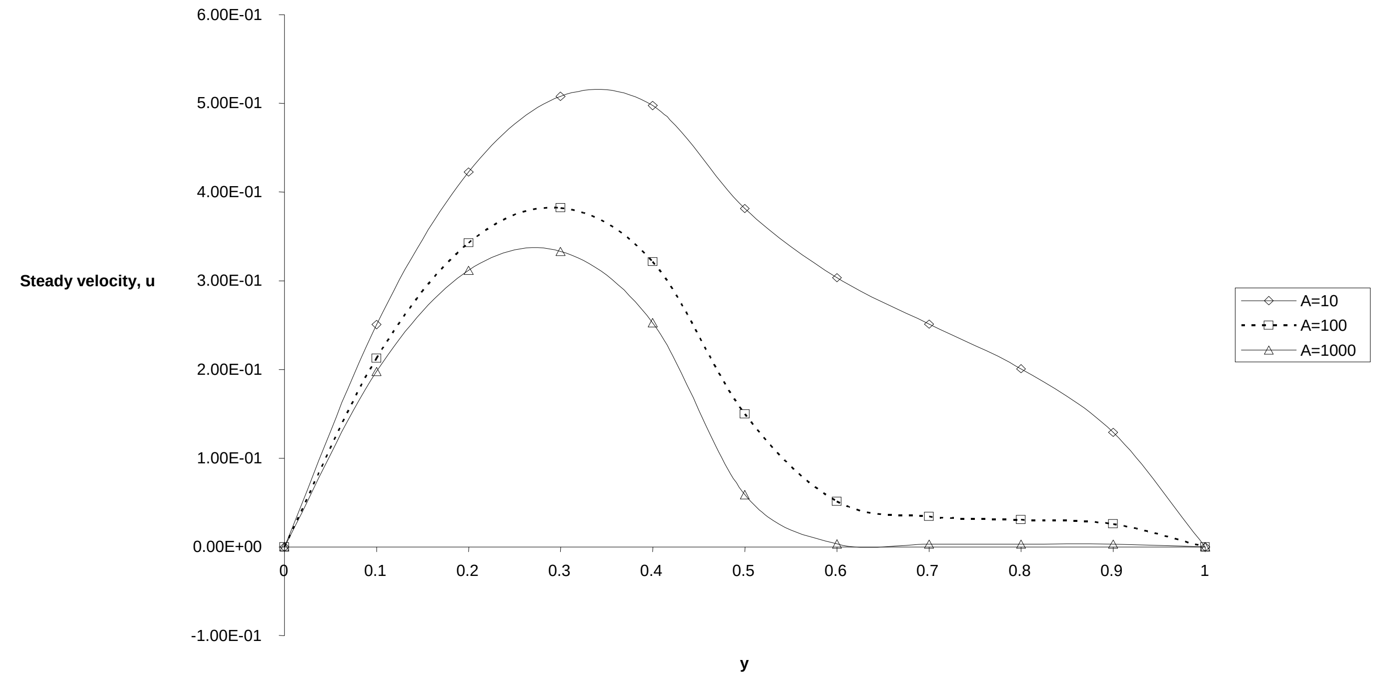

Figure 3(a) Transient velocity against y for different Da and A=10, s=1.5, b-0.6 and d=0.5

