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A Simple Algorithm for the Inventory Model With Random Demand Rate and Lead Time.

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Abstract

Osagiede had in a previous work examined an analytic procedure for inventory model with random demand rate and lead time and suggested for further studies that his model can be implemented on a computer. In this paper therefore, we propose a soft computing algorithm to determine the optimal inventory replenishment schedule with random demand. We developed an exact method using a computer program written in MATLAB and implemented on a PC to obtain the quantities, time intervals and the optimal number of replenishments for the system. We demonstrate this model with example.

Keywords: Inventory, delay cost, random demand, deliveries, lead time. .

1.0 Introduction

Demand is the major factor in inventory management. The demand of a product may increase with time due to incoming of a new product which may be technically good and attractive than the old one and also decreases with time [1]. Osagiede and Omosigho [2, 3] developed a simple algorithm for the inventory problem with a linear demand and a simple replenishment rule for the same system. In their model shortages are not permitted. Omosigho and Osagiede [4] proposed an inventory model with increasing demand, this time, followed by a constant demand. Osagiede and Omosigho [5] developed a computer aided analytical solution for inventory problems with linear increasing demand. Osagiede [6] examined the inventory replenishment schedule with random demand rate with lead time. In Osagiede model, he relaxed some of the assumptions of EOQ model by assuming that the rate of demand is random with probability distribution $f(x)$ and the replenishment rate is finite. Also over stock losses and penalties for shortages and carrying cost are negligible. A detail review can be found elsewhere [6, 7].

The paper is organized as follows. Section 2 gives the model notation, assumptions and statement of problem. Section 3 gives model development and solution. In section 4, we presented the optimal algorithm for determining the optimum stock and the number of replenishments, quantity to be ordered. Numerical illustration is presented in section 5 and in section 6 gives the remark and conclusion.

2.0 Model assumption and statement of problem:

The mathematical model in this paper is developed on the basis of the following notations and assumption: These assumptions are the same as those of Osagiede [6].

c_1 **:** unit loss for overstocks.

c_2 **:** unit loss for under stocks

p_0 **:** initial inventory (start of the first period).

$p_{i,s}$ **:** inventory at the end of period i;.

x_i **:** demand for period i;

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$q_{i's}$: the order quantity for period i;
$f(x)$: probability distributions for demand x
$\propto$: time intervals between orders
T_i : length of the i^{th} replenishment circle $i = 1(1)\, n$
k_2 : extra cost by the unit loss
$W(p)$: Total relevant inventory cost associated with p replenishments.
$f(t) = bt,\ b > 0,\ \ 0 \le t \le H$ the demand rate.

- **Assumptions:**

The demand x is random with overstock losses and penalties for shortages; lead time is not zero but equal to $\propto$; replenishment occurs at a finite rate; overstock losses, penalty for shortages and carrying cost is negligible; the quantity q_i for the $n-1$ periods that precede period n are known and have been regularly ordered is adopted.

- **Problem Description**

Osagiede [6] gave the following problem description. The inventory problem in a time interval α partitioned into n equal periods T_1. We assume that the lead time is equal to α. At the beginning of every period T, orders are placed so that deliveries will be made nth periods later. Let $f(x)$ be the probability distribution for the demand level x, covering a period (interval) T. Under this interval T, if x is less than inventory p the left over pieces will be sold at a unit loss of c_1; if x is greater the p, a special order for the backlog piece is made and the extra cost will be represented by a unit loss of k_2.

If the carrying cost is not substantial in comparison with c_1 *and* c_2, the interval T no longer has an effect, and we can say the management policy is time independent.

Let p be the quantity to be placed in stock. Two mutually exclusive situations are possible:

(a) $x \le p$: the stock covers the demand and the quantity $p - x$ is sold at a unit loss of c_1

(b) $x > p$: shortage exists and $x - p$ pieces must be specially ordered at a unit loss of k_2.

If $f(x)$, the probability distribution for demand x is known, the problem now is:

- What quantity q_i should be ordered in the nth periods before the period i so that the total cost for that period will be minima? We now attempt to find the optimum quantities q_n for the system, using our new optimal algorithm proposed.

