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Analysis of A Plane Strain Forging Process Using The Weighted Residual Finite Element Method

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Abstract

This paper reports on the analysis of the forging operation using the Bubnov-Galerkin weighted residual finite element model to get the stresses and pressure fields set up at various cross-section of a blank during forging operation. Four Lagrange quadratic elements were assembled to represent the blank. The governing equation is a one dimensional differential equation which describes the pressures and stresses exerted on a forging. In conducting the analysis, we split the blank into a finite number of elements and apply the Bubnov-Galerkin weighted residual scheme to obtain the weighted integral form, the finite element model is obtained in a matrix form from weighted residual boundary conditions are now applied to obtain the pressure distribution across the cross-section of the blank .Finite element results are obtained for a particular value of the co-efficient of friction, die angle ,and blank length and compare with the exact solution on a graph. Using numerical example, we show that the weighted residual finite element method is capable of accurately predicting the pressures and stresses in an open die forging operation.

Keywords: Bubnov-Galerkin, weighted residual, forging operation, finite element method, Lagrange, quadratics.

1.0 Introduction

Forging is one of the oldest and still remains one of the fastest method of shaping metal and other materials. The need therefore arises to critically and numerically analyze the drawing operation in order to predict the various stresses and pressures field set up at a particular cross section of a given blank material. The estimated pressure and stresses can thus be compared with the strength of the material and this aids the determination of the smallest pressure required to cause the bulk plastic flow of the material. Consequent upon this the fundamental and versatile forging process, a large number of research papers into metal forming process exist in the literature. Akpobi and Edobor [1] developed a model for analyzing forging process. Navarrete, et al [2] used a dimensional analysis approach to determine the die forging stress in open die forging. They proposed five dimensionless groups from the process variables in an attempt to simplify the forging stress determination. Alfozan and Gunasekera [3] proposed an upper bound element technique approach to the process design of axisymmetric forging by forward and backward simulation. Oviawe and Omorodion[4] determination of stresses in hot bar forging process using the weighted residual finite element model.Oviawe and Asikhia [5] determination of stresses in Wire- Drawing Operation using the finite Element model .Oviawe and Oviawe [6] analysis of axisymmetric forging operation using the Bubnov-Galerkin weighted residual finite element method .Oviawe and Aziegbemhen[7] developed a model for analyzing critical buckling load in structural member. Oviawe and Dibie [8] analysis of forging of flat lubricated disc with coulomb friction operation using the Bubnon-Galerkin finite element model.

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2.0 Materials and Methods

In this research, we employ the weighted residual finite element method as a numerical tool use in obtaining the forging stresses and pressures distribution on a material during forging. Due to symmetry, analysis is carried out on half the blank. The blank is represented by a mesh finite elements and Bubnov – Galerkin weighted residual scheme is applied to get the value of pressure at nodal points. Four quadratic elements were used to ensure an accurate solution. A numerical analysis is done to compare the finite element results with the exact solution.