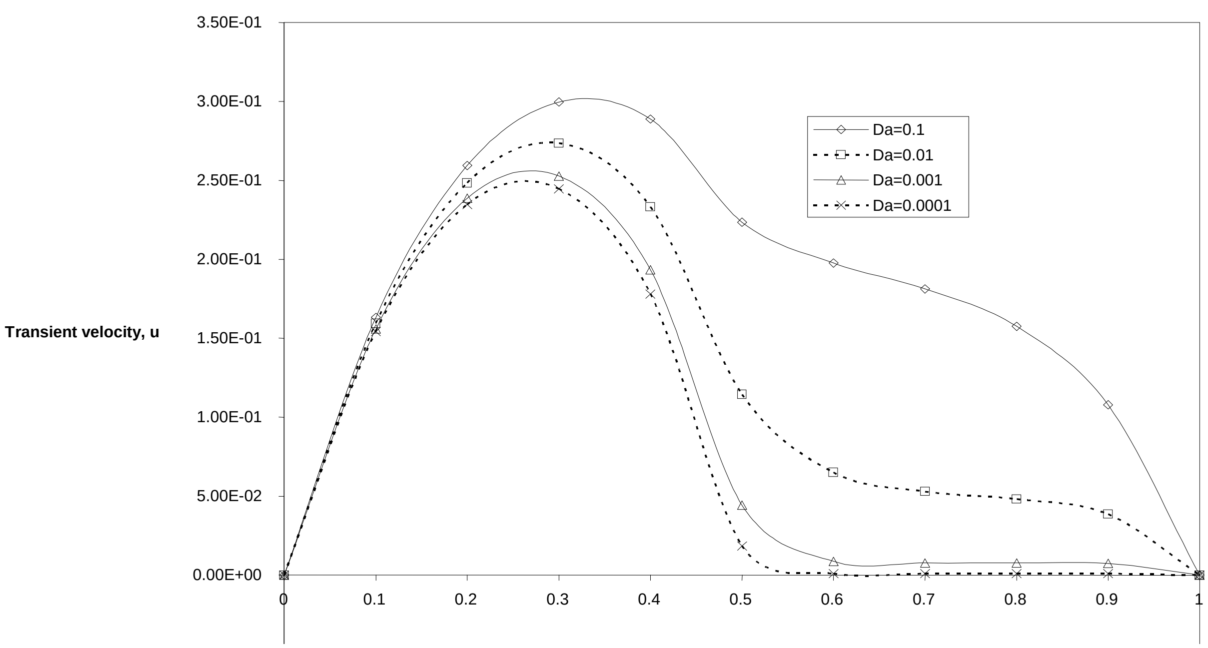

Figure 3 (b) Velocity at steady state against y for different Da and A=10, s=1.5, b=0.6 and d=0.5

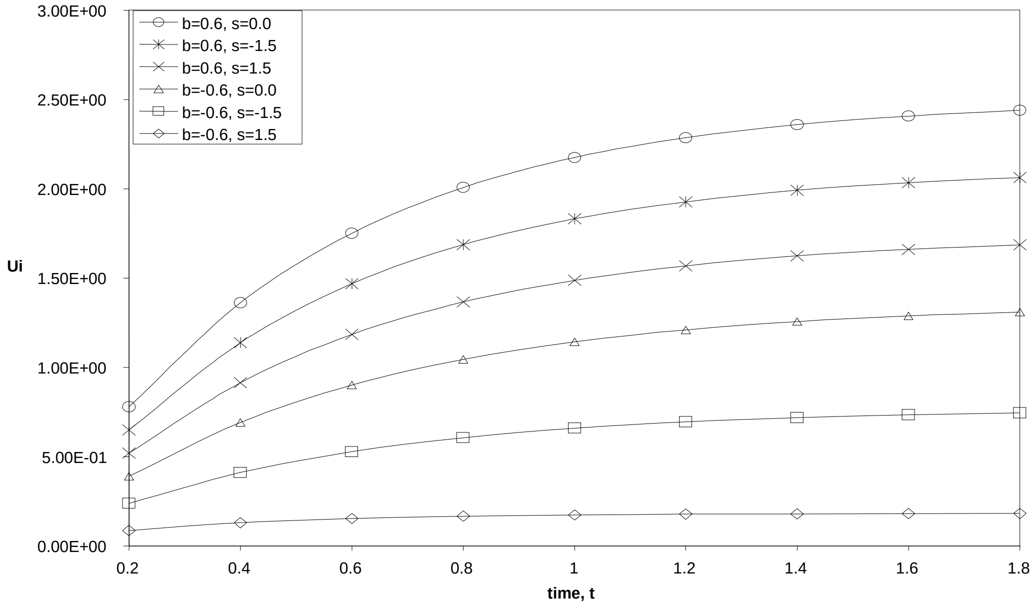

Figure 4 Interfacial velocity ui against time, t for different s and beta, b and d=0.5, Da=0.1, A=10

