

# Predicting the Compressive Strength of Concretes Made with Washed Gravel from Eastern Nigeria Using Artificial Neural Networks

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## Abstract

Cases of collapsed buildings and structures are prevalent in Nigeria. In most of these cases the cause of the collapse could be traced to the strength of the construction materials, mainly concrete. Secondly, experimental determination of the strength of concrete materials used in buildings and structures is quite expensive and time consuming. This research seeks to develop a computational model based on artificial neural networks for the determination of the compressive strength of concrete materials made from prevalent coarse aggregate components from Nigeria. The work involved building a multilayer perceptron neural network model which was trained using experimental data obtained from compressive strength test of concrete made from washed gravel. The compressive strength predictions were compared with predictions from an alternative model based on regression analysis. The results of the study show that for the washed gravel based concrete the regression model prediction has a sum of squares error of 10.683 and a mean absolute percentage (relative) error of 1.181, while the neural network model prediction has a sum of squares error of 0.028 and a mean absolute percentage (relative) error of 0.016. Generally, the models predicted well, but the neural network model predicted better than the regression model. The result of the study has ably demonstrated a cheap, simple, very quick and accurate alternative to experimental method of concrete strength determination. The method is also simpler and quicker than analytical methods based on regression analysis. This work is therefore expected to be of immense benefit to civil engineers and construction professionals, and would help in economically determining the strength, as well as economical selection of appropriate mix of construction materials, a prelude to building strong and cheap buildings and structures.

**Keywords:** Artificial Neural Network; Concrete; Washed gravel; Regression; Modelling

### Nomenclature, Symbols and Notations

$z_1$  = water/cement ratio

$z_2$  = water

$z_3$  = fine aggregate

$z_4$  = coarse aggregate

$X_1$  = fine aggregate

$X_2$  = coarse aggregate

$X_3$  = cement

$X_4$  = water

$Y$  = experimentally determined strength

$\hat{Y}$  = network prediction

$W$  = neural network input weight

$\gamma$  = network learning rate

$g$  = network activation function

$a_i$  = neural network input activations

$In$  = total weighted network input

$h_W$  = network weighting function

$t$  = time

ERR = error

## 1.0 Introduction

Strength being the most important property of concrete determines the quality of concrete. Traditionally, laboratory trial mixes have been used to determine the compressive strengths of concrete. Experimental determination of the strength characteristics of concrete materials is costly and time consuming. Often building contractors in Nigeria in their attempt to cut corners use low quality concrete materials in building constructions leading to cracking and in some cases total collapse of the building or structure. Here in Nigeria cases of collapsed buildings and structures are prevalent, and often lead to massive loss of lives and properties [1, 2]. Olujumoke et al. [1] identified weak concrete mixes as one the major

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reasons for the collapse of most buildings in Nigeria. This has disastrous socio-economic consequences for the country [1, 2].

Building structures with the right materials and proper strength characteristics would eliminate the incidences of collapsed buildings and structures in Nigeria. This will improve the socio-economic well-being of the citizens. Finding a potable low cost way of predicting the strength of concrete materials would help in solving the problem of collapsed buildings and structures in Nigeria. One way this could be done is by developing a computational model based on artificial neural network technology for predicting the strength of concrete materials. With mathematical and computational models a designer can easily find the best combination of constituent material to balance strength and cost.

An artificial neural network (ANN), usually called "neural network" (NN), is a mathematical model or computational model that is inspired by the structure and/or functional aspects of biological neural networks [3].

The concept of artificial neurons was first introduced in 1943 [4]. Russell and Norvig [3] stated that since 1943 when McCulloch and Pitts introduced the concept of neurons, much more detailed and realistic models have been developed both for neurons and for larger systems in the brain leading to the modern field of computational neuroscience. Since the work of McCulloch and Pitts in 1943, ANN has had wide application in many spheres of life. According to Maier and Dandy [5], in recent years, Artificial Neural Networks (ANNs) have become extremely popular for prediction and forecasting in a number of areas, including finance, power generation, medicine, water resources and environmental science.