Formulation of Governing Equation

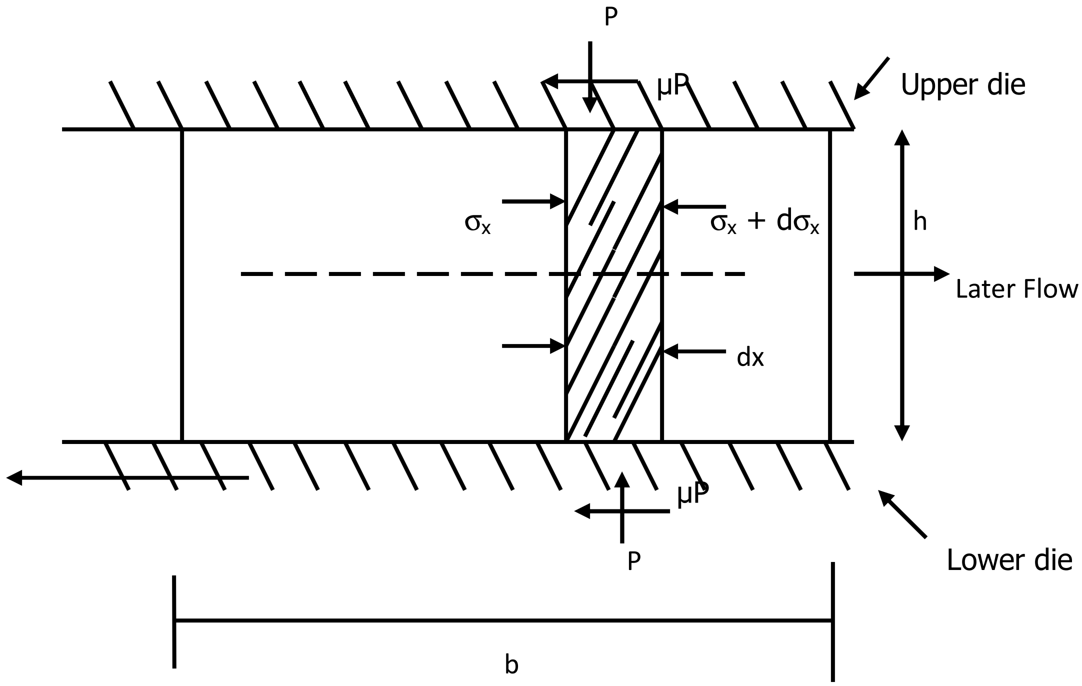

Fig 1.free body diagram of plain strain forging operation

In analysis the operation, it is assumed that the work piece material behaves like ideal plastic material, the entire work piece is in a plastic state and the stresses acting on the material do not vary with height. At any instant of forging, the equilibrium equation of small element of width dx in the x – direction as shown in ig. Fig 1. gives,

$$-(\sigma_x + d\sigma_x)h + \sigma_x h - 2\mu p dx = 0 \qquad (1)$$

$$-\sigma_x h + d\sigma_x h + \sigma_x h - 2\mu p dx = 0$$

$$-h d\sigma_x - 2\mu p dx = 0$$

Dividing through by hdx, gives

$$\frac{d\sigma_x}{dx} + \frac{2\mu p}{h} = 0 \qquad (2)$$

Generally, we consider a forging process where interfacial friction is involved and as such we assume coulomb friction with constant coefficient of friction and we apply Tresca's yield criterion.

We assume $\sigma_1 = \sigma_x$ and $\sigma_3 = -P$ are principal stresses.

i.e.

$$\sigma_x + P = \sigma_o \qquad (3)$$

Differentiating equation (3) and σ_o constant in cold forging as there is no work hardening.

$$\frac{d\sigma_x}{dx} = \frac{dp}{dx} \qquad (4)$$

Substituting equation (4) into equation (2)

$$\frac{dp}{dx} + \frac{2\mu P}{h} = 0 \qquad (5)$$

WEIGHTED INTEGRAL FORMULATION:

The weighted integral form of equation (5) is obtained by multiplying it by the weight function, W and integrating over the domain enclosing an element with respect to x

$$\int_o^L W\left(\frac{dp}{dx}+\frac{2\mu p}{h}\right)dx = 0 \qquad (6)$$

Where W = weight function, expanding equation (6) gives

$$\int_o^L \frac{Wdp}{dx}dx + \int_o^L \frac{W2\mu p}{h}dx = 0 \qquad (7)$$

An examination of equation (7) shows that the solution and hence the approximation function should be once differentiable with respect to x, Thus, the Lagrange family of interpolation function can be used satisfactorily.
Let us assume that the solution P is approximated as follows:

$$P \approx p^e = \sum_{j=1}^{n} p^e \psi_j^e(x) \qquad (8)$$

By adopting the Bubnov – Galerkin weighted residual method in which it is assumed that the weight function is equal to the interpolation function.

i.e $W = \psi_j^e(x)$ (9)

