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Heat Exchanger Network Synthesis using Non Linear Programming Superstructure Approach

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Abstract

The synthesis of a heat exchanger network using non linear programming (NLP) superstructure approach was carried out in this study. The minimum utility cost, minimum number of heat exchanger units and heat exchanger network superstructure corresponding to the minimum utility cost and minimum number of heat exchanger units were determined sequentially. The minimum utility cost was determined from the linear programming (LP) formulation while the minimum number of units was determined from the mixed integer linear programming (MILP) formulation. The LP and MILP mathematical formulations were implemented in GAMS (General Algebraic Modelling System), a high-level modelling system for mathematical programming problems. The minimum utility cost was determined to be $475,200 per annum while the minimum number of units was 13 with 7 heat exchanger units above the pinch point and 6 heat exchanger units below the pinch point. The heat exchanger network superstructure corresponding to minimum utility cost and minimum number of units was generated from an NLP formulation which was solved using GAMS to obtain the optimum heat exchanger network superstructure. The cost of the network configuration generated was estimated to be $536,102 per annum.

Keywords: NLP, Heat Exchanger, Pinch point, Transshipment model, Superstructure, Network synthesis

Nomenclature

A	Heat transfer area (m^2)
C	Cold stream
C_W	Cooling water
FC_p	Heat capacity flowrate (kW/K)
GAMS	General Algebraic Modelling System
H	Hot stream
HEN	Heat exchanger network
HP	High pressure
LMTD	Logarithmic mean temperature difference (K)
LP	Linear Programming
MILP	Mixed Integer Linear Programming
Q	Heat duty (kW)
q	Sub-network
R	Heat residual (kW)
T_{in}	Inlet temperature (K)
T_{out}	Outlet temperature (K)
U	Overall heat transfer coefficient (kW/m^2)
Y	Binary variable

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Greek Letters

$\forall$	For all
$\sum$	Sigma

1.0 Introduction

In most industrial processes, it is often desired to heat certain process streams while other streams need cooling. This often gives rise to high consumption of energy and this is the major challenge encountered in such processes as the cost of energy accounts for a substantial percentage of the total operating cost of such plants [1].

The heating and cooling of process streams is usually accomplished by utilising heat exchangers in which hot or cold utilities are used to heat cold process streams or cool hot process streams respectively. Heat Exchanger Network (HEN) synthesis is one of the most studied synthesis problems in chemical engineering and involves solving a three-way trade-off between energy (Q), heat transfer area (A), and how this total area is distributed into a number of heat transfer units (n) [2,3]. HEN synthesis has been the subject of much research over the past four decades. Various methodologies have been proposed to bring about energy recovery between process streams, minimizing hot or cold utility consumption, number of heat exchanger units required, annualized cost as well as an optimal network configuration corresponding to minimum utility cost.

The HEN synthesis process has some challenges which include the potentially explosive combinatorial problem of identifying the best pair of stream matches, required matches and restriction in matches, optimal selection of the heat exchanger network structure, type of heat exchanger unit to be used etc. The need to provide solutions to these problems has seen a lot of research efforts directed towards this area. Thermodynamic, heuristic and optimization approaches have been developed to tackle these difficulties [4-10]

In this work, the objective is to synthesize a heat exchanger network configuration (superstructure) with minimum number of units and minimum annual utility cost using non linear programming algorithms. The procedure adopted involves the formulation of optimisation problems which were subsequently solved to determine the best alternative or optimal choice in the form of minimum annualized cost, minimum number of heat exchanger units and optimal heat exchanger superstructure.

Problem Definition

Four hot process streams designated H1, H2, H3 and H4 are desired to be cooled and two cold process streams designated C1 and C2 are to be heated. As shown in Table 1, each hot and cold process stream has a specified heat capacity flow rate, inlet and outlet temperatures. Hot utilities such as fuel, high pressure steam and low pressure steam and a cold utility (cooling water) are also available along with their corresponding temperatures and costs. Additional data include overall heat transfer coefficient, operating time and cost of heat exchangers as a function of heat exchange area.

Table 1: Data for minimum annual utility cost problem (process streams).