The utility of artificial neural network models lies in the fact that they can be used to infer a function from observations. This is particularly useful in applications where the complexity of the data or task makes the design of such a function by hand impractical [3].

The tasks to which artificial neural networks are applied tend to fall within the following broad categories:

- i. Function approximation, or regression analysis, including time series prediction, fitness approximation and modelling.
- ii. Classification, including pattern and sequence recognition, novelty detection and sequential decision making.
- iii. Data processing, including filtering, clustering, blind source separation and compression.
- iv. Robotics, including directing manipulators, Computer numerical control.

Many papers have been written on the application of ANNs to the prediction of strength of engineering materials. Mukherjee and Biswas [6] in their paper applied artificial neural networks to the prediction of the mechanical behaviour of concrete materials at high temperature. Their results were very encouraging. Oretal and Kawashima [7] in their paper proposed an artificial neural network (ANN) based model, to predict the confined compressive strength and corresponding strain of circular concrete columns. Their study shows the importance of validating the ANN models in simulating physical processes especially when data are limited. The ANN model they developed was also compared to some analytical models and was found to perform well. Other papers on the prediction of concrete strength using neural networks include: Lee [8], Kasperkiewicz et al. [9], Ahmet et al. [10], Topcu and Saridemir [11] etc.

Maier and Dandy [5] reviewed 43 papers dealing with the use of neural network models for the prediction and forecasting of water resources variables in terms of the modelling process adopted. They identified inadequate model building as the obstacle militating against accurate predictions using artificial networks. They suggested that ANN models must be properly evaluated before its application in time series analysis.

Their assertion is corroborated by Chatfield [12] when commenting on the suitability of ANNs for time series analysis and forecasting, who commented thus: "when the dust has settled, it is usually found that the new technique is neither a miraculous cure-all nor a complete disaster, but rather an addition to the analyst's toolkit which works well in some situations and not in others".

It is important to note that a neural network modelling is purely a computational technique. Hence, if one wants to explain an underlying process or mathematical framework that produces the relationships between the dependent and independent variables, it would be better to use a more traditional statistical model like regression analysis. However, if model interpretability is not important, one can often obtain good model results more quickly using a neural network.

Properties of materials used in construction vary from region to region and from country to country. Hence, accordingly, the properties of building materials used in Nigeria are unique and differ significantly with what is obtained in other countries. Here we examined one major coarse aggregate component of concrete used as construction material in eastern Nigeria. This coarse aggregate is washed local gravel.

Concrete is a four component mix of water, cement, fine aggregate and coarse aggregate, of which the important properties are strength (compressive and flexural), deformation under load, durability, permeability and shrinkage. But strength, being considered the most important of these properties determines the quality of the concrete.

Neural network approach was used to predict the compressive strength of concrete materials produced from this coarse aggregate component (washed local gravel). Compressive strength prediction of concrete is necessary in structural design of buildings and structures [1, 2]. The neural network model developed has intuitive and theoretical appeal. It was developed based on the assumption that the experimental results were generated by a stochastic process. The model developed was in very good agreement with values obtained from experiment and the theoretical model based on Scheffe's (4, 2) regression equations [13, 14].

## 2.0 Experimental Technique and Regression Methodology

The materials for the mixing and production of concrete were obtained, prepared, and the concrete produced tested. The test results were used to determine the coefficients of the regression model. The material preparation and testing procedure, as well as the regression model development are hereby presented.

### 2.1 Preparation, Curing and Testing of Cube Samples

The aggregates were sampled in accordance with the methods prescribed in British Standards Institution (BS 882: Part 1: 1992) [15]. The test sieves were selected according to British Standards Institution (BS 410: Part 1: 1986) [16]. The water absorption, the apparent specific gravity and the bulk density of the coarse aggregates were determined following procedures prescribed in (BS 812: Part 2: 1975) [22]. The Los Angeles abrasion test was carried out in accordance with American Society for Testing and Materials (ASTM Standard C131:1976) [21]. The sieve analysis of the fine and coarse aggregate samples was done in accordance with British Standard Institution (BS 812: Part 1: 1975) [22] and satisfied British Standard Institution (BS 882:1992) [17]. The sieving was performed by a sieve shaker. The water used in the preparing the experimental samples satisfied the conditions prescribed in British Standard Institution (BS 3148: 1980) [20]. These specimens were cured for 28 days in accordance with British Standard Institution (BS 1881: Part 111: 1983) [18]. The testing was done in accordance with British Standard Institution (BS 1881: Part 116: 1983) [19] using compressive testing machine.