3.0 Model development and solution:

The objective of the inventory problem is to determine the optimal order quantity and the length of ordering cycle so as to keep the total relevant cost as low as possible. As a result of the conditions and assumptions made in this paper, then the formulation is as follows:

$$\left.\begin{aligned} p_1 &= p_0 - x_1 + q_1 \\ p_2 &= p_1 - x_2 + q_2 \\ &\ldots\ldots\ldots\ldots\ldots\ldots \\ p_{n-1} &= p_{n-2} - x_{n-1} + q_{n-1} \\ p_n &= p_{n-1} - x_n + q_n \end{aligned}\right\} \qquad (1)$$

But

$$p_n = p_{n-2} - x_{n-1} - x_n + q_{n-1} + q_n \qquad (2)$$

and a continuous substitution for $p_i,\ \ i = n-2,\ n-3, \cdots, 1$
yield

$$p_n = p_0 + q_1 + q_2 + \cdots + q_n - (x_1 + x_2 + \cdots + x_n) \tag{3}$$

Then equation (3) reduces to

$$p_n = p - x \tag{4}$$

The quantity p_n can either be the following. Negative if $p < x$ or positive if $p > x$

The mathematical expectation of the total cost for time interval $\alpha = nT$ is given by

$$W(p) = c_1 \sum_{x=0}^{p} (p-x) f(x) + c_2 \sum_{x=p+1}^{\infty} (x-p) f(x) \tag{5}$$

Osagiede [6] gave a simple proof of the result given by equation (5) by stating the next proposition.

***Proposition*:**

The minimum of the function given by equation (5) for $W(p_i)$ occurs for a value of p^* such that

$$f(x \le p^* - 1) < \frac{c_2}{c_1 + c_2} < f(x \le p^*) \tag{6}$$

and

$f(x \le p^*) = f(0) + f(1) + \cdots + f(p^*)$.

(*See proof of the proposition in Osagiede* [6]

where

$$\begin{aligned} W(p+1) &= W(p) + (c_1 + c_2) \sum_{x=0}^{p} f(x) - c_2 \\ &= W(p) + (c_1 + c_2) f(x \le p) - c_2 \end{aligned} \tag{7}$$

and

$$W(p-1) = W(p) - (c_1 + c_2) f(x \le p - 1) + c_2 \tag{8}$$

For verification of equations (7) and (8) (see Osagiede [6])

Suppose p^* is the optimal stock, then p^* is such that

$$[W(p^* - 1) - W(p^*)][W(p^*) - W(p^* + 1)] < 0 \tag{9}$$

Further, it can be more simplified as

$$W(p^* - 1) > W(p^*) < W(p^* + 1) \tag{10}$$

Which is equivalent to

$$\left.\begin{aligned} &\text{(a)} \quad W(p^* + 1) - W(p^*) > 0 \\ &\text{(b)} \quad W(p^* - 1) - W(p^*) > 0 \end{aligned}\right\} \tag{11}$$

Equation (11a) implies that equation (7) is

$$(c_1 + c_2) f(x \le p^*) - c_2 > 0 \tag{12}$$

and equation (11b) implies that equation (8) is

$$-(c_1 + c_2) f(x \le p^* - 1) + c_2 > 0 \tag{13}$$

Combining equations (12) and (13), we have,

$$f(x \le p^* - 1) < \frac{c_2}{c_1 + c_2} < f(x \le p^*) \tag{14}$$

The value p^* which satisfies (14) gives the value of the stock that minimize $W(p)$.

This equation (14) is the proposed model for an enumeration process for determining n, the number of replenishments. In the enumeration, he compare three successive values of $W(p)$. If $p^* > 1$ is the optimal number of replenishment, then p^* must satisfy (14).

Again, the optimal stock p^* having been determine, and the quantities q_1 , $q_2, \cdots, q_{n-1}$ is proposed as follows:

$$q_n^* = q_n = p^* - p_0 - (q_1 + q_2 + \cdots + q_{n-1})$$

$$\left.\begin{aligned} & q_{n+1}^* = p^* - p_1 - (q_2 + q_3 + \cdots + q_n^*) \\ \text{and} \quad & \\ & q_{n+2}^* = p^* - p_2 - (q_3 + q_4 + \cdots + q_{n+1}^*) \\ & \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ & q_{n+t}^* = p^* - p_t - (q_{t+1} + q_{t+2} + \cdots + q_{n+1-1}^*) \end{aligned}\right\} \quad (15)$$

Equation (15) is the expression for order quantity. This completes a slight summary of Osagiede [6] model for determining the optimum number of replenishments n, the order quantity when demand is random. He suggested for further studies that the model can be implemented on a computer.