Stream	T_{in} **(K)**	T_{out} **(K)**	FC_p **(kW/K)**
H1	700	420	20
H2	600	310	40
H3	460	310	70
H4	360	310	94
C1	350	650	50
C2	300	400	180

Table 2: Data for minimum annual utility cost problem (utilities)

Utility	**Temperature (K)**	**Cost (US$/kJ)**	**Maximum available**
Fuel	750	5×10^{-6}	No limit
HP Steam	510	3×10^{-6}	1000 kW
LP Steam	410	1.8×10^{-6}	500 kW
Cooling water	290-325	7×10^{-7}	No limit

Additional data:

1. Overall heat transfer coefficient, U: 1.4285kW/m^2
2. Operating time: 8000 hr/yr
3. Cost of heat exchangers: US4000A^{0.6}$/yr (A in m^2)

Task 1: Optimisation of utilities cost

The minimum utility cost problem was solved using the method proposed by Floudas [11] using a minimum temperature approach of 20K. According to Floudas [11], the minimum utility cost problem can be treated as a transport problem by regarding heat as a commodity that is transferred from the hot process streams and hot utilities (referred to as sources) to the cold process streams and cold utilities (referred to as destination nodes) via temperature intervals (referred to as warehouses) that guarantee feasible heat exchange. When there is excess heat or that which is not allocated to a destination node at a given temperature interval, it is cascaded down to lower temperature intervals as heat residuals. The minimum utility cost of the network was determined by formulating a linear programming (LP) transshipment model. The temperature interval representation of the problem is shown in Figure 1. The heat loads for the respective temperature intervals were calculated by using the expression presented in equation (1) for the hot streams and (2) for the cold streams.

$Q_{ik} = FCp\Delta T$ (1)

$Q_{jk} = FCp\Delta T$ (2)

Q_{ik} and Q_{jk} represent heat loads of hot and cold streams respectively in temperature interval k.

The LP Transshipment model was obtained by performing a heat balance about all the temperature intervals. A generalized representation of a given temperature interval is shown in Figure 2. The heat loads associated with the respective temperature intervals are shown in Table 3.

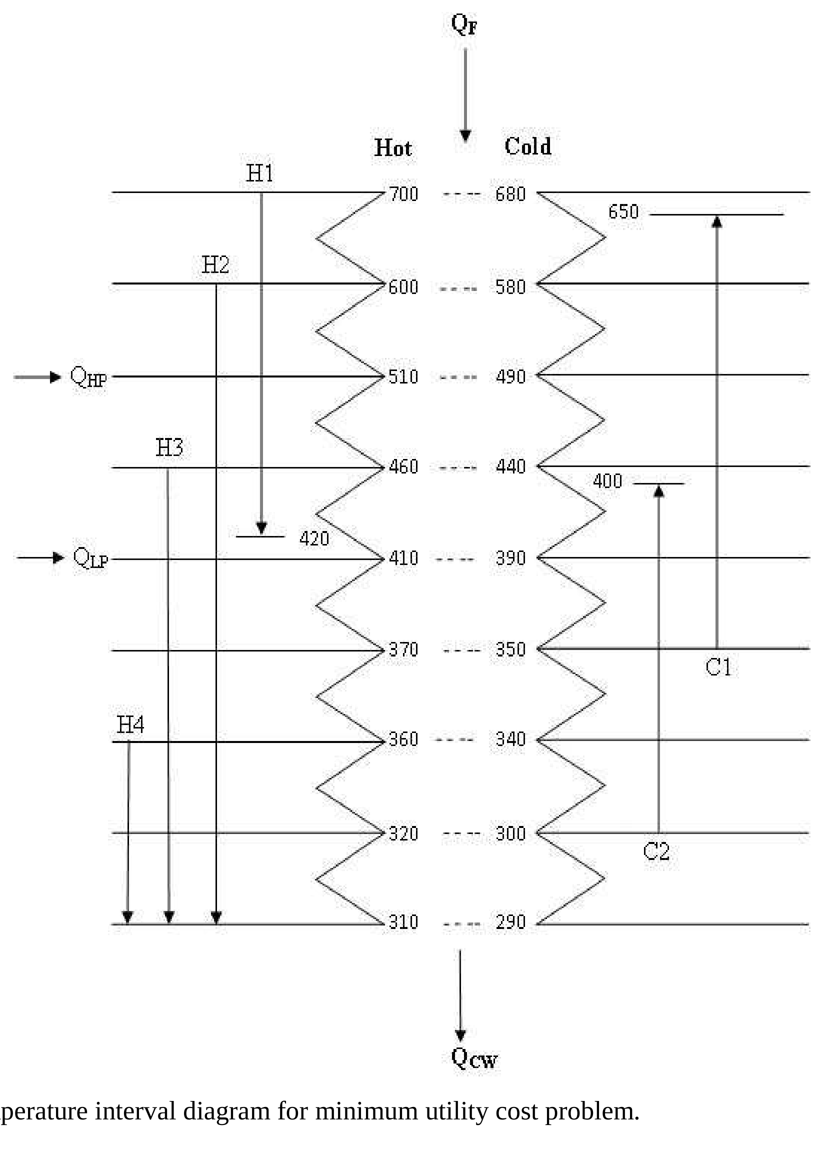

Figure 1: Temperature interval diagram for minimum utility cost problem.