### 2.2 Regression Model Development Methodology

The experimental results were fitted to a polynomial regression model based on Scheffe's (4, 2) regression model [13, 14]. The regression models were:

For washed gravel:

$$\hat{Y} = 23.46x_1 + 25.01x_2 + 14.83x_3 + 9.41x_4 + 2.06x_1x_2 - 0.78x_1x_3 - 4.38x_1x_4 - 2.32x_2x_3 - 6.44x_2x_4 + 9.28x_3x_4 \quad (1)$$

The regression model assumed that each of the components of concrete namely: water, cement, fine aggregate and coarse aggregate could be zero or one. But in reality none of these components could be zero or one. Hence, an appropriate transformation of the actual components  $z_1, z_2, z_3$  and  $z_4$  was used to determine the pseudo components  $x_1, x_2, x_3$  and  $x_4$  that was used in the regression equations above [13, 14].

## 3.0 Neural Networks

As has been previously mentioned, the origin of artificial neurons (ANNs) is based on the work of McCulloch and Pitts in 1943 [4]. Artificial neurons are building blocks for artificial neural networks. We shall discuss here the structure artificial neurons and neural network used in this research.

### 3.1 Artificial Neurons

Artificial neural networks make use of artificial neurons. Artificial neural networks (ANNs) simulate the manner of operation of natural neurons in the human body. The basic unit of operation of an ANN is the neuron shown in Figure 1.

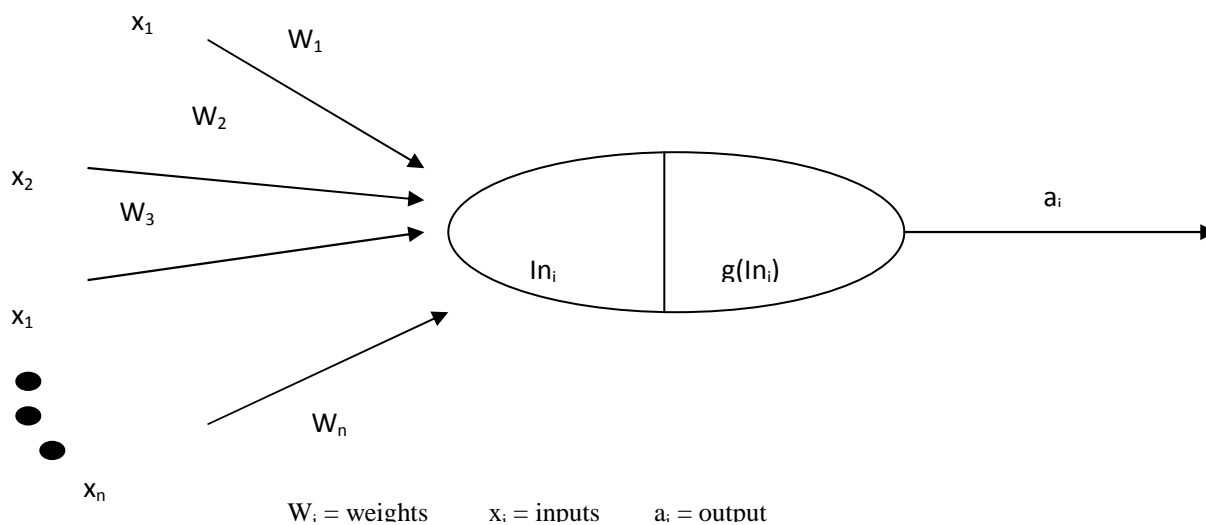


Figure 1: An artificial neuron

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In a typical neuron shown in Figure 1, the input to the neuron  $x_i$  is multiplied by a weighting function  $W_i$  to generate the transformed input  $W_i x_i$ . The transformed inputs are summed to obtain the summed input. The summed input constitutes the variables to the activation/transfer function,  $g$ , which generates the output  $a_i$ . The output of the transfer function is compared to a threshold value. If the output is greater than the threshold value, the neuron is activated and signal is transferred to the neuron output, alternatively, if it is less the signal is blocked.