Substituting equation (8) and (9) into equation (7)

$$\int_o^L \psi_j^e \frac{d}{dx}\sum_{j=i}^{n} P_j^e dx + \int_o^L \psi_j^e \frac{2\mu}{h}\sum_{j=i}^{n} P_j^e \psi_j^e dx = 0$$

$$\sum_{j=i}^{n}\left\{\int_o^L\left(\psi_j^e \frac{d}{dx}\psi_j^e + \frac{2\psi}{h}\psi_j^e\psi_j^e\right)\right\}dx\left[P_j^e\right] = 0 \qquad (10)$$

Equation (10) can be recast in the form;

$$\left\{K_{ij}^e\right\}\left\{P_j^e\right\} = 0 \qquad (11)$$

Equation (11) is the weighted residual finite element model of equation (2)
Where,

$$K_{ij}^e = \int_o^L\left(\psi_j^e \frac{d}{dx}\psi_j^e + \frac{2\mu}{h}\psi_j^e\psi_j^e\right)dx \qquad (12)$$

Using the Lagrange quadratic interpolation function

$$\psi_1^e = \left(1-\frac{x}{L}\right)\left(1-\frac{2x}{L}\right)$$

$$\psi_2^e = \frac{4x}{L}\left(1-\frac{x}{L}\right)$$

$$\psi_3^e = -\frac{x}{L}\left(1-\frac{2x}{L}\right)$$

$$K_{11}^e = \int_o^L\left(\psi_1^e \frac{d\psi_1^e}{dx} + \frac{2\mu}{h}\psi_1^e\psi_1^e\right)dx$$

$$K_{11}^e = \frac{-90h+4\mu L}{60h}$$

$$K_{12}^e = \int_o^L\left(\psi_1^e \frac{d\psi_2^e}{dx} + \frac{2\mu}{h}\psi_1^e\psi_2^e\right)dx$$

$$K_{12}^e = \frac{10h+2\mu L}{60h}$$

$$K_{13}^e = \int_o^L \left(\psi_1^e \frac{d\psi_3^e}{dx} + \frac{2\mu}{h} \psi_1^e \psi_3^e \right) dx$$

$$K_{13}^e = \frac{5h + 2\mu L}{60h}$$

$$K_{21}^e = \int_o^L \left(\psi_2^e \frac{d\psi_1^e}{dx} + \frac{2\mu}{h} \psi_2^e \psi_1^e \right) dx$$

$$K_{21}^e = \frac{-10h + 2\mu L}{60h}$$

$$K_{22}^e = \int_o^L \left(\psi_2^e \frac{d\psi_2^e}{dx} + \frac{2\mu}{h} \psi_2^e \psi_2^e \right) dx$$

$$K_{22}^e = \frac{16\mu L}{60h}$$

$$K_{23}^e = \int_o^L \left(\psi_2^e \frac{d\psi_3^e}{dx} + \frac{2\mu}{h} \psi_2^e \psi_3^e \right) dx$$

$$K_{23}^e = \frac{15h + 2\mu L}{60h}$$

Due to symmetry

$$K_{13}^e = -K_{31}^e = \frac{5h - 2\mu L}{60h}$$

$$K_{12}^e = -K_{32}^e = \frac{10h + 2\mu L}{60h}$$

$$K_{11}^e = -K_{33}^e = \frac{90h + 4\mu L}{60h}$$

Hence, for one Lagrange quadratic element.

$$K_{ij}^e = \frac{1}{60h} \begin{bmatrix} -90h + 4\mu L & 10h + 2\mu L & -5h - 2\mu L \\ -10h + 2\nu L & 16\mu L & -90h + 4\mu L \\ 5h - 2\mu L & 10h + 2\mu L & -90h + 4\mu L \end{bmatrix} \qquad (13)$$