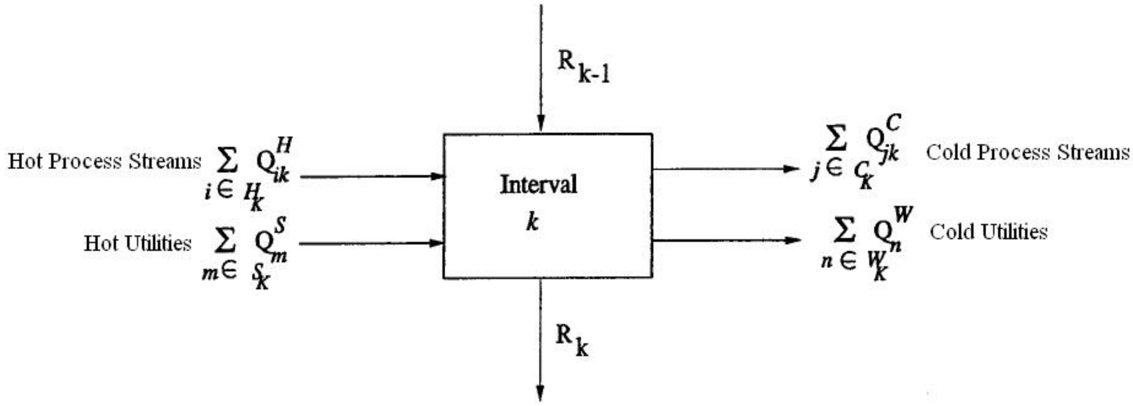

Figure 2: Heat flow pattern for a given temperature interval.