Given an input vector  $X = (x_1, x_2, x_n)$ , the activations of the input units are set to  $(a_1, a_2, a_n) = (x_1, x_2, x_n)$  and the network computes the following:

$$In_i = \sum_{j=1}^n W_{j,i} a_j \quad (2)$$

$$a_i = g(In_i) \quad (3)$$

The transfer function could be a threshold transfer function, a sin function, a sigmoid function, hyperbolic tangent function etc. Differentiable transfer functions are preferred. Similarly, non linear transfer functions perform better than linear transfer function. Bearing these in mind, in this particular application we chose the sigmoid function. The sigmoid activation function which is given by the equation:

$$a_i = g(In_i) = \frac{1}{1 - e^{-In_i}} \quad (4)$$

Training the network (learning) could be supervised or unsupervised training. In supervised training, the network is provided with the inputs and appropriate outputs; hence the network is trained with a set of examples in a specified manner. In unsupervised/adaptive learning, the network is provided with inputs but not the outputs. In this present application, we used the supervised learning, hence, the appropriate network architecture is the feedforward architecture.

### 3.2 The Feedforward Network Architecture

As has been mentioned, the developed neural network models are feed forward multi-layer perceptron networks (MLP). The hidden units as previously noted use the sigmoid activation function. The network model is shown in Figure 2.

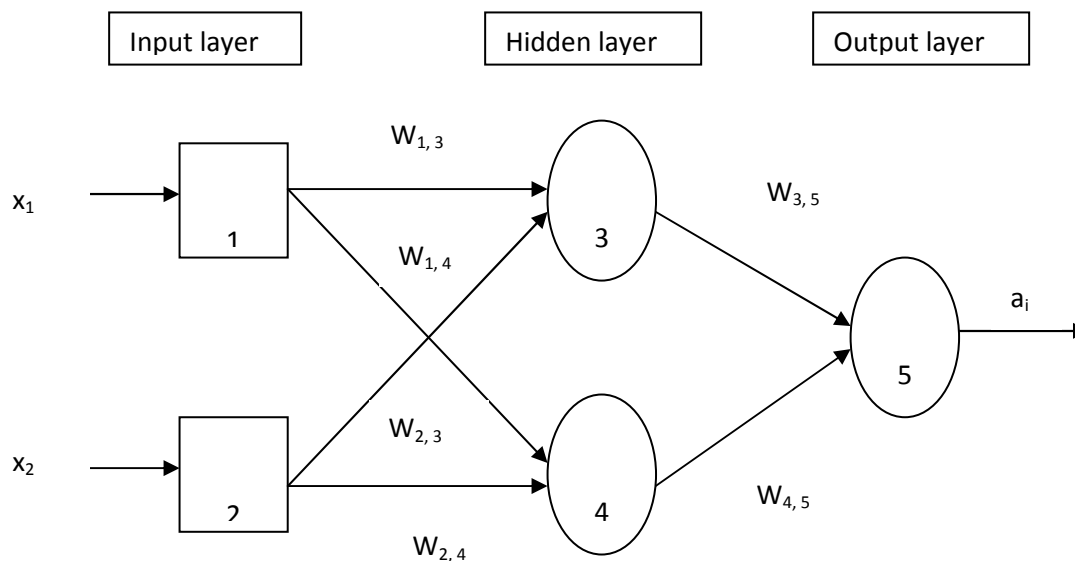


Figure 2: Two-input feed forward neural network model

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In the feed forward network shown in Figure 2, the output of the network is compared with the desired output. The difference between the output and the desired output is known as the error,  $E$ . ANNs learn by trying to minimize this error. The learning process uses optimisation algorithms such as gradient descent algorithm, genetic algorithm or some other natural optimisation algorithm. These algorithms work by adjusting the weights,  $W_i$ , such that the error,  $Err$ , is minimized. Most ANNs use the simple gradient descent optimisation algorithm. In this work we used this algorithm.