In order to ensure high accuracy, we used a mesh of four quadratic elements (9 nodes).
Dividing the domain into four 1 – D quadratic finite elements and the finite elements model over an element is given as:

$$K_{ij}^e = \begin{bmatrix} K_{11}^e & K_{12}^e & K_{13}^e \\ K_{21}^e & K_{22}^e & K_{23}^e \\ K_{31}^e & K_{32}^e & K_{33}^e \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} Q_1 \\ 0 \\ Q_3 \end{bmatrix} \qquad (14)$$

For a mesh of four 1 – D quadratic element the assembled equations are:

$$K_{ij}^{e}=\frac{1}{60}\begin{bmatrix} K_{11}^{1} & K_{12}^{1} & K_{13}^{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ K_{21}^{1} & K_{22}^{1} & K_{23}^{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ K_{31}^{1} & K_{32}^{1} & K_{33}^{1}+K_{11}^{2} & 0 & K_{12}^{2} & K_{13}^{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & K_{21}^{2} & 0 & K_{22}^{2} & K_{23}^{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & K_{31}^{2} & K_{32}^{2} & K_{33}^{2}+K_{11}^{3} & & K_{12}^{3} & K_{13}^{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & K_{21}^{3} & & K_{22}^{3} & K_{23}^{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & K_{31}^{3} & K_{32}^{3} & K_{33}^{3}+K_{11}^{4} & 0 & K_{12}^{4} & K_{13}^{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & K_{21}^{4} & 0 & K_{22}^{4} & K_{23}^{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & K_{31}^{4} & 0 & K_{32}^{4} & K_{33}^{4} \end{bmatrix}=\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \\ P_9 \end{bmatrix}=0 \quad (15)$$

Substituting into Equation (15),it becomes,

$$K_{ij}^{2}=\frac{1}{60}\begin{bmatrix} -90h+4\mu L & 10h+2\mu L & -5h-2\mu L & 0 & 0 & 0 & 0 & 0 \\ -10h+2\mu L & 16\mu L & 15h+2\mu L & 0 & 0 & 0 & 0 & 0 \\ -5h-2\mu L & 10h+2\mu L & 8\mu L & 10+2\mu L & -5h-2\mu L & 0 & 0 & 0 \\ 0 & 0 & -10h+2\mu L & 16\mu L & 15h+2\mu L & 10h+2\mu L & 0 & 0 \\ 0 & 0 & -5h-2\mu L & 10h+2\mu L & 8\mu L & 16\mu L & 15h+2\mu L & -5h-2\mu L \\ 0 & 0 & 0 & 0 & -10h+2\mu L & 8\mu L & 10h+2\mu L & 15h+2\mu L \\ 0 & 0 & 0 & -5h-2\mu L & 10h+2\mu L & -10h+2\mu L & 16\mu L & -90h+4\mu L \\ 0 & 0 & 0 & 0 & 0 & -5h-2\mu L & 10h+2\mu L & 0 \\ 0 & 0 & 0 & 0 & 0 & & & \end{bmatrix}=\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \\ P_9 \end{bmatrix} \quad (16)$$

The boundary condition is

$$AtX = L, \sigma_x = 0$$

From Tresca"s yield criterion, $\sigma_x + P = \sigma_o = 2k$

Therefore, $x = L,\ P_q = \sigma_o = 2k$

Hence, the only unknown pressures are P_1, P_2, P_3, P_4, P_5, P_6, P_7 and P_8.

Equation(16) becomes,

$$K_{ij}^{e}=\frac{1}{60h}\begin{bmatrix} -90h+4\mu L & 10h+2\mu L & -5h-2\mu L & 0 & 0 & 0 & 0 & 0 \\ -10h+2\mu L & 16\mu L & 15h+2\mu L & 0 & 0 & 0 & 0 & 0 \\ -5h-2\mu L & 10h+2\mu L & 8\mu L & 10h+2\mu L & -5h-2\mu L & 0 & 0 & 0 \\ 0 & 0 & -10h+2\mu L & 16\mu L & 15h+2\mu L & 0 & 0 & 0 \\ 0 & -5h-2\mu L & 10h+2\mu L & 8\mu L & 10h+2\mu L & -5h-2\mu L & 0 & \\ 0 & 0 & 0 & -10h+2\mu L & 16\mu L & 15h+2\mu L & 0 & \\ 0 & 0 & 0 & -5h-2\mu L & 10h+2\mu L & 8\mu L & 10h+2\mu L & \\ 0 & 0 & 0 & 0 & -10h-2\mu L & 0 & 16\mu L & \end{bmatrix}=\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \end{bmatrix}=\frac{\sigma_o}{60h}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -5h-2\mu L \\ 15h+2\mu L \end{bmatrix} \quad (17)$$