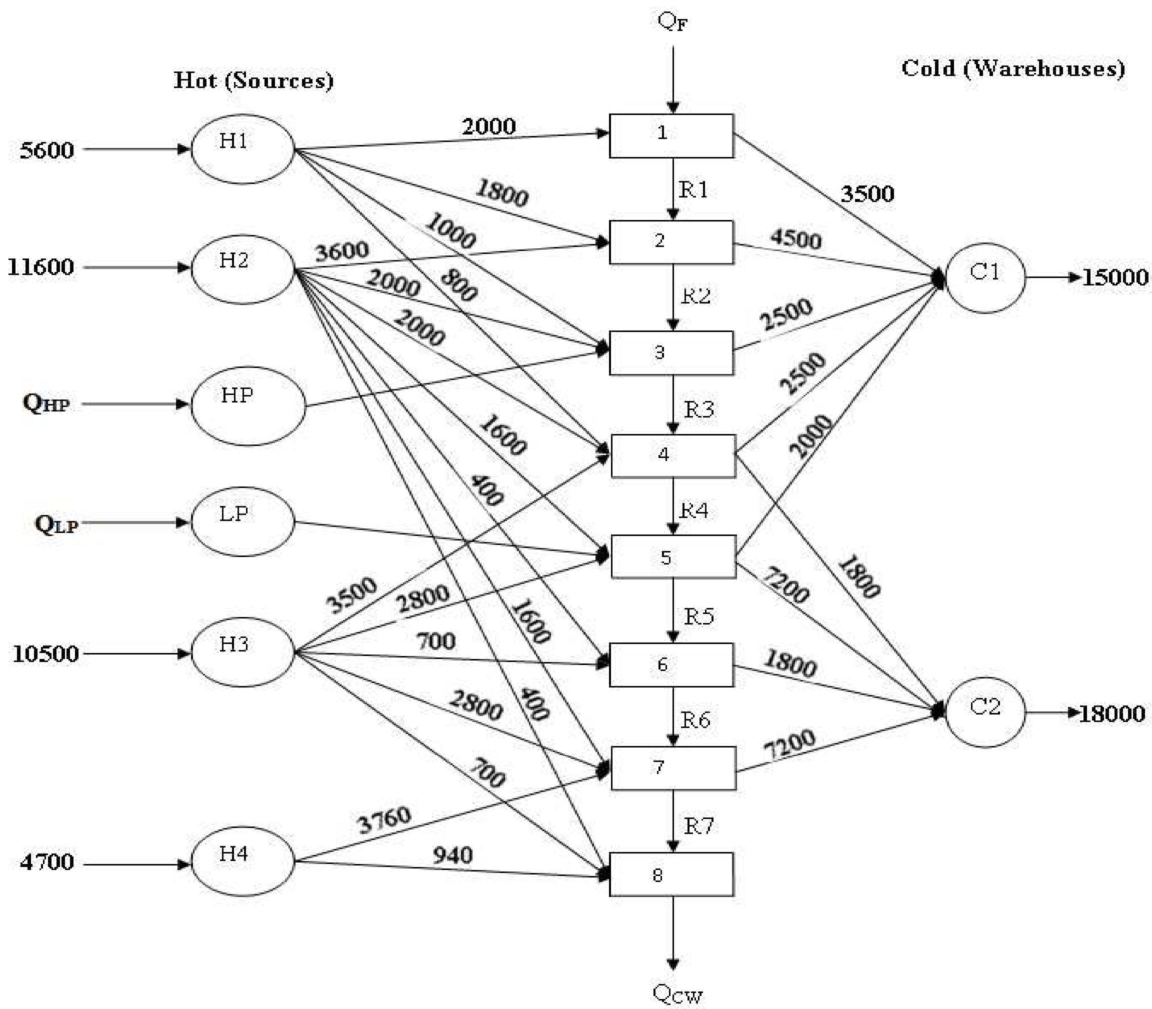

Figure 3: Transshipment representation of minimum utility cost problem

Table 3: Heat loads in respective temperature intervals

Inter	H1	H2	H3	H4 (kW)	C1	C2
1	2000	0	0	0	3500	0
2	1800	3600	0	0	4500	0
3	1000	2000	0	0	2500	0
4	800	2000	3500	0	2500	1800
5	0	1600	2800	0	2000	7200
6	0	400	700	0	0	1800
7	0	1600	2800	3760	0	7200
8	0	400	700	940	0	0
Total	**5600**	**11600**	**10500**	**4700**	**15000**	**18000**

The transshipment representation of the minimum utility cost problem is shown in Figure 3. It shows the heating and cooling loads (Q_F, Q_{HP}, Q_{LP} and Q_{CW}) and the heat residuals (R1, R2, R3, R4, R5, R6 and R7). Performing a heat balance about the 8 temperature intervals in Figure 3 in accordance with the model shown in Figure 2 results in the following:

Interval 1: $Q_F+2000+R1+3500$

$R1-Q_F=-1500$ (3)

Interval 2: $R1+1800+3600=R2+4500$

$R2-R1=900$ (4)

Interval 3:

$R2+1000+2000+Q_{HP}=R3+2500+4500$

$R3-R2-Q_{HP}=500$ (5)

Interval 4: $R3+800+2000+3500=R4+2500+1800$

$R4-R3=2000$ (6)

Interval 5:

$R4+1600+Q_{LP}+2800=R5+2000+7200$

$R5-R4-Q_{LP}=-4800$ (7)

Interval 6: $R5+400+700=R6+1800$

$R6-R5=-700$ (8)

Interval 7:

$R6+1600+2800+3760=R7+7200$

$R7-R6=960$ (9)

Interval 8:

$R7+940+700+400=Q_{CW}$

$R7-Q_{CW}=-2040$ (10)

The total annual cost is given as follows:

Maximum available high pressure steam is 1000 kW: $Q_{HP}\leq 1000$ (11)

Maximum available low pressure steam is 500 kW: $Q_{LP}\leq 500$ (12)

The total cost of utilities is given as:

$C=\sum C_i Q_i t_o$ (13)

Where C_i, Q_i and t_o are cost, heat load and operating time (in seconds) respectively.

For fuel:

$C_F=144Q_F$ (14)

For high pressure steam:

$C_{HP}=86.4Q_{HP}$ (15)

For low pressure steam:

$C_{LP}=51.84Q_{LP}$ (16)

For cooling water:

$C_{CW}=20.16Q_{CW}$ (17)

Total cost $C=C_F+C_{HP}+C_{LP}+C_{CW}$ (18)

$C=144Q_F+86.4Q_{HP}+51.84Q_{LP}+20.16Q_{CW}$ (19)

$R1,R2,R3,R4,R5,R6,R7,Q_F,Q_{HP},Q_{LP},Q_{CW}\geq 0$ (20)

The total cost is the objective function of the optimisation problem and the heat balances are the constraints. The objective function is minimised subject to the constraints provided.