Hence, the learning process uses the sum of squares error criterion  $E$  to measure the effectiveness of the learning algorithm.

$$E = \frac{1}{2} Err^2 \equiv \frac{1}{2} (y - h_W(x))^2 \quad (5)$$

Here

$y = Y$  = the true/experimental value

$$\hat{Y} = h_W(x) \quad (6)$$

$h_W(x)$  is the output of the perceptron.

The learning process, as previously mentioned, uses Cauchy's steepest descent or gradient descend algorithm optimisation method given by the formula [24]:

$$W_j(t+1) = W_j(t) + \gamma \times \nabla E(W_j) \quad (7)$$

Here  $\gamma$  = non negative scalar that minimizes the function,  $E(W_j)$ , in the direction of the gradient and it is equal to the network learning rate.

While

$$\nabla E(W_j) = \frac{\partial E}{\partial W_j} \quad (8)$$

But

$$\frac{\partial E}{\partial W_j} = \frac{\partial E}{\partial Err} \times \frac{\partial Err}{\partial W_j} \quad (9)$$

Since

$$\frac{\partial E}{\partial Err} = Err \quad (10)$$

Hence,

$$\frac{\partial E}{\partial W_j} = Err \times \frac{\partial Err}{\partial W_j} \quad (11)$$

But

$$\frac{\partial Err}{\partial W_j} = \frac{\partial}{\partial W_j} \left( y - g \left( \sum_{j=1}^n W_j x_j \right) \right) \quad (12)$$

$$\frac{\partial Err}{\partial W_j} = -g'(in) \times x_j \quad (13)$$

Substituting equation (13) into equation (11), equation (14) is obtained:

$$\frac{\partial Err}{\partial W_j} = -Err \times g'(in) \times x_j \quad (14)$$

$$W_j(t+1) = W_j(t) + \gamma \times Err \times g'(in) \times x_j \quad (15)$$

#### 4.0 The ANN for Predicting Compressive Strength of Concrete

Recall that our application is for concrete strength prediction, and we used supervised learning. Hence, Seventy five (75%) percent of the data was used for training, while twenty five (25%) percent was used for testing and validation. The number of epoch was set to 1000.