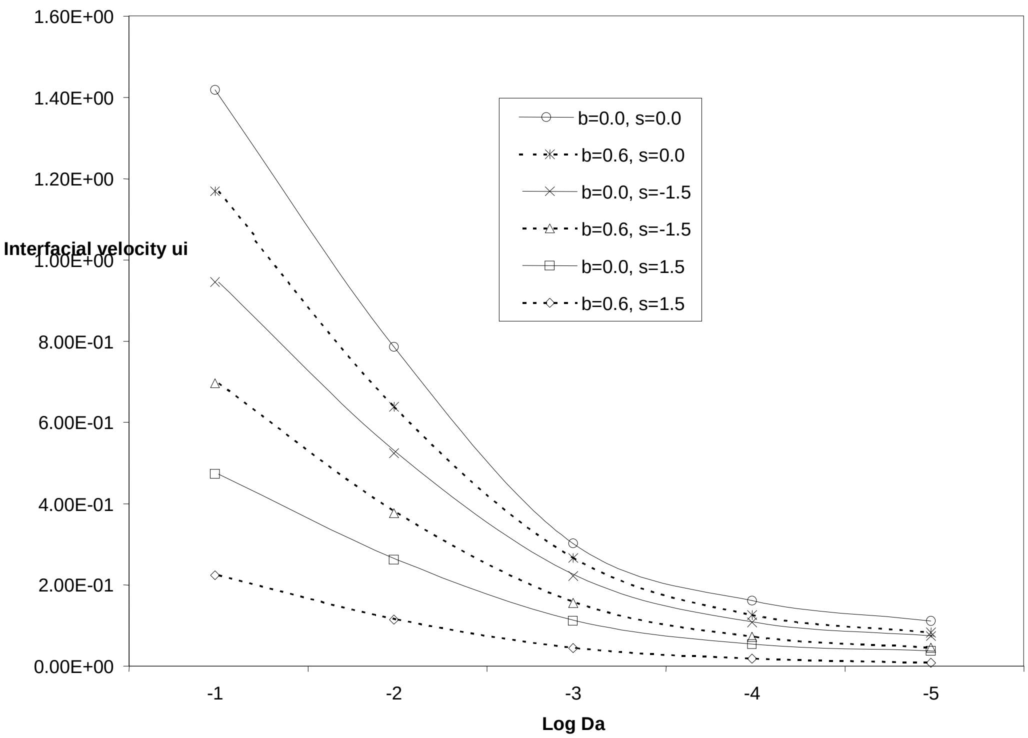

Figure 5 Interfacial velocity, ui against Log Da for different s and b for d=0.5, A=10

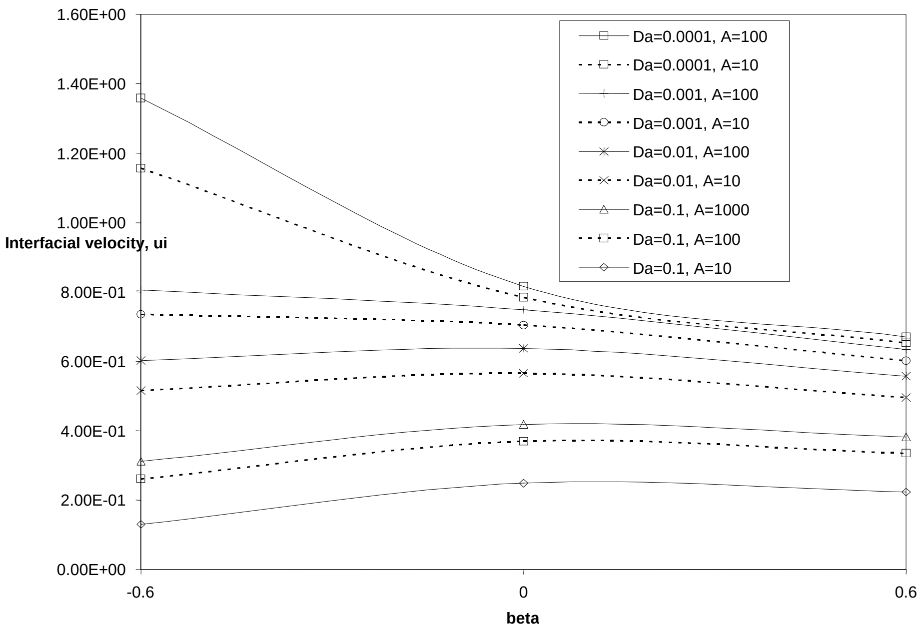

Figure 6 Interfacial velocity ui against beta, b for different Da and A and s=1.5, d=0.5, t=0.04