In the following, the LP transshipment model is presented: (All heat loads and heat residuals are non negative)

$$Min.\ C=144Q_F+86.4Q_{HP}+51.84Q_{LP}+20.16Q_{CW}$$

Subject to equations (3) to (12) and (20)

This model was implemented in GAMS® (General Algebraic Modelling System). GAMS is an equation oriented general modelling system with proven capabilities for the optimisation of highly complex processes. It allows the user to write equations almost as they would appear on paper. The following results were obtained after the model was implemented in GAMS.

- Minimum annual utility cost: $475,200 per year
- Heat loads of utilities: $Q_F = 2100kW$, $Q_{HP} = 1000kW$, $Q_{LP} = 500$, $Q_{CW} = 3000kW$.
- Heat residuals: *R1=600kW, R2= 1500kW, R3= 3000kW, R4= 5000kW, R5=700 kW, R6=0kW, R7= 960kW.*
- Location of Pinch point: the pinch point is located at the points where the residual is zero. This is found to be the point at which *R6=0kW* i.e. 360-340K interval.

Task 2: Optimisation of number of heat exchanger units

The minimum number of heat exchanger units present in the physical heat exchanger network is equal to the minimum number of stream matches [12,13]. For this work, one pinch point was identified; hence a mixed integer linear programming (MILP) optimisation problem was formulated and solved for two sub-networks which were derived from partitioning the main network in line with the position of the pinch point. The MILP was formulated for a given sub-network *q* as follows.

$$\min \sum_{i\in H_k}\sum_{j\in C_k} y_{ij}^{\;q} \tag{21}$$

$$\text{subject to}: \quad R_{i,k} - R_{i,k-1} + \sum_{j\in C_k} Q_{ijk} = Q^H{}_{ik}, \qquad i\in H_k,\ k=1,.....,K_q \tag{22}$$

$$\sum_{i\in H_k} Q_{ijk} = Q^C{}_{jk},\ j\in C_k, \qquad k=1,.....,K_q \tag{23}$$

$$\sum_{k\in K_q} Q_{ijk} - Q^U{}_{ij}\, y^q{}_{ij} \le 0, \qquad i\in H,\ j\in C \tag{24}$$

$$R_{ik}, Q_{ijk} \ge 0, \qquad \forall i, \forall j, \forall k \tag{25}$$

$$y^q{}_{ij} \in \{0,1\}, \qquad \forall i, \forall j \tag{26}$$

The terms are as defined below.

- $Q^U{}_{ij}$ is the maximum heat that can be exchanged between hot stream *i* and cold stream *j*.
- $Q^H{}_{ik}$ is the heat content of hot stream *i* in temperature interval *k*.
- $Q^C{}_{jk}$ is the heat content of cold stream *j* in temperature interval *k.*
- Q_{ijk} is the amount of heat exchanged between hot stream *i* and cold stream *j* in temperature interval *k.*
- $R_{i,k}$ is the heat residual of hot stream *i* leaving temperature interval *k.*

The binary variables were defined to represent the potential match between a given pair of hot and cold streams. They are defined as follows.

$$y_{ij}^q = \begin{cases} 1\ hot\ stream\ i, cold\ stream\ j\ exchange\ heat \\ 0\ hot\ stream\ i, cold\ stream\ j\ do\ not\ exchange\ heat \end{cases} \tag{27}$$

For each predicted match with a binary variable having a value of one, there will be associated with it, a single heat exchanger unit in the physical network. This means that the sum of the units in a given sub-network will simply be the number of binary variables with a value of one. Logical constraints were also defined to ensure that if a predicted stream match does not occur, the associated heat exchange between both streams must be zero. This constraint is given as:

$$\sum_{k\in K_q} Q_{ijk} - Q^U{}_{ij}\, y^q{}_{ij} \le 0 \qquad i\in H,\ j\in C \tag{28}$$

Heat Exchanger Subnetworks

Above the pinch point:

Above the pinch point, the following 12 matches were predicted to occur:

HF- C1, HF - C2, H1 - C1, H1 -C2, H2-C1, H2-C2, H3-C1, H3-C2, HP-C1, HP-C2, LP-C1 and LP-C2. The corresponding binary variables and heat loads are:

Binary variables: F1, YF2, Y11, Y12, Y21, Y22, Y31, Y32, YHP1, YHP2, YLP1, and YLP2.
Heat loads: QF1, QF2, Q11, Q12, Q21, Q22, Q31, Q32, QHP1, QHP2, QLP1, and QLP2 respectively

Tables 4 shows the values of the heat load of the hot and cold streams as well as utilities respectively in the temperature intervals above the pinch point. These values were obtained from the transshipment representation of Figure 3.