The stresses are obtained by substituting the values of the pressures into the equation:

$$\sigma_x = \sigma_o - P \quad (18)$$

Post – Processing of Solution:

In order to get the pressure at any point of the blank, we make use of the following finite element solution.

$$P(x) = P_1\psi_1^1 + P_2\psi_2^1 + P_3\psi_3^1 \qquad For\ 0 \le \times \le \frac{L}{4}$$

$$P(x) = P_3\psi_1^2 + P_4\psi_2^2 + P_5\psi_3^2 \qquad For\ \frac{L}{4} \le \times \le \frac{L}{2}$$

$$P(x) = P_5\psi_1^3 + P_6\psi_2^3 + P_7\psi_3^3 \qquad For\ \frac{L}{2} \le \times \le \frac{3L}{4}$$

$$P(x) = P_7\psi_1^4 + P_8\psi_2^4 + P_9\psi_3^4 \qquad For\ \frac{3L}{4} \le \times \le L$$

Where P is the pressure at node 1 and Ψ_i^e is the ith Lagrange length interpolation function for the eth element.

Exact solution:

Recall the equation (5)

$$\frac{dP}{dx}+\frac{2\mu P}{h}=0$$

Separating variables

$$\frac{dP}{dx}=-\frac{2\mu P}{h}$$

i.e. $$\frac{dP}{p}+\frac{-2\mu dx}{h} \qquad (19)$$

Integrating both sides

$$\text{In}\,P=-\frac{2\mu x}{h}+C \qquad (20)$$

$$P=C.e^{\left(-\frac{2\mu x}{h}\right)} \qquad (21)$$

From boundary condition;

$At \quad x=L,\sigma_x=0$ (stress free surface) and we have

$P=\sigma_o$

Substituting into equation (20)

$$\therefore \text{in}\ \sigma_o=-\frac{2\mu L}{h}+C$$

$$\therefore C=In\sigma_o+\frac{2\mu L}{h}$$

$$\text{In}\ \frac{P}{\sigma_o}=-\frac{2\mu}{h}(L-x) \qquad (22)$$

$$\therefore \frac{P}{\sigma_o}=e^{\frac{2\mu}{h}(L-x)} \qquad (23)$$

Recall that

$$\sigma_x+P=\sigma_o=2k \qquad (24)$$

$$\therefore \frac{P}{2k}=\frac{P}{\sigma_o}=e^{\frac{2\mu}{h}(L-x)}$$

$$\therefore \sigma_x=\sigma_o-P=\sigma_o\left[I-e^{\frac{2\mu}{h}(L-x)}\right]$$

$$\therefore \sigma_x=\sigma_o\left[I-e^{\frac{2\mu}{h}(L-x)}\right]$$

Numerical example;

Consider a plain strain forging operation in which length of billet = 100mm, width = 80mm and thickness = 3mm, μ = 0.25

Using MATHCAD soft ware and substituting these values into the developed model in equation (17) shown the graphical comparison of friction hills of finite element and exaction solution.