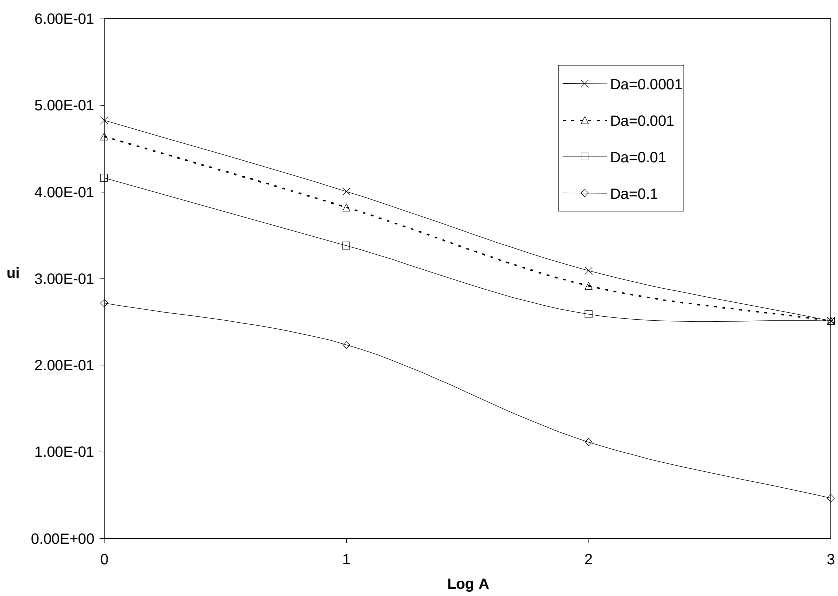

Figure 7 Interfacial velocity ui against Log A for different Da and d=0.5, s=1.5 and b=0.6

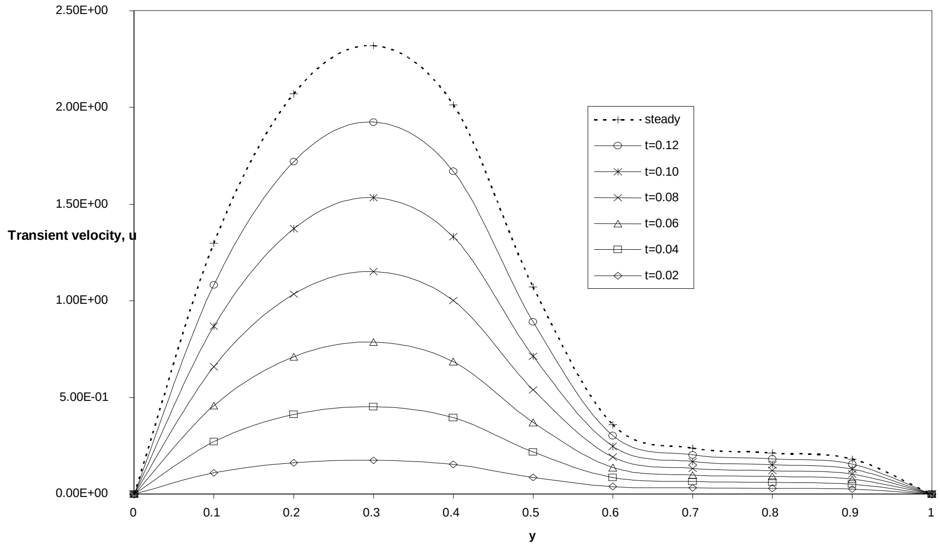

Figure 8 (a) Velocity against y for different time t and A=10, d=0.5, s=1.5 and b=0.6

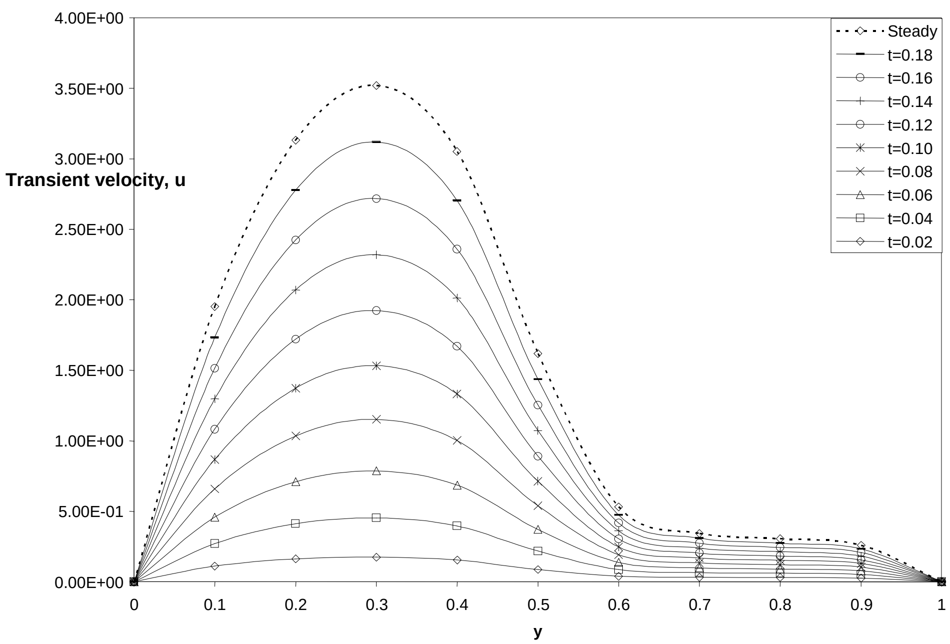

Figure 8 (b) Velocity against y for different time t and A=100, d=0.5, s=1.5 and b=0.6