Table 4: Heat load of hot streams, hot utilities, cold stream and cold utilities in different temperature intervals

	Interval 1	**Interval 2**	**Interval 3**	**Interval 4**	**Interval 5**	**Interval 6**
H1	2000	1800	1000	800	0	0
H2	0	3600	2000	2000	1600	400
H3	0	0	0	3500	2800	700
HF	2100	0	0	0	0	0
HP	0	0	1000	0	0	0
LP	0	0	0	0	500	0
C1	3500	4500	2500	2500	2000	0
C2	0	0	0	1800	7200	1800

The value of $Q^{U}{}_{ij}$ (heat exchange between hot stream *i* and cold stream *j*) can be evaluated from Tables 4 and the result is presented in Table 5.

Table 5: Maximum heat exchange between hot stream *i* and cold stream *j*.

	H1	**H2**	**H3**	**HF**	**HP**	**LP**
C1	5600	9600	7000	2100	1000	500
C2	5600	9600	7000	2100	1000	500

The model was formulated and solved using GAMS. From the results of the simulation, the following were obtained:
Binary variable: YF1=1, YF2=0, Y11=1, Y12=0, Y21=1, Y22=1, Y31=0, Y32=1, YHP1=0, YHP2=1, YLP1=1, YLP2=0.
Heat loads: QF1=2100, QF2=0, Q11=5600, Q12=0, Q21=7300, Q22=2300, Q31=0, Q32=7000, QHP1=0, QHP2=1000, QLP1=0, QLP2=500
Seven stream matches were obtained corresponding to the stream matches with a binary variable having a value of one. They are given below with their corresponding heat loads.
YF1=1, Y11=1, Y21=1, Y22=1, Y32=1, YHP2=1, YLP1=1
HF-C1 (QF1=2100kW), H1-C1 (Q11=5600kW), H2-C1 (Q21=7300kW), H2-C2 (Q22=2300kW), H3-C2 (Q32=7000kW), HP-C2 (QHP2=1000kW), LP-C2 (QLP2=500kW).

Below the pinch point

Below the pinch point, the following 6 matches were predicted to occur:
H2-C2, H3-C2, H4-C2, H2-CW, H3-CW, H4-CW
The corresponding binary variables and heat loads are:
Binary variables*:* Y22, Y32, Y42, Y2W, Y3W, Y4W
Heat loads: Q22, Q32, Q42, Q2W, Q3W, and Q4W respectively
Tables 6 show the values of the head load of the hot and cold streams respectively in the temperature intervals below the pinch point. These values were also obtained from the transshipment representation of Figure 3.

Table 6: Heat load of hot streams and hot utilities in different temperature intervals

	Interval 7	**Interval 8**
H2	1600	400
H3	2800	700
H4	3760	940
C2	7200	400
CW	0	3000

The heat exchange between hot stream *i* and cold stream *j* was evaluated from Table 6 and the result is presented in Table 7.

Table 7: Maximum heat exchange between hot stream *i* and cold stream *j*

	H2	**H3**	**H4**
C2	5600	9600	7000
CW	5600	9600	7000

The model was formulated and solved using GAMS. From the solution of the model, the following were obtained:
Binary variables: Y22=1, Y32=1, Y42=1, Y2W=1, Y3W=1, Y4W=1
Heat loads: Q22=640 kW, Q32=2800 kW, Q42=3760 kW, Q2W=1360 kW, Q3W=700 kW, and Q4W=940 kW
Six stream matches were obtained corresponding to the stream matches with a binary variable having a value of one. They are given below with their corresponding heat loads.
H2-C2 (Q22=640kW), H3-C2 (Q32=2800kW), H4-C2 (Q42=3760kW), H2-CW (Q2W=1360kW), H3-CW (Q3W=700kW), H4-CW (Q4W=940kW)
As reported by Floudas et al (1986), the minimum number of heat exchanger units is the sum of the number of units in both sub-networks (7+6) i.e. thirteen (13) as shown in Table 8.