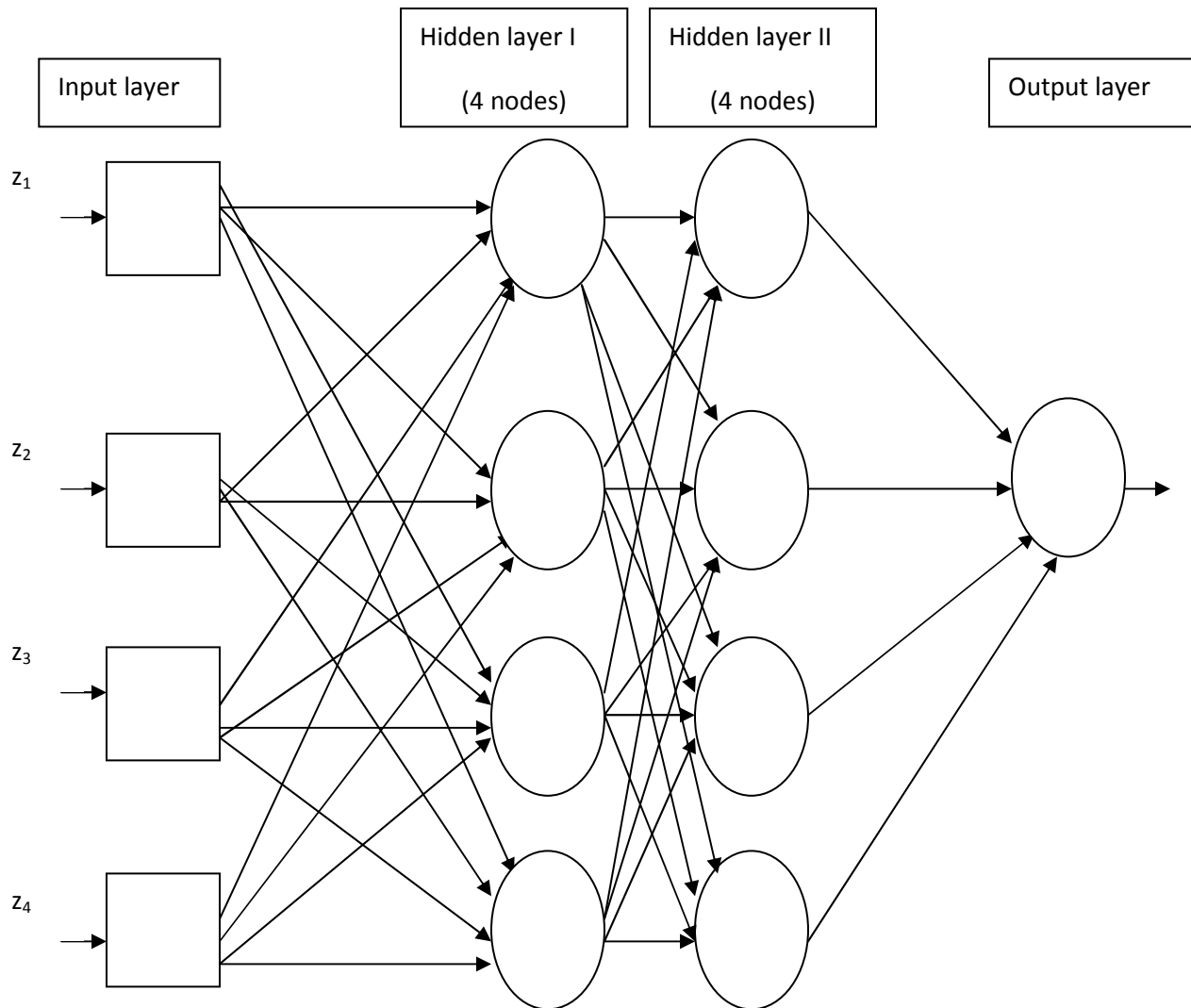


Figure 3: The Four-input, 4 layers feed foreword neural network model

Single network architecture was used in the study. The network architecture consists of four input units, two hidden layers with four hidden units (nodes) and one output unit. The network structure is shown in Figure 3. The inputs  $Z_1$ ,  $Z_2$ ,  $Z_3$  and  $Z_4$  to the neural network consists of water/cement ratio, cement, fine aggregate and coarse aggregate respectively.

## 5.0 Results

Table 1 shows the experimentally determined strength for different mixture ratios for washed-gravel concrete mixtures represented by Y.

Table 1: Results of compressive strengths obtained experimentally

Sample No	Y	Z <sub>1</sub>	Z <sub>2</sub>	Z <sub>3</sub>	Z <sub>4</sub>
1	23.46	0.6	1	1.5	2
2	25.01	0.5	1	1	2
3	14.83	0.55	1	2	5
4	9.41	0.65	1	3	6
5	24.75	0.55	1	1.25	2
6	18.95	0.575	1	1.75	3.5
7	15.34	0.625	1	2.25	4
8	19.34	0.525	1	1.5	3.5
9	15.6	0.575	1	2	4
10	14.44	0.6	1	2.5	5.5
11	22	0.5625	1	1.5	2.75
12	17.39	0.6	1	2	3.75
13	18.09	0.55	1	1.75	3.75
14	18.2	0.575	1	1.875	3.75
15	24.1	0.575	1	1.375	2
16	21	0.5875	1	1.625	2.75
17	19.08	0.6125	1	1.875	3
18	22.1	0.5125	1	1.25	2.75
19	19.59	0.5375	1	1.5	3
20	14.99	0.585	1	2.25	5.25

### 5.1 Physical and Mechanical Properties of Aggregates

Sieve analyses of both the fine and coarse aggregates were performed and the grading curves are shown in Figures 4 and 5. These grading curves showed the particle size distribution of the aggregates. The physical and mechanical properties are summarized in table 2.