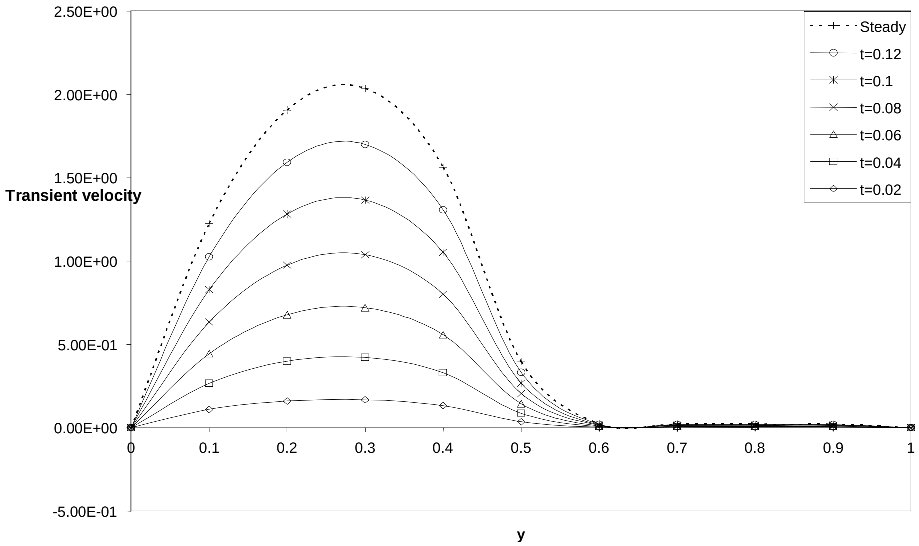

Figure 8(c) Transient Velocity against y for different time t and A=1000, d=0.5, s=1.5 and b=0.6

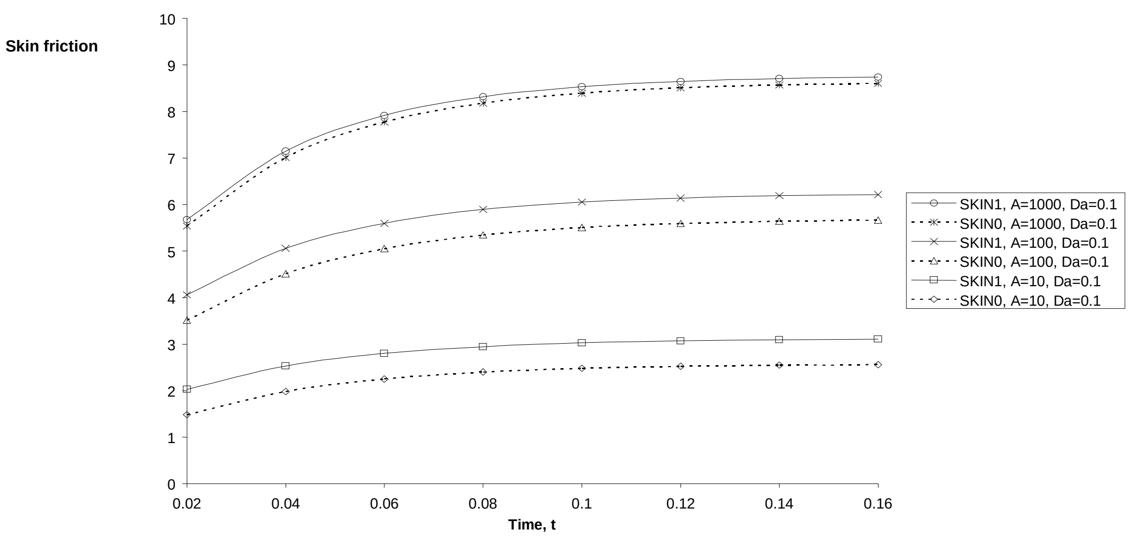

Figure 9 Skin friction against time t for different A and Da at clear fluid wall (SKINO) and porous wall (SKIN1)

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