Table 8: Number of heat exchanger units above and below the pinch point

Above the pinch point (7)			**Below the pinch point (6)**		
Binary variable	Stream match	Heat Load (kW)	Binary variable	Stream match	Heat Load (kW)
YF1=1	HF-C1	QF1=2100	Y22=1	H2-C2	Q22=640
Y11=1	H1-C1	Q11=5600	Y32=1	H3-C2	Q32=2800
Y21=1	H2-C1	Q21=7300	Y42=1	H4-C2	Q42=3760
Y22=1	H2-C2	Q22=2300	Y2W=1	H2-CW	Q2W=1360
Y32=1	H3-C2	Q32=7000	Y3W=1	H3-CW	Q3W=700
YHP2=1	HP-C2	QHP2=1000	Y4W=1	H4-CW	Q4W=940
YLP2=1	LP-C2	QLP2=500	**Total = 13 units**		

Task 3: Synthesis of optimal heat exchanger network superstructure and cost

Since it is rather difficult to synthesise a network that corresponds to the solution of the MILP transshipment model i.e. minimum number of units and minimum cost, it is formulated as an optimisation problem [12]. The heat exchanger superstructure configuration was generated by formulating an NLP optimisation model from results obtained from the solution of the MILP transshipment model [12]. Each stream match as derived from the MILP transshipment model was assigned a heat exchanger unit in the physical network. The heat loads of the heat exchangers are same as those predicted by the MILP transshipment model. For the network above and below the pinch point, the objective function was the total annualised cost of the network. This was minimised subject to a set of constraints as shown in the following.

Min. $C=4000A_i^{0.6}$

Subject to:

- Mass balance around the stream splitters
- Mass and heat balance around the stream mixers
- Heat balance around the heat exchangers
- Temperature feasibility constraints
- Area definition constraints

All variables are non-negative.

The NLP was implemented in GAMS and the optimal solution obtained was used to generate the heat exchanger network superstructure.
Figures 5 and 6 show the optimised superstructure configuration for stream matches above and below the pinch point respectively.