Table 2: Physical and mechanical properties of the local gravel

PROPERTIES	LOCAL GRAVEL
Water absorption	4.55 %
Moisture content	53.25 %
Apparent specific gravity	1.88
Los Angeles abrasion	60 %
Impact resistance	22.9 %
Bulk density	1302.7 Kg/m <sup>3</sup>
Fineness modulus	70.05

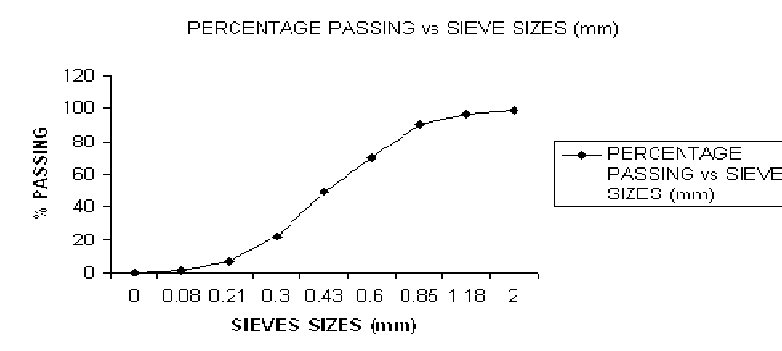


Figure 4: Grading curve for the fine aggregate

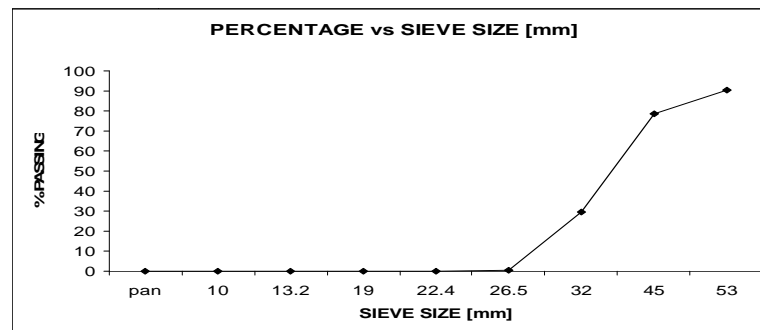


Figure 5: Grading curve for the local gravel

## 5.2 ANN Prediction Results

The results of the experimentally determined concrete strength, analytically determined strength using the regression model, and the strength prediction using neural network models are presented in this section.

The strengths (responses) of washed-gravel concrete were a function of the proportions of its ingredients: water, cement, fine aggregate, and coarse aggregates. As shown in Figure 6, the experimental values were in very good agreement with theoretical values obtained from the Scheffes's regression model and the neural network model. Table 3 shows the comparison of the sum of squares error versus the relative error for the regression and neural network models.