This is the basis of our new optimal algorithm which we implemented on a computer using a program written in MATLAB.

We now propose a new optimal algorithm to solve the problem encountered by Osagiede [6] in the next section.

4.0 Optimal Algorithm:

Step 1: Initialization:

Given the values of parameters, compute the economic stock level p^* using

$$f(x \le p^* - 1) < \frac{c_2}{c_1 + c_2} < f(x \le p^*)$$

Step 2. Calculation of the total inventory cost:

Starting with p = 0, and increase p by 1 and calculate

(a) $\sum x \ fx$ where $f(x)$ is the probability distributions for demand x

(b) $W(1)$ *and* $W(2)$ using equation (5) $W(p) = c_1 \sum_{x=0}^{p} (p-x) f(x) + c_2 \sum_{x=p+1}^{\infty} (x-p) f(x)$

If $W(1) < W(2)$ means that only one replenishment is obtained, else repeat step 2 again if $W(1) > W(2)$.

Step 3. Optimality test: Compare three successive values of W (p)

Allow $p^* = p - 1$ be a positive integer. If p^* satisfies equation (10) $W(p^* - 1) > W(p^*) < W(p^* + 1)$, then this value is the optimal number of replenishment and return to step 4, otherwise go back to step 2.

Step 4. Determination of replenishment points and quantities

Let p^* be the optimal number replenishment obtained in step 3. Determine the quantities using $q_{n+t}^* = p^* - p_t - (q_{t+1} + q_{t+2} + \cdots + q_{n+1-1}^*)$.

This ends the new proposed optimal algorithm for the random demand incorporating lead time.

5.0 Numerical Illustration.

In this section, we shall illustrate our new proposed algorithm solution model with some hypothetical example. This example is from Osagiede [6]

Example: A particular warehouse receives stock replenishment supply of an article every month in a multiple of 100 with the following parameters as shown in Table 1

Table 1: Parameters for the problem.

q	q	q	q	q	p	c_1	c_2	T	α
100	200	200	200	300	100	N1,500	N18,000	One month	6 months

The probability distribution of demand x, and the lead time been six months is shown in Table 2;

Table 2: Probability distribution for demand x

x	$f(x)$	$f(x \leq p)$
0	0.000	0.000
100	0.002	0.002
200	0.008	0.010
300	0.022	0.032
400	0.046	0.078
500	0.078	0.156
600	0.109	0.265
700	0.131	0.396
800	0.138	0.534
-	-	-
-	-	-
-	-	-
> 2000	0.004	1.000

The $f(x)$ in Table 2 was carefully selected so as to satisfies the properties of a probability mass function (pmf) and to suit his propose model.

Our interest in our new proposed algorithm solution model is to find the optimal value q_6^* that should be ordered in the sixth month. With the algorithm and the implementation of the parameters in the computer, the results obtained are the same as those obtained in Osagiede [6] where $\lambda = 0 \cdot 9226$ while Osagiede [6] result is 0.923 from step 1 of our algorithm, the optimal stock level for the period α six months is

$$[f(x < 1200 < 0 \cdot 9226 < f(x < 1{,}300)]$$

This means that, the probability lies between 0.910 and 0.947.

Thus $p^* = 1{,}300$ units.

From step 4 we obtain q_6^* using equation (15), q_6^* = 199.60 units and Osagiede [6] obtained q_6^* = 200 units

6.0 Remark and conclusion:

The differences observed in the values obtained is insignificant, however if we approximate, we also obtain same results as those values of Osagiede [6]. This means that at the sixth month 200 units of stock is ordered. The use of our new optimal algorithm saves time and reduces the computational task in calculating the quantities for the system at every interval.

A simple algorithm for the inventory model with random demand and lead time is examined. A soft computing method for solving this problem is proposed. Clearly the new optimal algorithm solution procedure reported in this paper is computationally efficient.

This work is motivated by the fact that Osagiede [6] suggested that his work can be implemented in a computer in his recommendation for further studies. This we have done.

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