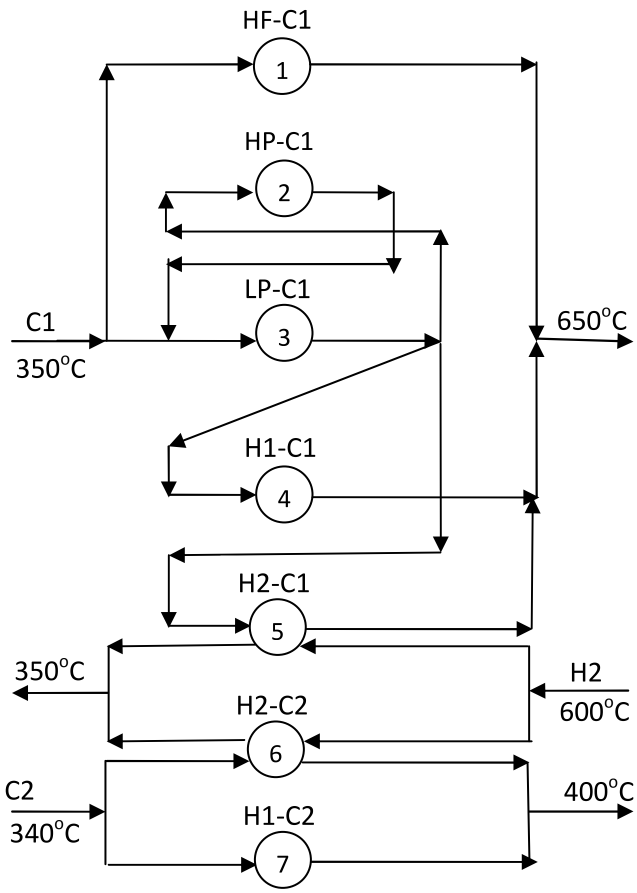

Figure 5: Heat exchanger superstructure for stream matches above the pinch point

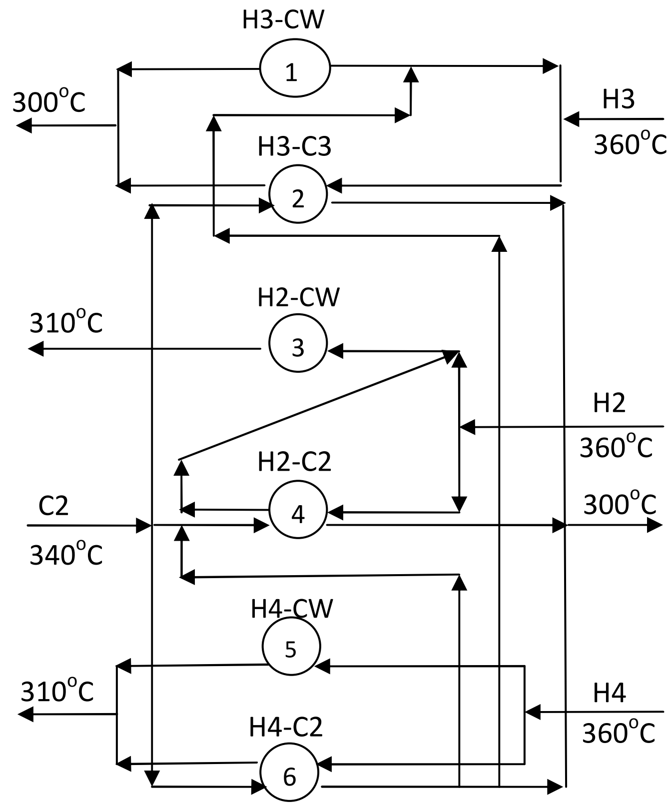

Figure 6: Heat exchanger superstructure for stream matches below the pinch point

Heat load of each heat exchanger is given as: $Q = UA\Delta T_m$ (29)

Where U, A, and ΔT_m are overall heat transfer coefficient, heat transfer area and logarithmic mean temperature difference (LMTD). The Chen approximation was used to evaluate the LMTD [14]. This was preferred to other approximations because it has the advantage that when the temperature approaches ΔT_1 and ΔT_2 equals zero, the LMTD is approximated to be zero unlike some other approximations that yield a nonzero value. The Chen approximation is given as:

$$\Delta T_m = \left[\Delta T_1 . \Delta T_2 \left(\frac{\Delta T_1 + \Delta T}{2} \right) \right]^{\frac{1}{3}} \quad (30)$$

Where
$$\Delta T_1 = T_{in}^H - T_{out}^C$$
$$\Delta T_2 = T_{out}^H - T_{in}^C \quad (31)$$

The optimum annualised cost of the heat exchanger network superstructure is the sum of the values in the last column of Table 9 and this was determined to be $536102 per annum.

Table 9: Summary of Network configuration

	T_{in}^{H} (K)	T_{out}^{H} (K)	T_{in}^{C} (K)	T_{out}^{C} (K)	ΔT_1 (K)	ΔT_2 (K)	ΔT_M (K)	Q (kW)	A (m^2)	$Cost$ **($/yr)**
ABOVE PINCH POINT										
H1-C1	700	420	350	680	20	70	39.79	5600	98.52	62829
H2-C1	600	417.5	350	571.01	28.99	67.5	45.53	7300	112.22	67938
H2-C2	418	360	340	384.54	32.96	20	25.94	2300	62.06	47618
H3-C2	460	360	340	396.45	63.55	20	37.59	7000	130.37	74330
SP-C1	750	750	608	650	100	142	119.77	2100	12.27	18006
HP-C2	510	510	394.44	400	110	115.56	112.78	1000	6.21	11963
LP-C2	410	410	340	384.54	25.46	70	43.98	500	7.96	13885
BELOW PINCH POINT										
H2-C2	360	320	300	340	20	20	20	640	22.40	25835
H2-CW	320	310	290	290	30	20	24.66	1360	38.60	35812
H3-C2	360	333.7	300	340	20	33.7	26.25	2800	74.65	53198
H3-CW	334	310	290	290	43.7	20	30.31	700	16.17	21245
H4-C2	360	320	300	340	20	20	20	3760	131.60	74750
H4-CW	320	310	290	290	30	20	24.66	940	26.68	28693

Conclusion

The synthesis of a heat exchanger network superstructure was carried out in this study. The minimum utility cost which was determined from the formulation of the LP transshipment model was $475200 per year while the minimum number of units which was determined from the formulation of the MILP transshipment model was 13 with 7 heat exchanger units above the pinch point and 6 heat exchanger units below the pinch point. The network configuration corresponding to minimum utility cost and minimum number of units was generated from an NLP formulation which was solved using GAMS to obtain the optimum heat exchanger network superstructure. The cost of the network configuration generated was estimated to be $536,102 per annum.

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