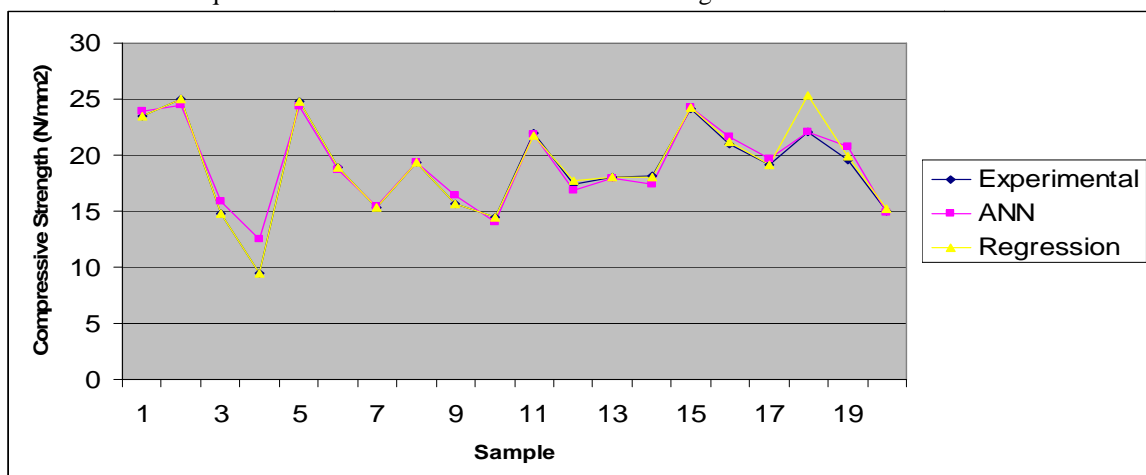


Figure 6: Comparison of ANN with experimental results and regression model prediction (washed gravel)



Table 3: Model type vs errors

Description	Sum of Squares Error	Relative Error (%)
Regression Model	10.683	1.181
Neural network model	0.028	0.016

## 6.0 Discussion

From the analysis in this work we have seen that the strength of concrete materials depends on the proportion of its ingredients: water, cement, fine aggregate, and coarse aggregates. Figure 7 shows the plot of the artificial neural network predictions and regression model predictions for the washed gravel concrete. Generally, the plots are in good agreement with the experimentally determined compressive strength. Table 2 shows that for the washed gravel based concrete the regression model prediction has a sum of squares error of 10.683 and a mean absolute percentage (relative) error of 1.181. Similarly, for the washed gravel based concrete the neural network model prediction has a sum of squares error of 0.028 and a mean absolute percentage (relative) error of 0.016.

Generally, the neural network models predicted better than the regression models. According to Mukherjee and Biswas [6], guidelines on the configuration of ANNs are not well established. Therefore a trial and error approach is adopted in the selection of network size, training examples and test problems [6]. Past experience plays an important role for selection of the various attributes of the network [6]. Hence, we tested various network configurations before we settled on the most appropriate configuration. The variations in the sum of squares error and the relative error of the neural network models tested confirm the fact that the performance of the network depends on the network design architecture ([5], [6]).

Generally, the neural network models were bereft of the messy mathematics and statistical analysis required in building the regression model, while at the same time giving good model predictions. Hence, would be preferable when the underlying mathematical structure behind the model predictions is irrelevant to the modeller/analyst, and model building is required quickly.

Concrete compressive strength determination is very important in civil engineering and in the construction industry [1, 7]. It is obvious that neural network models will help in efficient and accurate determination of concrete strength for building and construction purposes using local materials obtained from Nigeria.

## Conclusion

The construction industry is a major component of the economy of any nation. Building and structures are indispensable in any modern society. Concrete is the primary building material in Nigeria. As had noted by Olajumoke et al. [1] and Arum [2], cases of collapsed building and structures is endemic in Nigeria. These have resulted in loss of lives and properties, in addition to these, the economy is impacted negatively [1, 2]. Often poor concrete mixtures and inadequate knowledge of the role of concrete mixture properties to its strength are to blame [1].

Computational models using neural networks offer a very promising solution to the problem of concrete strength prediction. As we have demonstrated in this application, artificial neural network method for the prediction of compressive strength of concrete from local materials in Nigeria compares favourably with an equivalent mathematical model based on regression analysis. Computational models are simple because it does not involve complex mathematical analysis. Hence, what the engineer needs is good and reliable computer software and a matching hardware to do his analysis. The ubiquity of various computing platforms ranging from desktop PCs, laptops, palmtops, tablets etc means that such analysis is made even easier. The present application was done using a computer laptop which ran the artificial neural network software.

If the recommendations of this research are implemented by adopting this novel concrete strength prediction aid and making sure construction engineers and technicians stick to it, there will be resultant reduction in cases of collapsed buildings and structures in Nigeria. This will have a very positive effect on the socio-economic situation of the country which has been declining over the years. The same applies to other countries that are in similar situation as Nigeria.

Finally, neural network models for other common materials used in construction in Nigeria should be developed by engineers and scientists. This will further boost the quality of our building constructions.

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