Forging pressure against distance from centre of blank (x)

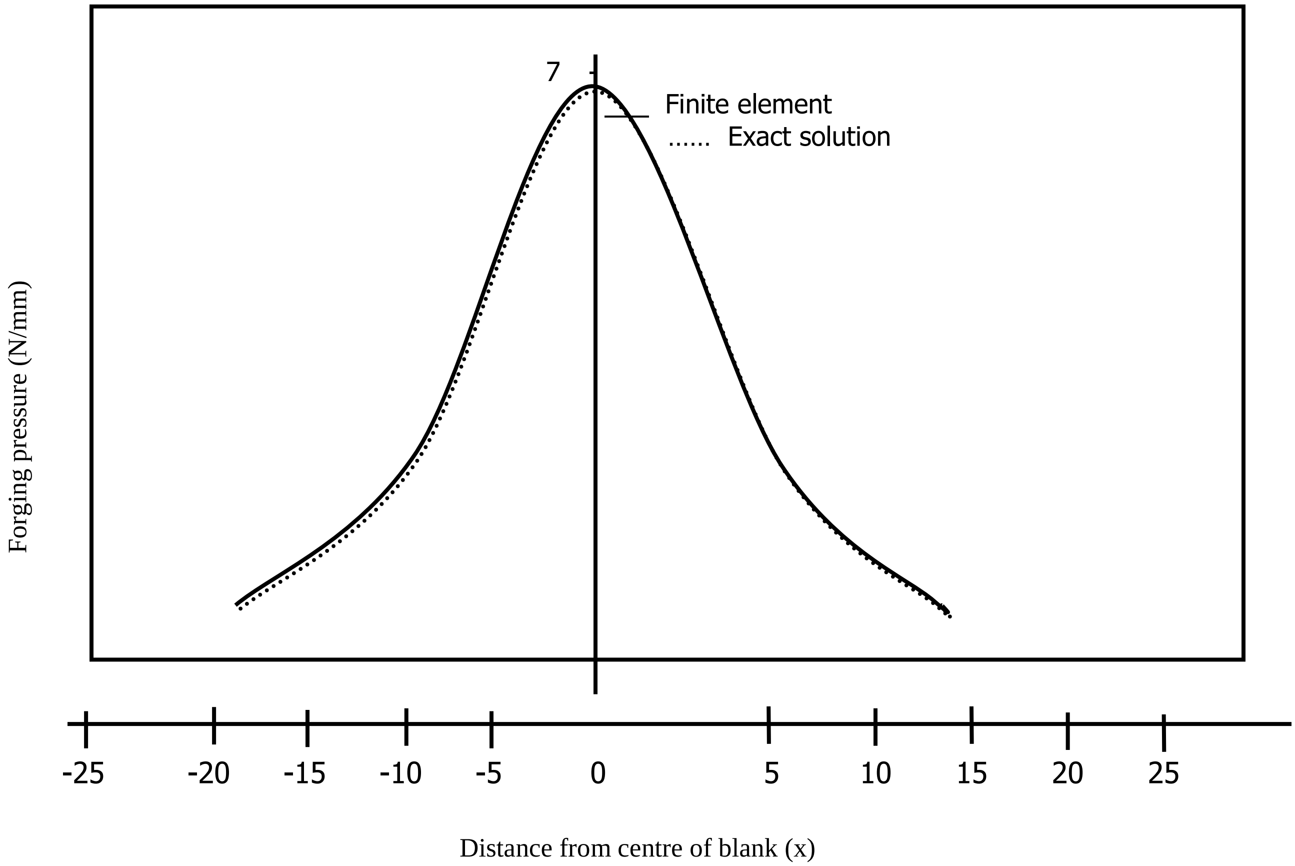

Fig 2. Graphical comparison of friction hill of the exact solution and finite element solution

Discussion of result:

The solution obtained can be applied to all open die forging problems. This advantage is as a result of the fact that numerical values of the pressures and stresses of such problems can be determined by simply substituting the appropriate values of the co-efficient of friction height of forging and half the length of the forging into the model provided. Also, a careful examination of Fig. 2 show that the differences between the friction hills described by the finite element solution and the exact solution is infinitesimal and negligible.

Conclusion

From the discussion, it can therefore be concluded that the weighted residual finite element method is capable of adequately and accurately predicting the stresses and pressure fields set up in a particular axi symmetric forging operation.

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